# Image Processing in Python

## 1 Introduction

During this exercise, the goal is to become familiar with Python and the NumPy library. You should also get a better feeling for how images are represented as matrices as well as the connection between mathematical expressions and the Python code to implement them.

### 1.1 Matlab vs Python

Unlike Matlab, Python is a general programming language used for many things, such as web servers and games aside from the more mathematical subjects covered in this course. In practice this means slightly more typing to specify what you want to do, it but also allows you to structure programs more easily than Matlab. For example, in Python the numerical operations are not included in the language, rather they are implemented as a library called NumPy that must be imported explicitly. This library is written in C/C++ and is highly optimized. Just like in Matlab one can of course perform explicit for-loops over data, but this will almost certainly take more time than figuring out how to correctly call an optimized function.

### 1.2 Preparations

Before the exercise you should have read trough this document. If you are not familiar with Python syntax in general, you should look try to look into this more throughly than can be covered in this guide.

On the lab computers at ISY, you will need to include the OpenCV module to use the opencv functions, and the course module to use the course-provided utility functions:

```
$ module load prog/opencv/3.4.3
$ module load courses/TSBB15
```

Start the standard Python interpreter by typing:

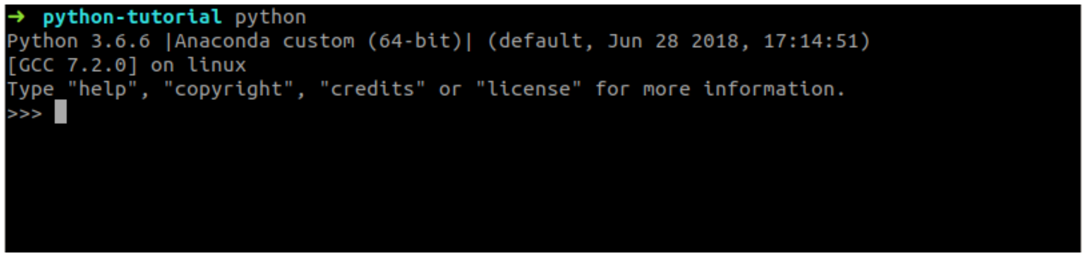
```
$ python3
```

or for the alternative Ipython interpreter:

```
$ ipython3
```

Note that the commands `python` and `ipython` will also work, but will instead start an interpreter for an older version of the Python language.

Starting the Ipython interpreter should print something like:



```
→ python-tutorial python
Python 3.6.6 |Anaconda custom (64-bit)| (default, Jun 28 2018, 17:14:51)
[GCC 7.2.0] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> 
```

Figure 1: The standard python interpreter

In general, Ipython is better for interactive use as it has some additional user-friendliness compared to the standard interpreter. Running scripts on the other hand is better done using the standard interpreter.

Python files are text files with the ending `.py`, and can be written in any text editor. To run such a file, type: `python3 thefile.py` in the terminal (not the Python interpreter)

## 2 Getting started with Python

The first thing we need to do after having started a Python interpreter is to import some libraries that we will use, write this in a terminal, or at the top of a .py file:

```
import numpy as np
import scipy
from matplotlib import pyplot as plt
```

The first line imports the NumPy library, and renames it to the more compact `np`. This is useful as we will often call functions from NumPy, and all such calls will be prefixed with `np`, for example, `np.power(x,2)` corresponds to: $x^2$ element wise, when $x$ is a NumPy array. The built-in Python function `pow` will also work, but `np.power` is more explicit, and therefore better.[1]

The second line imports SciPy while maintaining its original name, the final line imports the sub-module pyplot from the module Matplotlib and giving it the name `plt`, again to make it easy to call often.

To see what a module can do, you can call the built-in help function on the imported name, for example:

```
help(np)
```

This will open a text browser with a description of all the functions and objects in the `np` namespace. In practice it is often more convenient to look up the documentation on the web, for NumPy/SciPy it can be found at: `https://docs.scipy.org/doc/`. This documentation is generally up-to date and very well written. Exit the text browser by pressing `q`, this should bring you back to the Python prompt.

## 3 Simple array operations and creation and visualization

A NumPy array, more or less equivalent to a matrix in Matlab, is created by calling the constructor for the array object like:

```
row_vector = np.array([1,2,3], dtype='float32')
small_matrix = np.array([[1,2,3],[4,5,6],[7,8,9]], dtype='float64')
column_vector = np.array([[1,2,3]])
black_image = np.zeros((64,128,3))
```

This creates two vectors, one matrix and a 3-dimensional array containing zeros. In practice the one dimensional arrays can be treated as linear algebra vectors, the two dimensional array as a matrix. The higher dimensional arrays are a kind of generalization of a matrix to have more than two indices. Such objects are sometimes called Tensors, but this is not strictly correct mathematically and not really used in this course. Instead we will call a something a Tensor if it is a tensor, and an array if it is an "indexable lump of numbers".

Some simple operations that can be performed on the objects created above are:

```
# Scalar product:
scalar = row_vector @ column_vector
print("Scalar: {}".format(scalar))

# Outer product:
matrix = column_vector @ row_vector
print("Matrix:\n {}".format(matrix))

# matrix * vector product:
v1 = small_matrix @ column_vector
print("matrix*vector:\n {}".format(v1))
```

[1]`https://www.python.org/dev/peps/pep-0020/`

```
# vector * matrix product:
v2 = row_vector @ small_matrix
print("vector*matrix:\n {}".format(v2))

# Element wise multiplication
em = matrix * small_matrix
print("Element wise:\n {}".format(em))
```

Here the @ symbol denotes matrix multiplication, and the * symbol denotes element-wise multiplication.

As we are working with images it is sometimes useful to visualize them, this can be done using Matplotlib as:

```
plt.imshow(black_image)
plt.show()
```

Here the first line tells Matplotlib to enqueue an image to be displayed at some point in the future, and the second one blocks the interpreter to show the window with the image (i.e. flush the display queue). Since all the image values are zero the image will be black. Close the window to continue / unblock the interpreter.

Now we will make the image a bit more interesting to look at, by changing the pixel values. Changing a single pixel can be done by indexing that pixel like:

```
# Change the red channel of the top most pixel to one
black_image[0,0,0] = 1
# Change all color channels of the pixel beside it to one, setting it to white
black_image[0,1,:] = 1
# Change the bottom right square of the image to be blue:
black_image[-1:-10:-1,-1:-10:-1, 1] = 1
```

The first line above indexes a single value from the array representing our image, the second line indexes row 0 and column 1, but all color values and sets them to one. The third line indexes the pixels from the last (or the first one indexed backwards), to the middle one, with a step of -1, for the second channel (the one with the blue color).

**Now, display the image to make sure you did not make any mistakes.**
In general it is useful to check the result of intermediate steps of algorithms to make sure the values obtained are sane.

## 4 Fancy Python syntax and image reading

An important difference in Python compared to other languages is that code blocks are defined using whitespace and indentation, where C++ instead uses curly braces. This means that correct indentation is important, and not just something that will make others angry when you get wrong. If you have problems with this, set whatever text editor you are using to show whitespace characters. Try to stick with using 4 blank spaces as one indentation level.

Python syntax has some features that make it possible to, for example iterate over a list of objects:

```
things_list = ['a', 'b', 1, 2, 3, 4, 1.2, 1.3, 1.4]
numbers_list = [item for item in things_list if isinstance(item, (int, float))]
letters_list = [item for item in things_list if isinstance(item, str)]
print("=====")
for number in numbers_list:
    print(number)
print("======")
for idx, number in enumerate(letters_list):
    print("Letter number: {}, is {}".format(idx,letters_list))
```

The part in square brackets is a list-comprehension, when it is possible to express something as one it is typically easier to read, and runs much faster than a traditional for-loop. This can be verified easily when using ipython, by typing:

```
%timeit somefunction
```

Ipython has a few ”magic” commands that are prefixed by the % sign. When using the standard interpreter you need to write your own timing function. Since the timeit command only works for a single statement you will need to wrap the code to benchmark in a function.

```
def loop_function(N):
    """ Build a list using a loop """
    l = []
    for n in range(N):
        l.append(n)
    return n

def listcomprehension_function(N):
    """ Build a list using a list comprehension """
    return [n for n in range(0,N)]

%timeit loop_function(100)
%timeit listcomprehension_function(100)
```

Reading an image can be done using pillow, scikit-image or opencv.

```
image_filename = 'someimagefile'

# using pillow
from PIL import Image
im = np.array(image.open(image_filename))
print(im.shape)

# Using opencv
import cv2
im = cv2.imread(image_filename)
print(im.shape)

# using scikit-image
import skimage.io as skio
im = skio.imread(image_filename)
print(im.shape)

# using matplotlib
import matplotlib.image as mpimage
im = mpimage.imread(image_filename)
print(im.shape)

plt.imshow(im)
plt.show()
```

Note that depending on what you use to read the image, the image size will be different, as the libraries do not treat the alpha channel the same way. Also note that OpenCV by default permutes the colour channels to the order BGR, instead of the usual RGB.

## 5 Exercises:

### 5.1 Plotting

Begin by creating a sinus function:

```
sinus_curve = np.sin((np.pi*2) * np.linspace(0,1,10000))
plt.plot(sinus_curve)
plt.show()
```

The plot function can take a large number of different parameters. Look them up at: `http://matplotlib.org/api/pyplot_api.html`.

There is also a lot of example plots with the code to generate them at: `https://matplotlib.org`

Question: How would you make a plot with a magenta dash-dotted line with hexagrams at each data point?

### 5.2 Test-pattern generation

We start by generating two simple gradients (note that arange does not include its endpoint, thus the seemingly assymetric range):

```
(x,y) = np.meshgrid(np.arange(-128,129,1),np.arange(-128,129,1))

plt.figure(1)
plt.imshow(x)
plt.figure(2)
plt.imshow(y)
```

Now, use these arrays to generate an image of the function $r = \sqrt{x^2 + y^2}$.

Question: Write down the commands you use here:

Question: What is the function r?

We will now use this function to generate our first test image:

Generate the function $p1 = \cos(r/2)$ and display it.

Generate the Fourier transform of this pattern as variable, `P1`, using `np.fft.fft2`, and plot it.

Question: What does the Fourier transform look like? Explain why:

Question: What is the difference between `np.fft.fftshift` and `np.fft.ifftshift`

We will now create a test pattern with a slightly more interesting frequency content. Use the variable `r` to create the function $p_2 = \cos(r^2/200)$.

Plot the image `p2`, and its Fourier transform `P2`.

Question: The function $p_2 = \cos(r^2/200)$ appears to be rotationally symmetric, and its Fourier transform should then be rotational symmetric. Why then isn't the Fourier transform of `p2` rotationally symmetric?

To get a slightly better result, we now cut off the outermost periods of the cos() signal:

```
p2 = np.cos((r**2 / 200.0)) * ( (r**2 / 200.0) <  (22.5 * np.pi ) )
P2 = np.fft.fft2(p2)
#** is the exponentiation operation, so x**2 = x^2
```

You should now get a somewhat more correct representation of the transform `P2`, but note that the superimposed ringings are still there.

To generate a test image with a controlled frequency content is actually not that easy. The main problem is to create a test image which locally contains one frequency and has a nice behavior in the transition from the dc part to the low frequency part.

## 6 Filtering

We will now low-pass filter our test image. We start with filtering in the Fourier domain. Here the following variables will probably come handy:

```
u=x/256*2*np.pi
v=y/256*2*np.pi
```

Question: How do you generate a low-pass filter in the Fourier domain?

Assuming that the Fourier domain spans the range $[-\pi, \pi]$, use the method you describe above to generate an ideal LP-filter with the cut-off frequency at $\rho = \pi/4$ (Hint: A scaled $r$, and logical expressions are useful here.).

Use your filter on the transform `P2` of your test image `p2`.

Question: How is filtering in the Fourier domain computed?

Compute the inverse Fourier transform of the result from your filtering, and compare this with the original image `p2`.

Question: Since the image `p2` contains radially increasing frequency, one could expect that the result was attenuated outside a certain radius, and largely unaffected in the image centre. Why didn't this happen here?

We will now apply a box filter in the spatial domain. Instead of multiplication, the filtering now consists of a convolution:

```
#import convolution function
from scipy.signal import convolve2d as conv2

lp = np.ones((1,9),dtype='float32')/9.0
p2fs = conv2(lp, np.transpose(lp))
#Transpose can also be called like:
#lp.T
```

Question: This is not good either. Why does this filter behave differently in different parts of the image?

Now, try a Gaussian filter instead:

```
sigma = 3.0
lp = np.atleast_2d(np.exp(-0.5 * np.square(np.arange(-6,7,1)/sigma)))
lp = lp / np.sum(lp) #normalize the filter
```

Question: Why does this filter work better? (Hint: Look at the filter `lp'*lp`.)

## 7 Multi-dimensional arrays

We will now read a colour image from disk:

```
  image = mpimage.imread('some_image_file.png').astype('float32') / 255
```

The `astype('float32')` part converts the image to float, and the division with 255 makes sure the range is $[0, 1]$ rather than $[0, 255]$.

Check the size of the array containing the image

```
  print(image.shape)
```

```
plt.imshow(image)
plt.imshow(image[:,:,1])
```

Question: Which component is which?

Convert the image to grayscale using the formula $gray = 0.299R + 0.587G + 0.114B$, and plot it where the colour image now is. Compare the grey-scale image with the RGB components. (This is best done in a function)

## 7.1 Complex valued images

We will now combine the results of two Gaussian derivative filterings into one complex-valued field describing image orientation. Computing the derivative of a discrete signal is in general impossible, since it is the limit of a difference quotient, and we only have a sampled signal. It is however possible to compute a *regularised derivative*, i.e. the derivative convolved with a smoothing kernel:

$$\frac{\partial}{\partial x}(f * g(\sigma))(x) = (f * \frac{\partial}{\partial x}g(\sigma))(x) = (f * \frac{-x}{\sigma^2}g(\sigma))(x)$$

Where $g = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2}{2\sigma^2}}$. Thus, the regularised 1D derivative filter becomes:

```
df = np.atleast_2d(-1.0/np.square(sigma) * np.arange(-6,7,1) * lp)
```

Using `lp` and `df`, we can compute regularised partial derivatives along the $x$ and $y$ axes. Apply them on the test pattern `p2`. Now we store the results of filtering with these kernels as a complex field, and visualise the result using the `gopimage` command:

```
dx = conv2(conv2(paprika_red,df,mode='same'),lp.T,mode='same')
dy = conv2(conv2(paprika_red,lp,mode='same'),df.T,mode='same')
z = dx + 1i*dy

from cvllabs import gopimage

gopimage(z)
```

Each complex value in z can now be seen as a vector $(\mathrm{Re}(z)\ \ \mathrm{Im}(z))^T$ that approximates the image gradient.

Use the zoom tool to locate pixels of different colors, then extract the corresponding complex values from `z`. For instance you could try:

```
print(z[139,92])
```

Question: Describe what the colors mean:

Now, double the argument of the gradient image:

```
z2 = np.abs(z) * np.exp(1i * 2 * np.angle(z))
gopimage(z2)
```

Question: Compare the results, and explain why the double-angle representation is useful.

Have a look at the gradient magnitude, by computing the absolute value of the `z` variable.

Question: Why are there periodic variations in the magnitude?

## 7.2 2D tensor images

If we want to estimate the local orientation, the periodic variation in magnitude we saw above is not desirable. One way to remove the periodic variations is to average the double angle representation **z2**. Why is it not possible to average **z**? Another way to avoid this problem is to replace the complex valued image **z** with a tensor image **T**. Each position in the tensor image **T** contains the outer product of the vector $\mathbf{f} = \begin{pmatrix} f_x & f_y \end{pmatrix}^T$, i.e. $\mathbf{T}(\mathbf{x}) = \mathbf{f}(\mathbf{x})\mathbf{f}(\mathbf{x})^T$. If we perform averaging in the tensor representation, we can smear out the magnitude such that it is largely independent of the local signal phase.

$$\mathbf{T} = \begin{pmatrix} f_x \\ f_y \end{pmatrix} \begin{pmatrix} f_x & f_y \end{pmatrix} = \begin{pmatrix} f_x f_x & f_x f_y \\ f_y f_x & f_y f_y \end{pmatrix} \quad \text{and} \quad \mathbf{T}_{\mathrm{lp}}(\mathbf{x}) = (\mathbf{T} * g_x * g_y)(\mathbf{x})$$

Since **T** is symmetric, we only need to store three of the components in NumPy.

Now generate the tensor.

```
  T = zeros((256,256,3))
  T[:,:,0] = dx * dx
  T[:,:,1] = dx * dy
  T[:,:,2] = dy * dy
```

In order to be sure we implemented this correctly, it is advised to look at the produced tensor values. While calling imshow might work, it will not behave correctly since the tensor is not an RGB image. It is better to look at each channel separately as grayscale images.

Next we apply Gaussian smoothing with the appropriate standard deviation $\sigma$ on all the tensor components. Try to find a $\sigma$-value that gives just enough smoothing to remove the magnitude variation.

```
sigma = 3.0
lp = np.atleast_2d(np.exp(-0.5 * (np.arange(-10,11,1)/sigma)**2))
lp = lp / np.sum(lp) #normalize the filter

Tlp[:,:,0] = conv2(conv2(T[:,:,0], lp, mode='same'), lp.T, mode='same')
Tlp[:,:,1] = conv2(conv2(T[:,:,1], lp, mode='same'), lp.T, mode='same')
Tlp[:,:,2] = conv2(conv2(T[:,:,2], lp, mode='same'), lp.T, mode='same')
```

Question: Write down the required `sigma` value:

Question: The actual local orientation property is an angle modulo $\pi$. How is this angle represented in the tensor?

## 7.3 Images as vectors

This last exercise emphasizes the fact that we can treat an image as a vector instead of a multidimensional matrix. Treating an image as a vector has the advantage that we only need to keep track of one index instead of several. Load `mystery_vector.npy` to obtain an image stored as a vector. The exercise is to find the maximum value and the corresponding position of this value given in image coordinates. Finally reshape the vector to the original image. The original image consists of 320 columns and 240 rows.
**HINT:** One useful command is `unravel_index`.

Question: What is the position of the maximum value in image coordinates?