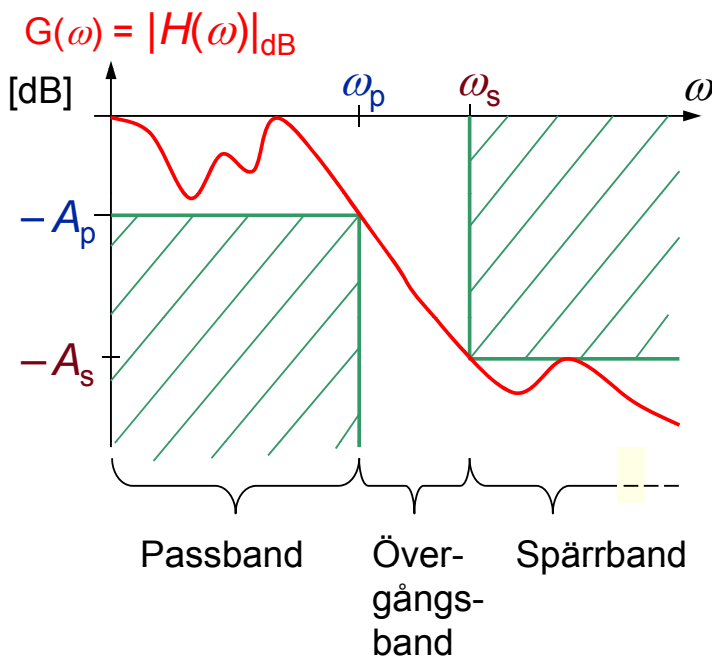


Frekvensselektiva Passiva Filter

Exempel, dämpningskrav för amplitudnormerat LP-filter:



$$|H(\omega)_{\text{max}}| = 1 \Rightarrow 0 \text{ dB}$$

$$|H(\omega)|_{\text{dB}} = 20 \cdot 10 \log |H(\omega)| \text{ dB}$$

A_p : Största passbandsdämpningen

ω_p : Passbandsgränsen (gränsvinkelfrekvensen)

A_s : Minsta spärrbandsdämpningen

ω_s : Spärrbandsgränsen

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Syntes av praktiska filter (LP, HP, BP & BS)

1. Överför frekvenser och dämpningskrav från önskat filter

$H_{\text{önskat}}(s)$ till ett motsvarande normerat LP-filter $H_{\text{norm}}(\omega)$

($|H_{\text{norm}}(\omega)|_{\text{max}} = 1$ ($\Leftrightarrow 0 \text{ dB}$) Ofta är $\omega_p (= \omega_c) = 1 \text{ rad/s}$)

Standardfilter har vanligen

$$|H_{\text{norm}}(\omega)| = \frac{1}{\sqrt{1 + L_n^2(\omega)}}$$

2. LP-filtret: A_p, A_s, ω_p & $\omega_s \Rightarrow n_{\text{min}} \Rightarrow |H_{\text{norm}}(\omega)|$

3. Tabell (filter med gränsv.frekv. = 1 rad/s) $\Rightarrow H_{\text{norm}}(s)$ (gränsv.frekv. = ω_p)

4. Filtertransformera: $H_{\text{norm}}(s) \rightarrow H_{\text{önskat}}(s)$

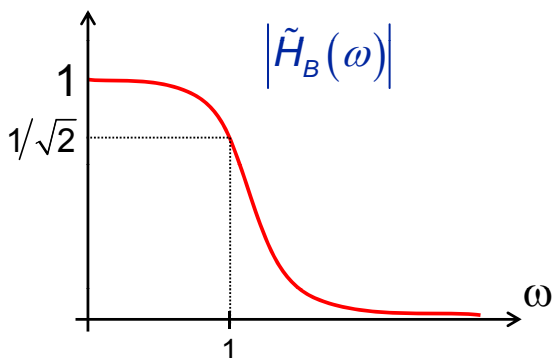
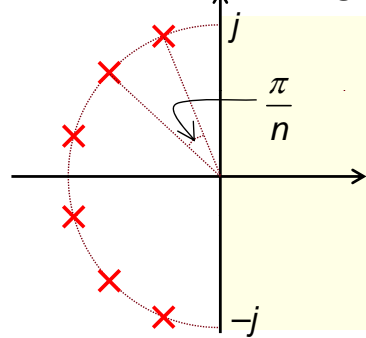
($H_{\text{önskat}}(s)$ är t.ex LP-, HP-, BP- eller BS-filter)

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Butterworthfilter, 3 dB-gränsv.frekv. $\omega_{3dB} = 1$ rad/s

Amplitudkaraktäristik

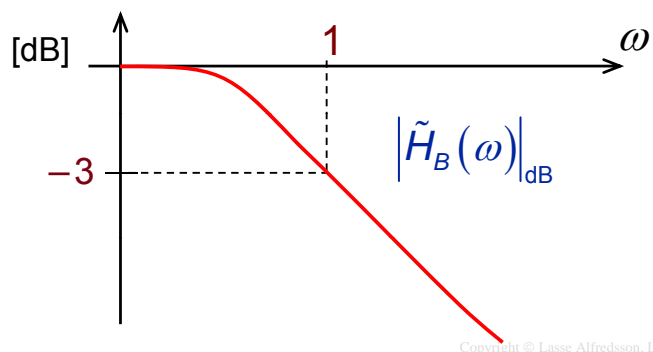
$$|\tilde{H}_B(\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}$$

Pol-nollställediagram för $\tilde{H}_B(s)$ 

Poler på halvcirkel
i vänster halvplan,
på radien 1:

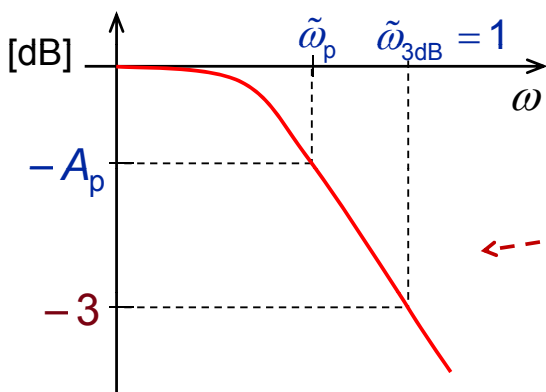
$$p_i = e^{j\left(1+\frac{m}{n}\right)\frac{\pi}{2}}$$

$$m = 1, 3, 5, \dots, 2n-1$$



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Butterworthfilter, A_p dB-gränsvinkelfrekvens $\tilde{\omega}_p$



Om strängare dämpningskrav,
max A_p dB dämpning i passbandet:

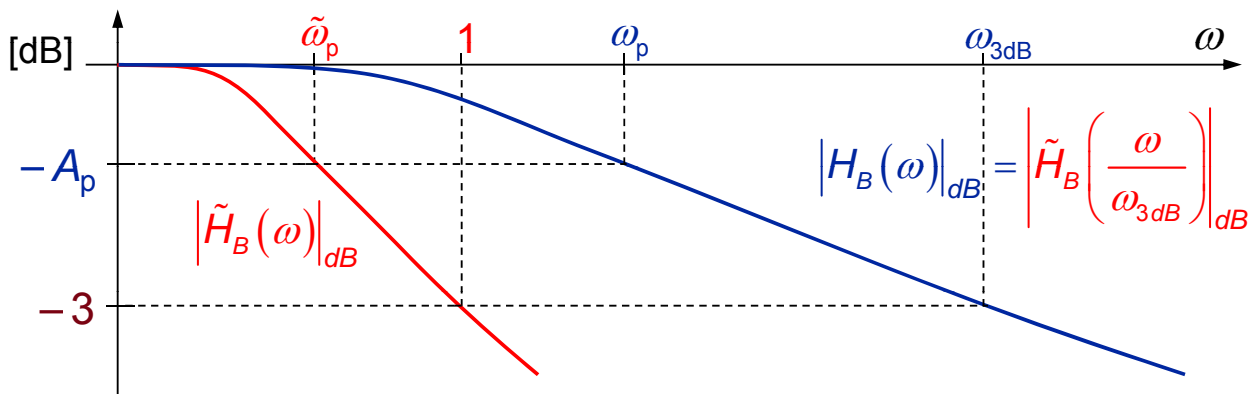
$$\begin{aligned} |\tilde{H}_B(\omega)|_{dB} &= 20 \cdot 10 \log |\tilde{H}_B(\omega)| \\ &= -10 \cdot 10 \log(1 + \omega^{2n}) \end{aligned}$$

$$\left. \begin{aligned} \underline{A_p} &= -|\tilde{H}_B(\tilde{\omega}_p)|_{dB} = 10 \cdot 10 \log(1 + \tilde{\omega}_p^{2n}) \\ / = \text{konstant, oberoende av } n / &= 10 \cdot 10 \log(1 + \varepsilon^2) \end{aligned} \right\}$$

$$\Rightarrow \boxed{\tilde{\omega}_p = \varepsilon^{1/n}} \quad \text{där} \quad \boxed{\varepsilon = \sqrt{10^{0.1A_p} - 1}}$$

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Butterworthfilter, godtycklig gränsvinkelfrekvens ω_p



dvs. $\omega_p = \tilde{\omega}_p \cdot \omega_{3dB}$ med $\tilde{\omega}_p = \varepsilon^{1/n} \Rightarrow \omega_p = \omega_{3dB} \cdot \varepsilon^{1/n}$

$$\Rightarrow |H_B(\omega)| = \tilde{H}_B\left(\frac{\omega}{\omega_{3dB}}\right) = \tilde{H}_B\left(\varepsilon^{1/n} \cdot \frac{\omega}{\omega_p}\right)$$

\Rightarrow (se nästa sida...)

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Butterworthfilter – sammanfattning

$$|H_{\text{norm}}(\omega)| = |H_B(\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2n}}}$$

$$\left(L_n(\omega) = \varepsilon \cdot \left(\frac{\omega}{\omega_p}\right)^n \right)$$

$$\varepsilon = \sqrt{10^{0.1A_p} - 1}$$

där $|H_B(\omega)|$ erhålls från

$$(\varepsilon = 1 \Leftrightarrow A_p \approx 3 \text{ dB})$$

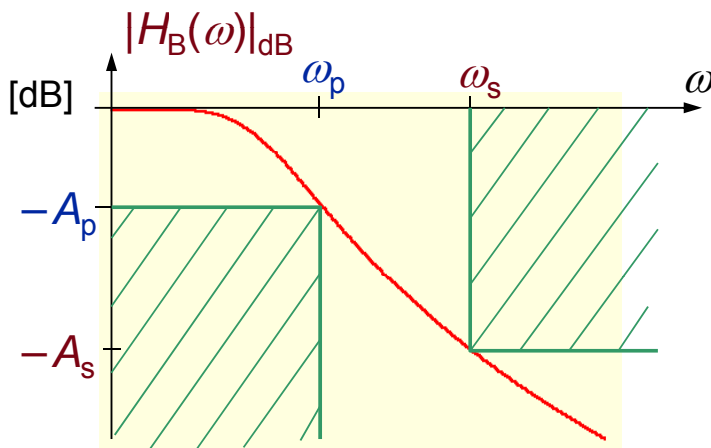
$$H_B(s) = \frac{1}{B_n(s)} \Big|_{S=\varepsilon^{1/n} \cdot \frac{s}{\omega_p}} = \frac{1}{S^n + a_{n-1}S^{n-1} + \dots + a_1S + 1} \Big|_{S=\varepsilon^{1/n} \cdot \frac{s}{\omega_p}}$$

Butterworthpolynomet av ordning n

Filtrets A_p dB-gränsvinkelfrekvens är $\omega_p = \omega_{3dB} \cdot \varepsilon^{1/n}$

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Butterworthfilter, forts



Butterworthfilter har maximalt flat amplitud-karaktäristik i passbandet!

(dvs ger bästa passbands-approximation)

Filterkraven uppfylls om $n \geq \frac{\log\left(\frac{10^{0.1A_s} - 1}{10^{0.1A_p} - 1}\right)}{2 \cdot 10 \log\left(\frac{\omega_s}{\omega_p}\right)}$ (n heltal)

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Chebyshev I-filter

$$|H_{\text{norm}}(\omega)| = |H_C(\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 T_n^2\left(\frac{\omega}{\omega_p}\right)}}$$

$$\left(L_n(\omega) = \varepsilon \cdot T_n\left(\frac{\omega}{\omega_p}\right) \right)$$

$$\varepsilon = \sqrt{10^{0.1A_p} - 1}$$

där $T_n(\omega)$ är chebyshevpolyomet av ordning n

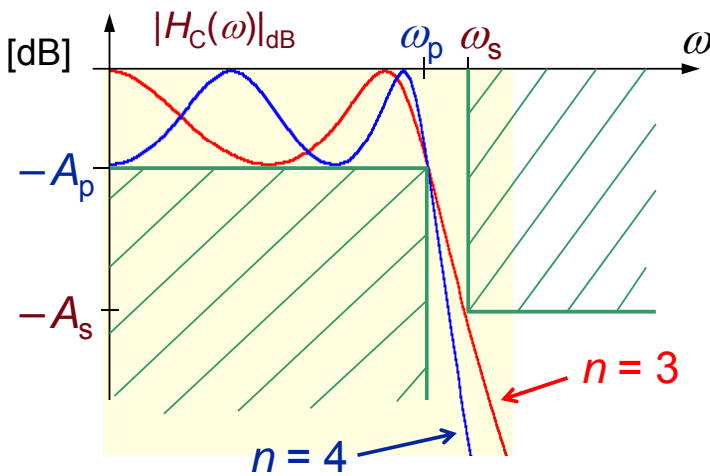
Motsvarande systemfunktion:

$$H_{\text{norm}}(s) = H_C(s) = \frac{K}{\left(\frac{s}{\omega_p}\right)^n + a_{n-1}\left(\frac{s}{\omega_p}\right)^{n-1} + \dots + a_1 \frac{s}{\omega_p} + a_0}$$

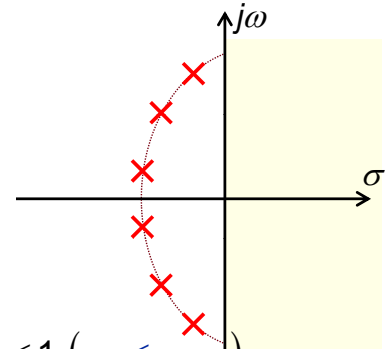
$$|H_C(0)| = \left| \frac{K}{a_0} \right| = \begin{cases} 1 & ; n \text{ udda} \\ \frac{1}{\sqrt{1+\varepsilon^2}} & ; n \text{ jämn} \end{cases} \quad |H_C(\omega_p)| = \frac{1}{\sqrt{1+\varepsilon^2}} \quad (\text{motsv. } -A_p \text{ dB})$$

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Chebyshev I-filter, forts



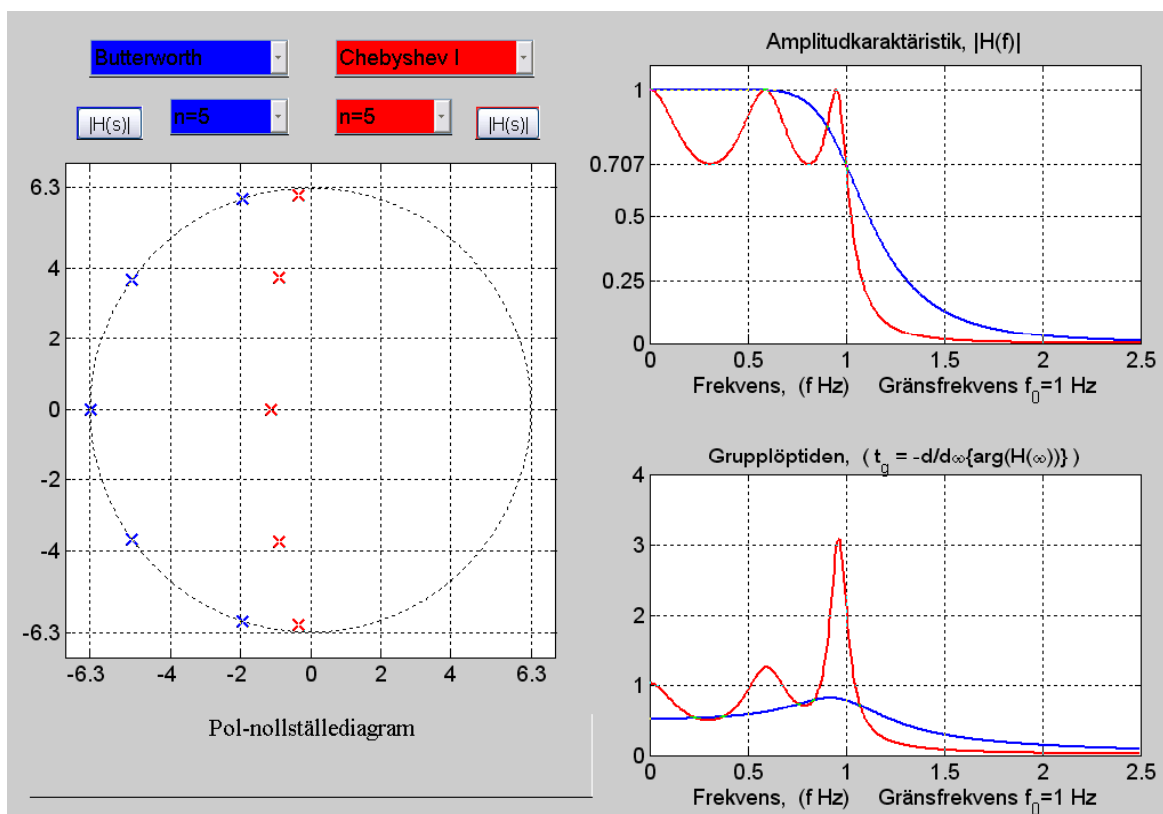
- Rippel (A_p dB) i passbandet!
- Optimalt m.a.p. brantheten i övergångsbandet
- Polerna ligger längs en halv-ellips i vänster halvplan



Filterkraven uppfylls om $n \geq \frac{\text{arcosh}\left(\sqrt{\frac{10^{0.1A_s} - 1}{\epsilon^2}}\right)}{\text{arcosh}\left(\frac{\omega_s}{\omega_p}\right)}$
 (n heltal)

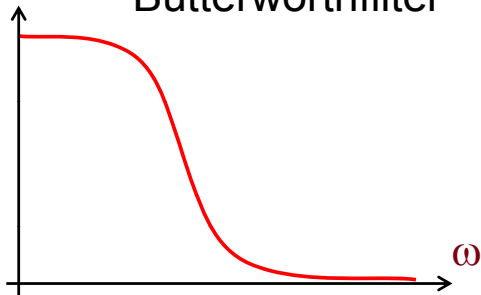
Om $\epsilon \leq 1$ ($\omega_p \leq \omega_{3dB}$)
 $\Rightarrow \omega_{3dB} = \omega_p \cosh\left(\frac{1}{n} \cosh^{-1} \frac{1}{\epsilon}\right)$

Kretsdemo1 – "Passiva LP-filter":

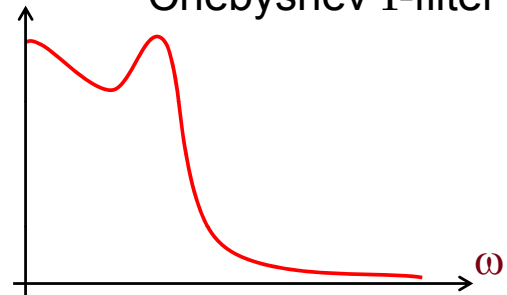


Klassiska ideala LP-approximationer

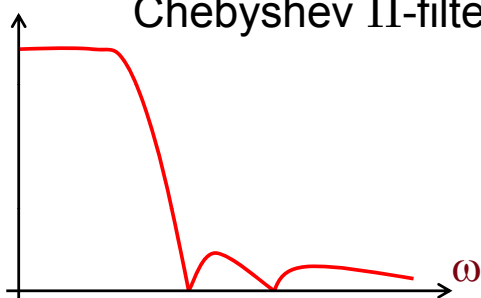
$|H(\omega)|$ Butterworthfilter



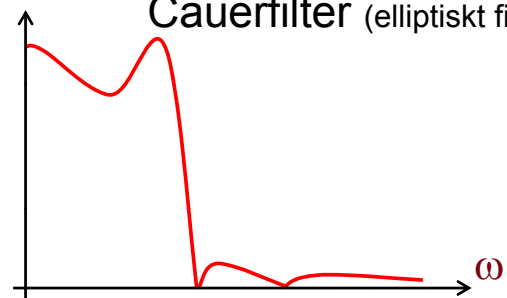
$|H(\omega)|$ Chebyshev I-filter



$|H(\omega)|$ Chebyshev II-filter



$|H(\omega)|$ Cauerfilter (elliptiskt filter)



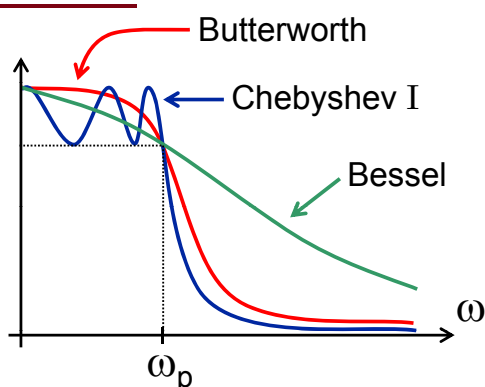
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Besselfilter

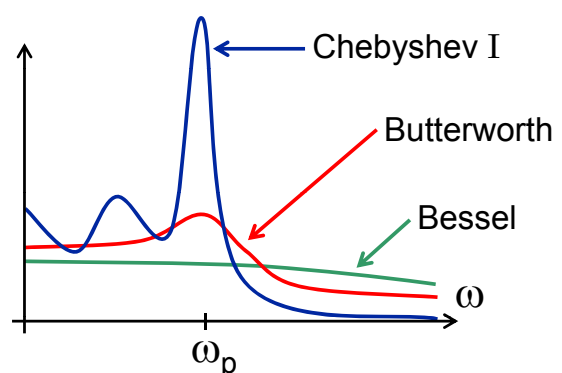
Butterworth, Chebyshev I & II och Cauerfilter ger bra approximationer till ideala LP-filtrets amplitudkaraktäristik.

Besselfiltret har istället goda faskaraktäristikegenskaper !

$|H_{\text{norm}}(\omega)|$



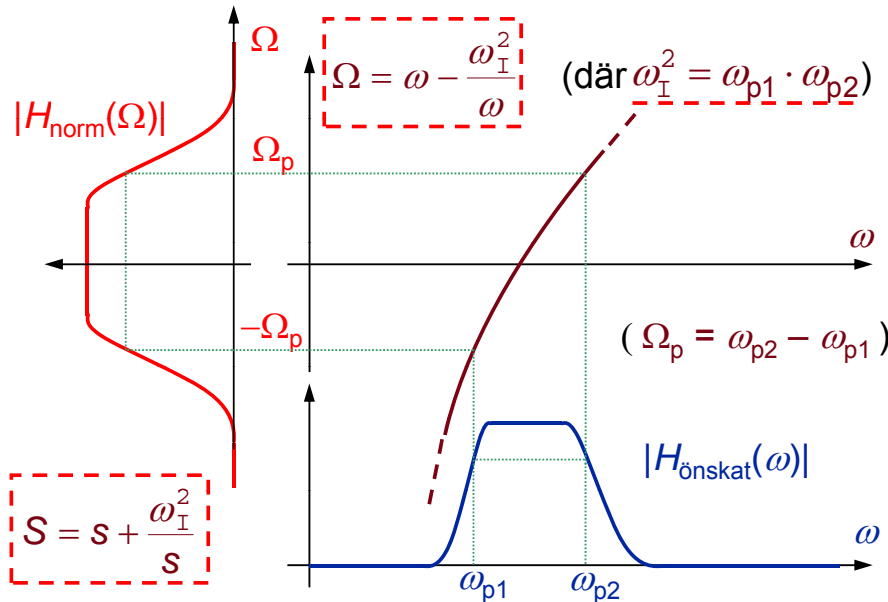
$t_g(\omega)$; grupplöptiden



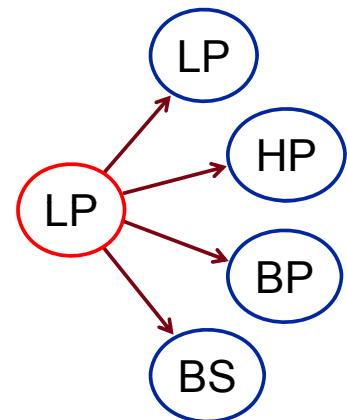
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Frekvenstransformation (filtertransf.)

1. Syntes av normerat referens(LP-)filter som uppfyller ställda dämpningskrav (tag fram $H_{norm}(S)$)
2. Filtertransformera till önskat filter ($H_{norm}(S) \rightarrow H_{önskat}(s)$)

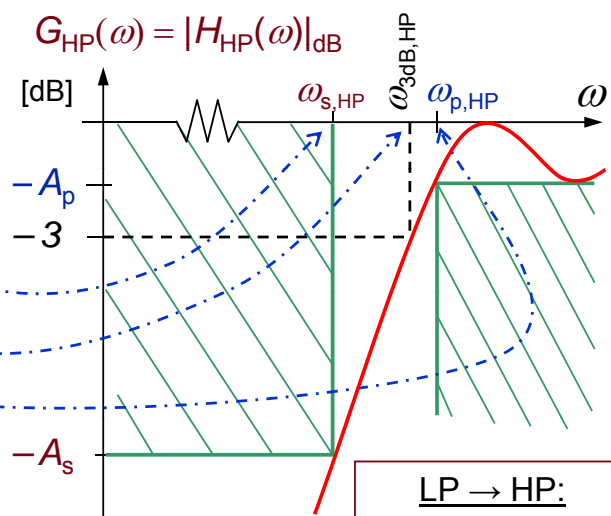
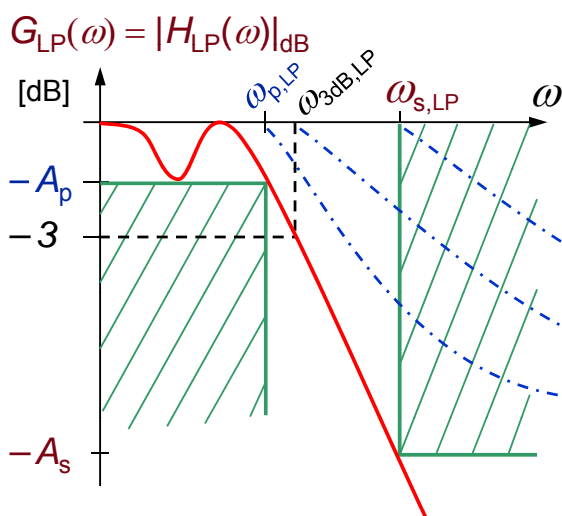


Filtertransformationer (kap 6.6):



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Räkneuppgift, Chebyshev I-filter



LP → HP:
 $\omega_1^2 = \omega_{s,LP} \cdot \omega_{s,HP}$

Referensfilter, Chebyshev, LP:

$A_p = 1 \text{ dB}, \omega_{p,LP} = 1000 \text{ rad/s}$

$A_s \geq 20 \text{ dB vid } \omega_{s,LP} = 2000 \text{ rad/s}$

Önskat Chebyshevfilter, HP:

$f_{3dB,HP} = 2 \text{ kHz}$

$\Rightarrow \omega_{3dB,HP} = 2\pi f_{3dB,HP} = 4\pi \text{ krad/s}$

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