Image enhancement

Computer Vision, Lecture 14
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Why image enhancement?

- Example of artifacts caused by image encoding

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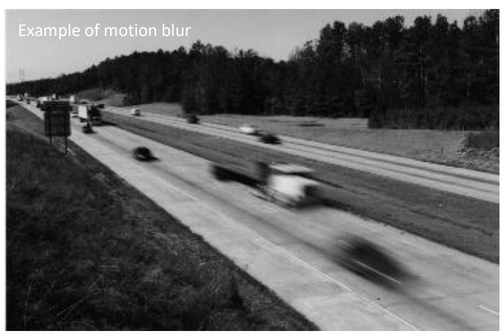
Why image enhancement?


Example of motion blur

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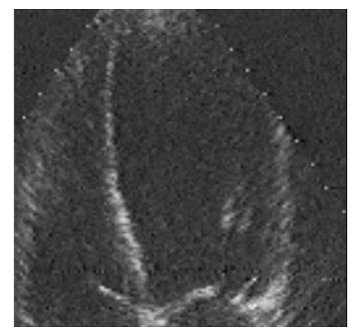
Why image enhancement?

- Example of an image with sensor noise
  - ultrasound image of a beating heart



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Why image enhancement?

- IR-image
  - fixed pattern noise = spatial variations in gain and offset
  - Possibly even variations over time!
  - Hot/dead pixels
- A digital camera with short exposure time
  - Shot noise (photon noise)

Methods for image enhancement

- Inverse filtering: the distortion process is modeled and estimated (e.g. motion blur) and the *inverse* process is applied to the image
- Image restoration: an *objective* quality (e.g. sharpness) is estimated in the image. The image is modified to increase the quality
- Image enhancement: modify the image to improve the visual quality, often with a subjective criteria

Additive noise

- Some types of image distortion can be described as
  - Noise added on each pixel intensity
  - The noise has the identical distribution and is independent at each pixel (i.i.d.)
- Not all type of image distortion are of this type:
  - Multiplicative noise
  - Data dependent noise
  - Position dependent
- The methods discussed here assume additive i.i.d.-noise

What about pixel shot noise?

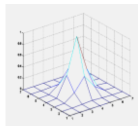
Removing additive noise

- Image noise typically contains higher frequencies than images generally do

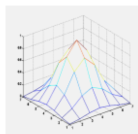
  $\Rightarrow$ a low-pass filter can reduce the noise
- BUT: we also remove high-frequency signal components, e.g. at edges and lines
- HOWEVER: A low-pass filter works in regions without edges and lines (ergodicity)

Example: LP filter

Image with some noise



Filter, $\sigma = 1$



Filter, $\sigma = 2$

Basic idea

The problem of low-pass filters is that we apply the same filter on the whole image

We need a filter that locally adapts to the image structures

A space-variant filter

Ordinary filtering / convolution

- Ordinary filtering can be described as a convolution of the signal f and the filter g:

$$h(\mathbf{x}) = (f * g)(\mathbf{x}) = \int f(\mathbf{x} - \mathbf{y}) g(\mathbf{y}) \, d\mathbf{y}$$

For each **x**, we compute the integral between the filter g and a shifted signal f

Adaptive filtering

- If we apply an adaptive (or position dependent, or space-variant) filter $g_{\mathbf{x}}$, the operation cannot be expressed as a convolution, but instead as

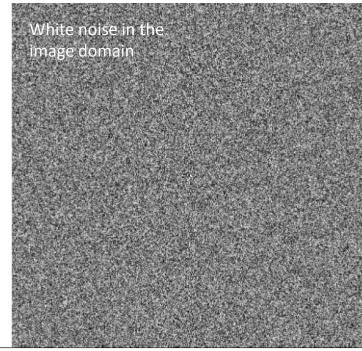
$$h(\mathbf{x}) = \int f(\mathbf{x} - \mathbf{y}) g_{\mathbf{x}}(\mathbf{y}) \, d\mathbf{y}$$

For each **x**, we compute the integral between a shifted signal f and the filter $g_{\mathbf{x}}$ where the filter depends on **x**

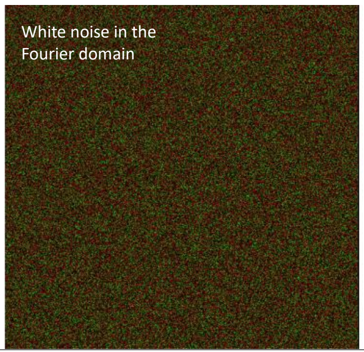
Orientation-selective g_x

- If the signal is ≈ i1D the filter can maintain the signal by reducing the frequency components orthogonal to the local structure
- The human visual system is less sensitive to noise along linear structures than to noise in the orthogonal direction
- Results in good subjective improvement of image quality

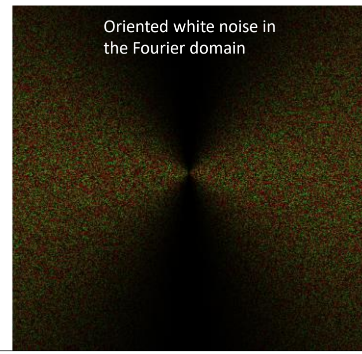
Oriented noise



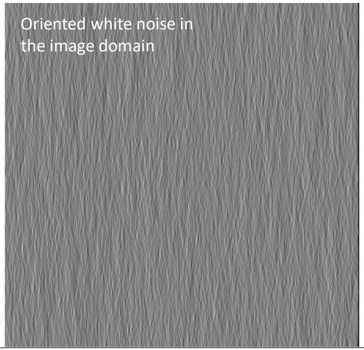
Oriented noise



Oriented noise



Oriented noise



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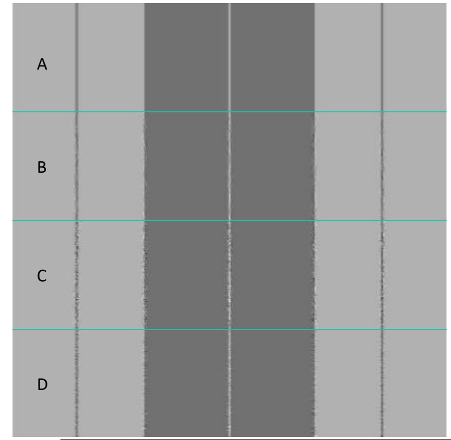

Oriented noise

Edges and lines

A. Without noise

B. With oriented noise along

C. With isotropic noise

D. With oriented noise across

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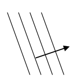
Local structure information

- We compute the local orientation tensor $\mathbf{T}(\mathbf{x})$ at all points $\mathbf{x}$ to control / steer $g_{\mathbf{x}}$
- At a point $\mathbf{x}$ that lies in a locally i1D region, we obtain

$$\mathbf{T}(\mathbf{x}) = A\hat{\mathbf{e}}\hat{\mathbf{e}}^T$$



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ê is normal to the linear structure

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Scale space recap (from lecture 2)

- The linear Gaussian scale space related to the image f is a family of images $L(x,y;s)$

$$L(x, y; s) = (g_s * f)(x, y)$$

Convolution over (x,y) only!

parameterized by the scale parameter s, where

$$g_s(x, y) = \frac{1}{2\pi s} e^{-\frac{x^2+y^2}{2s}}$$

A Gaussian LP-filter with $\sigma^2 = s$

Note: $g_s(x,y) = \delta(x,y)$ for $s = 0$

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Scale space recap (from lecture 2)

- $L(x,y;s)$ can also be seen as the solution to the PDE

$$\frac{\partial}{\partial s}L = \frac{1}{2}\nabla^2 L$$

$$\frac{\partial}{\partial s}L = \frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) L$$

with boundary condition $L(x,y;0) = f(x,y)$

The diffusion equation

Example:

L = temperature

s = time

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Repetition: Vector Analysis

- Nabla operator $\nabla = \begin{bmatrix}\partial_x \\ \partial_y\end{bmatrix} = \begin{bmatrix}\frac{\partial}{\partial x} \\ \frac{\partial}{\partial y}\end{bmatrix}$
- On a scalar function $\nabla f = \mathrm{grad} f = \begin{bmatrix}\partial_x f \\ \partial_y f\end{bmatrix} = \begin{bmatrix}f_x \\ f_y\end{bmatrix}$
- On a vector field $\langle \nabla | \mathbf{f} \rangle = \nabla^T \mathbf{f} = \mathrm{div}\, \mathbf{f} = \partial_x f_1 + \partial_y f_2$
- Laplace operator $\Delta = \nabla^2 = \langle \nabla | \nabla \rangle = \mathrm{div}\,\mathrm{grad} = \partial_x^2 + \partial_y^2$
  note: $\partial_x^2 f = f_{xx} \neq f_x^2$

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Enhancement based on linear diffusion

- This means that $L(x,y;s)$ is an LP-filtered version of $f(x,y)$ for $s > 0$.
- The larger s is, the more LP-filtered is f
  - High-frequency noise will be removed for larger s
- Also high-frequency image components (e.g. edges) will be removed
- We need to control the diffusion process such that edges remain - How?

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Step 1

- Modify the PDE by introducing a parameter μ:

$$\frac{\partial}{\partial s}L = \frac{\mu}{2}\nabla^2 L$$

- This PDE is solved by

$$L(x,y;s) = (g_s * f)(x,y)$$

$$g_s(x,y) = \frac{1}{2\pi\mu s}e^{-\frac{x^2+y^2}{2\mu s}}$$

μ can be seen as a "diffusion speed":

Small μ: the diffusion process is slow when *s* increases

Large μ: the diffusion process is fast when *s* increases

Same as before

Slightly different

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Step 2

- We want the image content to control μ
  - In flat regions: fast diffusion (large μ)
  - In non-flat region: slow diffusion (small μ)
- We need to do *space-variant* diffusion
  - μ is a function of position (x,y)

We will introduce another space-variant filter g_x in adaptive filtering

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Inhomogeneous diffusion

- Perona & Malik suggested to use

$$\mu(x,y) = \frac{1}{1 + |\nabla f|^2/\lambda^2}$$

where ∇f is the image gradient at (x,y)
and λ is fixed a parameter

- Close to edges: $|\nabla f|$ is large $\Rightarrow \mu$ is small
- In flat regions: $|\nabla f|$ is small $\Rightarrow \mu$ is large

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Inhomogeneous diffusion

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Inhomogeneous diffusion

- Noise is effectively removed in flat region
- Edges are preserved
- Noise is preserved close to edges

We want to be able to LP-filter along but not across edges, same as for adaptive filtering

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Step 3

- The previous PDEs are all isotropic
  $\Rightarrow$ The resulting filter g is isotropic
- The last PDE can be written:

$$\frac{\partial}{\partial s} L = \frac{\mu}{2} \nabla^2 L = \frac{1}{2} \text{div}(\mu \, \text{grad}\, L)$$

Divergence of (...) maps 2D vector field to scalar field

Gradient of *L*, a 2D vector field

Step 3

- Change μ from a scalar to a 2×2 symmetric matrix $\mathbf{D}$

$$\frac{\partial}{\partial s} L = \frac{1}{2} \text{div}(\mathbf{D} \, \text{grad}\, L)$$

- The solution is now given by

$$L(\mathbf{x}; s) = (g_s * f)(\mathbf{x})$$

$\Leftarrow$ Same as before

$$g_s(\mathbf{x}) = \frac{1}{2\pi \det(\mathbf{D})^{1/2} s} e^{-\frac{1}{2s} \mathbf{x}^T \mathbf{D}^{-1} \mathbf{x}}$$

Ansiotropic diffusion

- The filter g is now anisotropic, i.e., not necessary circular symmetric
- The shape of g depends on $\mathbf{D}$
- $\mathbf{D}$ is called a *diffusion tensor*
  - Can be given a physical interpretation, e.g. for anisotropic heat diffusion

The diffusion tensor

- Since $\mathbf{D}$ is symmetric 2×2:

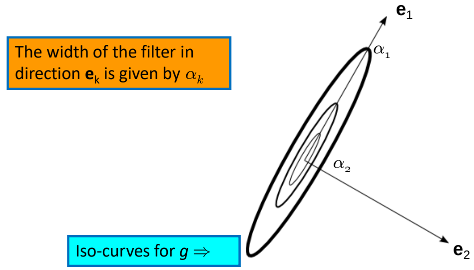
$$\mathbf{D} = \alpha_1 \mathbf{e}_1 \mathbf{e}_1^T + \alpha_2 \mathbf{e}_2 \mathbf{e}_2^T$$

where α_1, α_2 are the eigenvalues of $\mathbf{D}$, and $\mathbf{e}_1$ and $\mathbf{e}_2$ are corresponding eigenvectors

$\mathbf{e}_1$ and $\mathbf{e}_2$ form an ON-basis

The filter *g*

- The corresponding shape of g is given by



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Step 4

- We want g to be narrow across edges and wide along edges
- This means: **D** should depend on (x,y)
  - A space variant anisotropic diffusion
- This is referred to as *anisotropic diffusion* in the literature
- Introduced by Weickert

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Anisotropic diffusion

- Information about edges and their orientation can be provided by an orientation tensor, e.g., the structure tensor **T** in terms of its eigenvalues λ_1, λ_2
- However:
  - We want α_k to be close to 0 when λ_k is large
  - We want α_k to be close to 1 when λ_k is close to 0

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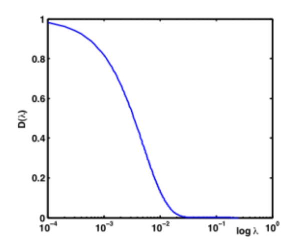
From **T** to **D**

- The diffusion tensor **D** is obtained from the orientation tensor **T** by modifying the eigenvalues and keeping the eigenvectors, e.g.

$$\alpha_k = \exp(-\lambda_k / m)$$

For example

m is a control parameter



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Anisotropic diffusion: summary

1. At all points:
   1. compute a local orientation tensor $\mathbf{T}(\mathbf{x})$
   2. compute $\mathbf{D}(\mathbf{x})$ from $\mathbf{T}(\mathbf{x})$
2. Apply anisotropic diffusion onto the image by locally iterating

Left hand side: the change in *L* at (*x,y*) between *s* and *s*+∂*s*

$$\frac{\partial}{\partial s}L = \frac{1}{2}\mathrm{div}(\mathbf{D}\,\mathrm{grad}\,L)$$

Right hand side: can be computed locally at each point (x,y)

This defines how scale space level *L*(*x,y;s*+∂*s*) is generated from *L*(*x,y;s*)

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Implementation aspects

- The anisotropic diffusion iterations can be done with a constant diffusion tensor field $\mathbf{D}(\mathbf{x})$, computed once from the original image (faster)
- Alternatively: re-compute $\mathbf{D}(\mathbf{x})$ between every iteration (slower)

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Simplification

- We assume **D** to have a slow variation with respect to **x** (cf. adaptive filtering)
- This means (see [EDUPACK – ORIENTATION (22)])

$$\frac{\partial}{\partial s}L = \frac{1}{2}\nabla^T\mathbf{D}\nabla L \approx \frac{1}{2}\langle\mathbf{D}|\nabla\nabla^T\rangle L = \frac{1}{2}\mathrm{tr}[\mathbf{D}\,(\mathbf{H}L)]$$

The Hessian of *L* = second order derivatives of *L*

$$\mathbf{H}L = \begin{pmatrix} \frac{\partial^2}{\partial x^2}L & \frac{\partial^2}{\partial x \partial y}L \\ \frac{\partial^2}{\partial x \partial y}L & \frac{\partial^2}{\partial y^2}L \end{pmatrix}$$

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Numerical implementation

- Several numerical schemes for implementing anisotropic diffusion exist
- Simplest one:
  - Replace all partial derivatives with finite differences

$$L(x,y;s+\Delta s) = L(x,y;s) + \frac{\Delta s}{2}\mathrm{tr}[\mathbf{D}\,(\mathbf{H}L)]$$

The Hessian of *L* can be approximated by convolving *L* with:

$$H_{11} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad H_{12} = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \quad H_{22} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

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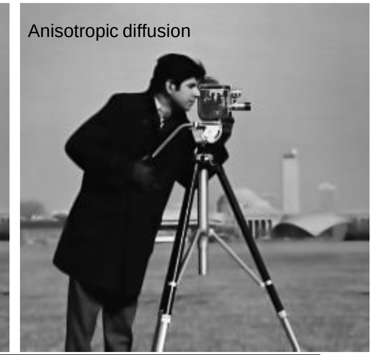
Algorithm Outline

1. Set parameters
   e.g.: k, Δs, number of iterations, ...
2. Iterate
   1. Compute orientation tensor $\mathbf{T}$
   2. Modify eigenvalues $\Rightarrow$ $\mathbf{D}$
   3. Computer Hessian $\mathbf{H}\,L$
   4. Update L according to:

$$L(x, y; s + \Delta s) = L(x, y; s) + \frac{\Delta s}{2}\mathrm{tr}[\mathbf{D}\,(\mathbf{H}L)]$$

Comparison



A note

- The image f is never convolved by the space-variant anisotropic filter g
- Instead, the effect of g is generated incrementally based on the diffusion eq.
- In adaptive filtering: we never convolve f with $g_{\mathbf{x}}$ either, instead several fixed filters are applied onto f and their results are combined in a non-linear way

How to choose $g_{\mathbf{x}}$?

According to the discussion in the introduction, we choose $g_{\mathbf{x}}$ such that:

- It contains a low-pass component that maintains the local image mean intensity **Independent of x**
- It contains a high-pass component that depends on the local signal structure **Dependent of x**
- Also: the resulting operation for computing h should be simple to implement **Computational efficient**

Ansatz for $g_\mathbf{x}$

We apply a filter that is given in the Fourier domain as

$$G_{\text{HP}}(\mathbf{u}) = G_\rho(u)(\hat{\mathbf{u}}^T\hat{\mathbf{e}})^2 \qquad \mathbf{u} = u\hat{\mathbf{u}}$$

- G_{HP} is *polar separable*
- It attenuates frequency components that are $\perp$ to $\hat{\mathbf{e}}$
- It maintains all frequency components that are || to $\hat{\mathbf{e}}$

How to implement $g_\mathbf{x}$?

- We know that [EDUPACK – ORIENTATION (20)]

$$(\hat{\mathbf{u}}^T\hat{\mathbf{e}})^2 = \langle \hat{\mathbf{u}}\hat{\mathbf{u}}^T | \hat{\mathbf{e}}\hat{\mathbf{e}}^T \rangle = \langle \hat{\mathbf{u}}\hat{\mathbf{u}}^T | \mathbf{T}(\mathbf{x}) \rangle$$

where $\mathbf{T}(\mathbf{x}) = \hat{\mathbf{e}}\hat{\mathbf{e}}^T$ (assume A = 1!)

- Using a N-D tensor basis $\hat{\mathbf{N}}_k = \hat{\mathbf{n}}_k\hat{\mathbf{n}}_k^T$ and its dual $\tilde{\mathbf{N}}_k$, we obtain:

$$\mathbf{T}(\mathbf{x}) = \sum_{k=1}^N \langle \mathbf{T}(\mathbf{x}) | \tilde{\mathbf{N}}_k \rangle \hat{\mathbf{N}}_k$$

How to implement $g_\mathbf{x}$?

$$\begin{aligned}
(\hat{\mathbf{u}}^T\hat{\mathbf{e}})^2 &= \sum_{k=1}^N \langle \hat{\mathbf{u}}\hat{\mathbf{u}}^T | \hat{\mathbf{N}}_k \rangle \langle \mathbf{T}(\mathbf{x}) | \tilde{\mathbf{N}}_k \rangle \\
&= \sum_{k=1}^N \langle \hat{\mathbf{u}}\hat{\mathbf{u}}^T | \hat{\mathbf{n}}_k\hat{\mathbf{n}}_k^T \rangle \langle \mathbf{T}(\mathbf{x}) | \tilde{\mathbf{N}}_k \rangle \\
&= \sum_{k=1}^N (\hat{\mathbf{u}}^T\hat{\mathbf{n}}_k)^2 \langle \mathbf{T}(\mathbf{x}) | \tilde{\mathbf{N}}_k \rangle
\end{aligned}$$

depends on **u** but not on **ê**

depends on **ê** but not on **u**

How to implement $g_\mathbf{x}$?

- Plug this into the expression for G_{HP} :

$$G_{\text{HP}}(\mathbf{u}) = \sum_{k=1}^N \langle \mathbf{T}(\mathbf{x}) | \tilde{\mathbf{N}}_k \rangle G_\rho(u)(\hat{\mathbf{u}}^T\hat{\mathbf{n}}_k)^2$$

depends on **ê** but not on **u**

depends on **u** but not on **ê**

How to implement $g_{\mathbf{x}}$?

Consequently, the filter G_{HP} is a linear combination of N filters, where each filter has a Fourier transform:

$$G_{\mathrm{HP},k}(\mathbf{u}) = G_\rho(u)(\hat{\mathbf{u}}^T \hat{\mathbf{n}}_k)^2$$

Independent of **x**

and N scalars:

$$\langle \mathbf{T}(\mathbf{x}) | \tilde{\mathbf{N}}_k \rangle$$

Dependent of **x**

How to implement $g_{\mathbf{x}}$?

Summarizing, the adaptive filter can be written as

$$g_{\mathbf{x}} = g_{\mathrm{LP}} + \sum_{k=1}^{N} \langle \mathbf{T}(\mathbf{x}) | \tilde{\mathbf{N}}_k \rangle g_{\mathrm{HP},k}$$

A fixed LP-filter

N fixed HP-filters

N position dependent scalars

How to implement $g_{\mathbf{x}}$?

If the filter is applied to a signal, we obtain

$$\begin{aligned} h(\mathbf{x}) &= \int f(\mathbf{x}-\mathbf{y}) \left[g_{\mathrm{LP}}(\mathbf{y}) + \sum_{k=1}^{N} \langle \mathbf{T}(\mathbf{x}) | \tilde{\mathbf{N}}_k \rangle g_{\mathrm{HP},k}(\mathbf{y}) \right] d\mathbf{y} \\ &= (f * g_{\mathrm{LP}})(\mathbf{x}) + \sum_{k=1}^{N} \langle \mathbf{T}(\mathbf{x}) | \tilde{\mathbf{N}}_k \rangle (f * g_{\mathrm{HP},k})(\mathbf{x}) \end{aligned}$$

Standard convolution

Standard convolutions

Position dependent scalars

Outline Adaptive Filtering v.1

1. Estimate the orientation tensor $\mathbf{T}(\mathbf{x})$ at each point $\mathbf{x}$
2. Apply a number of fixed filters to the image: one LP-filter g_{LP} and the N HP-filters $g_{\mathrm{HP},k}$
3. At each point $\mathbf{x}$:
   1. Compute the N scalars $\langle \mathbf{T}(\mathbf{x}) | \tilde{\mathbf{N}}_k \rangle$
   2. Form the linear combination of the N HP-filter responses and the N scalars and add the LP-filter response
4. At each point $\mathbf{x}$, the result is the filter response $h(\mathbf{x})$ of the locally adapted filter $g_{\mathbf{x}}$

The filter $g_{\mathbf{x}}$ is also called a *steerable filter*

Observation

- $\mathbf{T}$ can be estimated for any image dimension
- The filters g_{LP} and $g_{\text{HP},k}$ can be formulated for any image dimension

$\Rightarrow$ The method can be implemented for any dimension of the signal (2D, 3D, 4D, ...)

Remaining questions

1. What happens in regions that are not i1D, i.e., if $\mathbf{T}$ has not rank 1?
2. What happens if $A \neq 1$?
3. How to choose the radial function G_ρ?

Non-i1D signals

- The tensor's eigenvectors with non-zero eigenvalues span the subspace of the Fourier domain that contains the signal's energy
- Equivalent: For a given local region with orientation tensor $\mathbf{T}$, let $\hat{\mathbf{u}}$ define an arbitrary orientation. The product $\hat{\mathbf{u}}^{\mathrm{T}}\mathbf{T}\,\hat{\mathbf{u}}$ is a measure of how much energy in this orientation the region contains.

Non-i1D signals

- But

$$\hat{\mathbf{u}}^T\mathbf{T}\hat{\mathbf{u}} = \langle \hat{\mathbf{u}}\hat{\mathbf{u}}^T | \mathbf{T} \rangle$$

which means that the adaptive filtering should work in general, even if the signal is non-i1D

How about A = 1?

- Previously we assumed $A = 1$, but normally A depends on the local amplitude of the signal (depends on $\mathbf{x}$)
- In order to achieve $A = 1$, $\mathbf{T}$ must be pre-processed
- The resulting tensor is called the control tensor $\mathbf{C}$
- Replace $\mathbf{T}$ with $\mathbf{C}$ in all previous expressions!

Pre-processing of T

- The filter $g_{\mathbf{x}}$ is supposed to vary slowly with $\mathbf{x}$, but $\mathbf{T}$ contains high-frequency noise that comes from the image noise
- This noise can be reduced by an initial LP-filtering of $\mathbf{T}$ (i.e., of its elements)
- The result is denoted $\mathbf{T}_{\mathrm{LP}}$

Pre-processing of T

$\mathbf{T}_{\mathrm{LP}}$ must be *normalized* (similar to the diffusion tensor $\mathbf{D}$):

Eigen-decomposition of $\mathbf{T}_{\mathrm{LP}}$

$$\mathbf{T}_{\mathrm{LP}} = \sum_{k=1}^{n} \lambda_k \hat{\mathbf{e}}_k \hat{\mathbf{e}}_k^T \qquad \lambda_k \geq \lambda_{k+1}$$

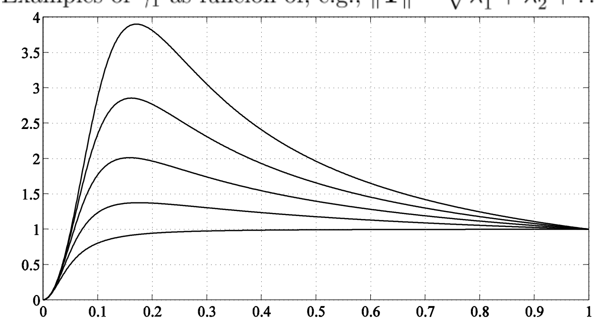
$$\mathbf{C} = \sum_{k=1}^{n} \gamma_k \hat{\mathbf{e}}_k \hat{\mathbf{e}}_k^T \qquad \gamma_k \geq \gamma_{k+1}$$

$$\gamma_k = \gamma_k(\lambda_1, \ldots, \lambda_n)$$

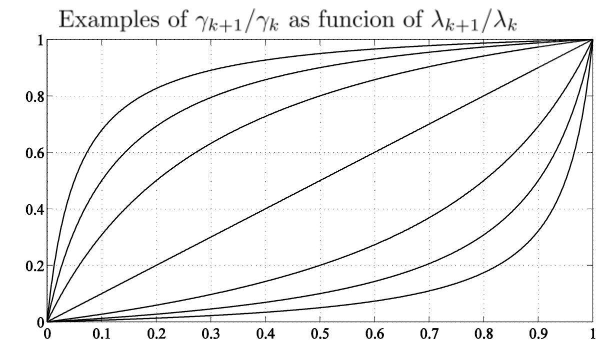
Same eigenvectors as $\mathbf{T}_{\mathrm{LP}}$, but different eigenvalues

Modification of the eigenvalues

Examples of γ_1 as funcion of, e.g., $\|\mathbf{T}\| = \sqrt{\lambda_1^2 + \lambda_2^2 + \ldots}$

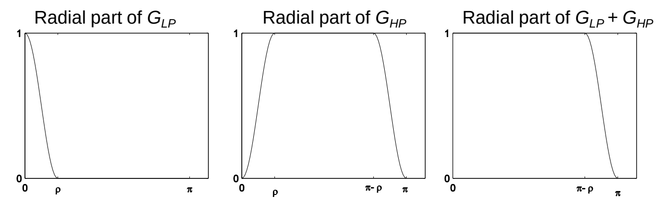


Modification of the eigenvalues

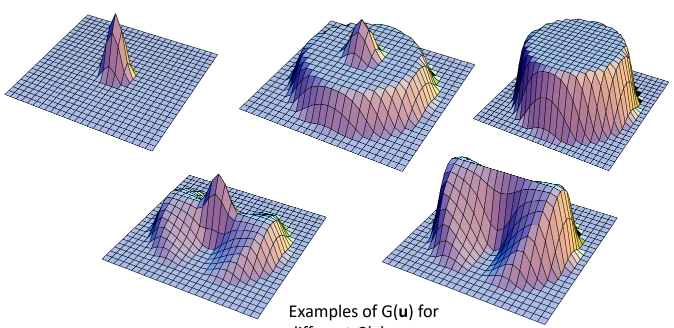


The radial function G_ρ

- Should "mainly" be equal to 1
- Should tend to 0 for u = π
- Together with the LP-filter g_{LP}: an all-pass filter



The adaptive filter in 2D



Examples of G(**u**) for different **C**(**x**)

Outline Adaptive Filtering v.2

1. Estimate the local tensor in each image point: $\mathbf{T}(\mathbf{x})$
2. LP-filter the tensor: $\mathbf{T}_{LP}(\mathbf{x})$
3. In each image point:
   1. Compute the eigenvalues and eigenvectors of $\mathbf{T}_{LP}(\mathbf{x})$.
   2. Map the eigenvalues λ_k to γ_k.
   3. Re-combine γ_k and the eigenvectors to form the control tensor $\mathbf{C}$
   4. Compute the scalars $\langle \mathbf{C} | \tilde{\mathbf{N}}_k \rangle$ for all $k = 1,\ldots, N$
4. Filter the image with g_{LP} and the N HP-filters $g_{HP,k}$
5. In each image point: form the linear combination of the filter responses and the scalars

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Example

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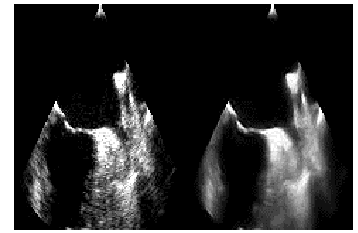
Example



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An iterative method

- Adaptive filtering can be iterated for reducing the noise
- If the filter size is reduced at the same time, a close-to continuous transition is achieved (evolution)
- This is closely related to the previous method for image enhancement: *anisotropic diffusion*

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