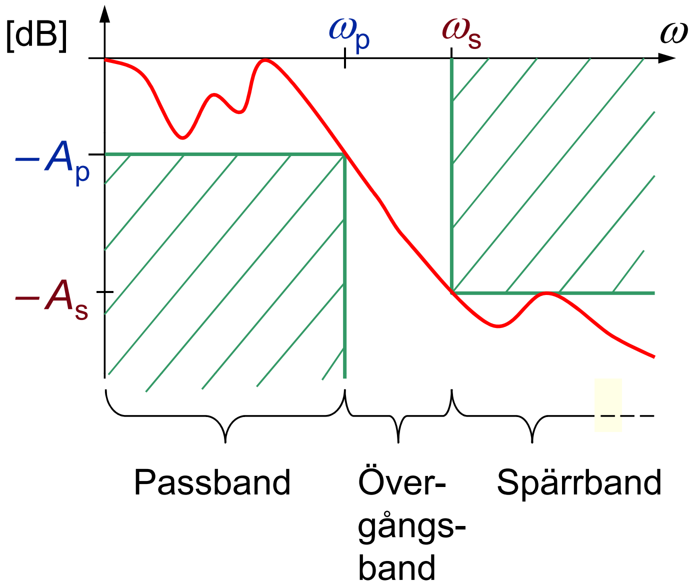
Frekvensselektiva Passiva Filter

<u>Exempel, dämpningskrav för amplitudnormerat LP-filter:</u>

$G(\omega) = |H(\omega)|_{dB}$

$|H(\omega)_{max}| = 1$

$\Rightarrow$ 0 dB



$$|H(\omega)|_{dB} = 20 \cdot {}^{10}\log |H(\omega)| \text{ dB}$$

A_p : Största passbands-dämpningen

ω_p : Passbandsgränsen (gränsvinkelfrekvensen)

A_s : Minsta spärrbands-dämpningen

ω_s : Spärrbandsgränsen

Syntes av praktiska filter (LP, HP, BP & BS)

1. Överför frekvenser och dämpningskrav från önskat filter $H_{\text{önskat}}(s)$ till ett motsvarande normerat LP-filter $H_{\text{norm}}(\omega)$

   ($|H_{\text{norm}}(\omega)|_{\max} = 1$ ($\Leftrightarrow$ 0 dB) Ofta är ω_p (= ω_c) = 1 rad/s)

   Standardfilter har vanligen $$|H_{\text{norm}}(\omega)| = \frac{1}{\sqrt{1+L_n^2(\omega)}}$$

2. LP-filtret: A_p, A_s, ω_p & ω_s $\Rightarrow$ $n_{\min}$ $\Rightarrow$ $|H_{\text{norm}}(\omega)|$

3. Tabell (filter med gränsv.frekv. = 1 rad/s) $\Rightarrow$ $H_{\text{norm}}(s)$ (gränsv.frekv. = ω_p)

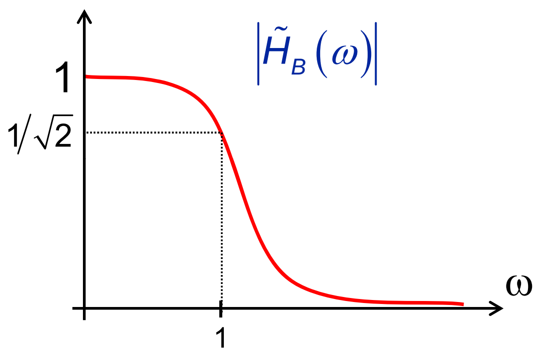
4. Filtertransformera: $H_{\text{norm}}(s) \rightarrow H_{\text{önskat}}(s)$

   ($H_{\text{önskat}}(s)$ är t.ex LP-, HP-, BP- eller BS-filter)

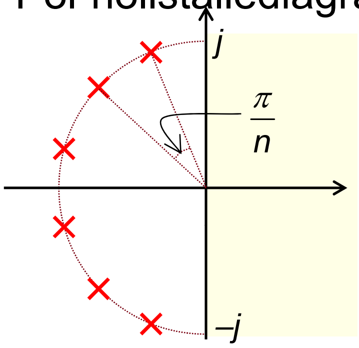
Butterworthfilter, 3 dB-gränsv.frekv. ω_{3dB} = 1 rad/s

Amplitudkaraktäristik

$$\left|\tilde{H}_B(\omega)\right| = \frac{1}{\sqrt{1+\omega^{2n}}}$$



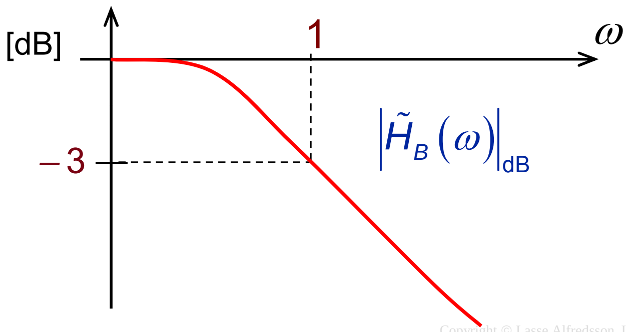
Pol-nollställediagram för $\tilde{H}_B(s)$



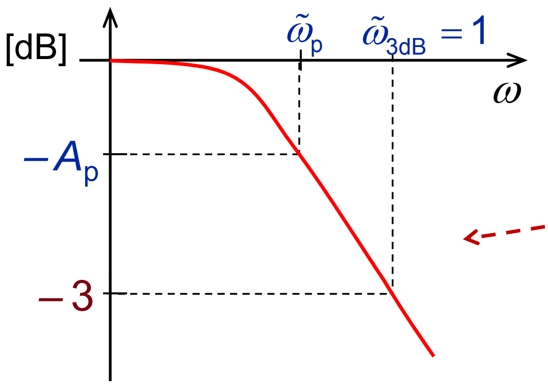
Poler på halvcirkel i vänster halvplan, på radien 1:

$$p_i = e^{j\left(1+\frac{m}{n}\right)\frac{\pi}{2}}$$

$m = 1, 3, 5, \ldots, 2n-1$



Butterworthfilter, A_{p} dB-gränsvinkelfrekvens $\tilde{\omega}_{\text{p}}$



Om strängare dämpningskrav,

max A_{p} dB dämpning i passbandet:

$$\left|\tilde{H}_{\text{B}}(\omega)\right|_{dB} = 20 \cdot {}^{10}\log\left|\tilde{H}_{\text{B}}(\omega)\right|$$

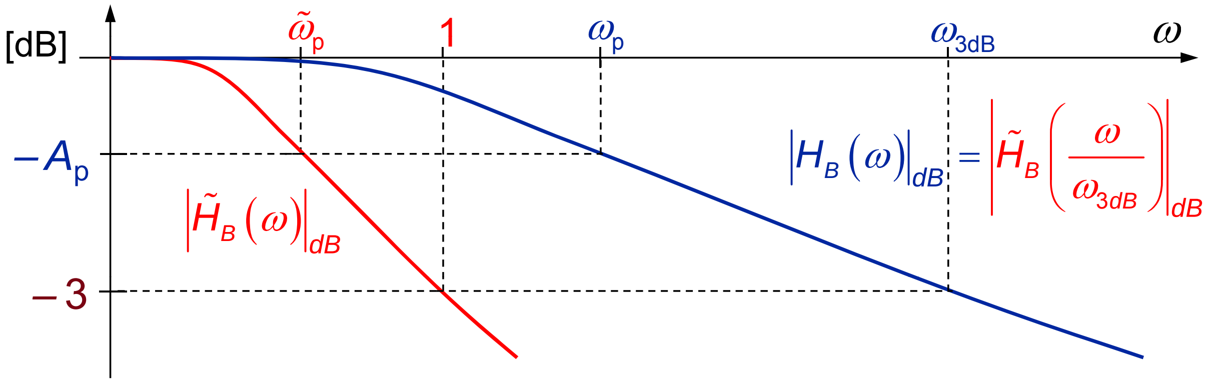
$$= -10 \cdot {}^{10}\log\left(1 + \omega^{2n}\right)$$

$$A_{\text{p}} = -\left|\tilde{H}_B(\tilde{\omega}_{\text{p}})\right|_{\text{dB}} = 10 \cdot {}^{10}\log\left(1 + \tilde{\omega}_{\text{p}}{}^{2n}\right)$$

/= konstant, oberoende av n/ $= 10 \cdot {}^{10}\log\left(1 + \varepsilon^2\right)$

$$\Rightarrow \boxed{\tilde{\omega}_{\text{p}} = \varepsilon^{1/n}} \quad \text{där} \quad \boxed{\varepsilon = \sqrt{10^{0.1A_{\text{p}}} - 1}}$$

Butterworthfilter, godtycklig gränsvinkelfrekvens ω_p



dvs. $\omega_\mathrm{p} = \tilde{\omega}_\mathrm{p} \cdot \omega_\mathrm{3dB}$ med $\tilde{\omega}_\mathrm{p} = \varepsilon^{1/n}$ $\Rightarrow$ $\boxed{\omega_\mathrm{p} = \omega_\mathrm{3dB} \cdot \varepsilon^{1/n}}$

$$\Rightarrow \left|H_B(\omega)\right| = \tilde{H}_B\left(\frac{\omega}{\omega_\mathrm{3dB}}\right) = \tilde{H}_B\left(\varepsilon^{1/n} \cdot \frac{\omega}{\omega_\mathrm{p}}\right)$$

$\Rightarrow$ *(se nästa sida…)*

Butterworthfilter – sammanfattning

$$|H_{\text{norm}}(\omega)| = |H_{\text{B}}(\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\left(\dfrac{\omega}{\omega_{\text{p}}}\right)^{2n}}}$$

$$\left(L_n(\omega) = \varepsilon \cdot \left(\frac{\omega}{\omega_{\text{p}}}\right)^n\right)$$

$$\varepsilon = \sqrt{10^{0.1A_{\text{p}}} - 1}$$

$$(\varepsilon = 1 \Leftrightarrow A_{\text{p}} \approx 3 \text{ dB})$$

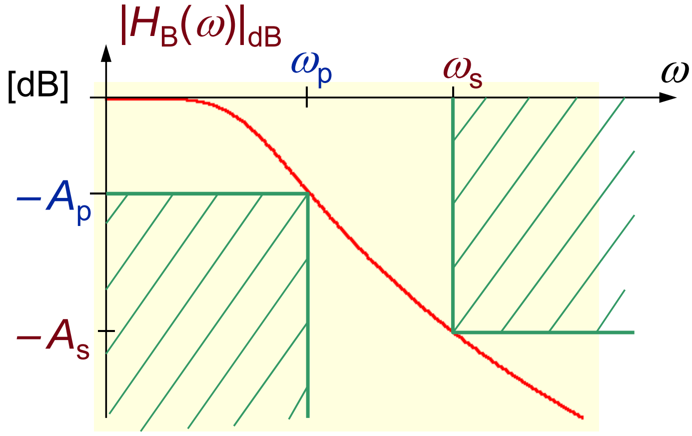
där $|H_{\text{B}}(\omega)|$ erhålls från

$$H_{\text{B}}(s) = \left.\frac{1}{B_n(S)}\right|_{S=\varepsilon^{\frac{1}{n}}\cdot\frac{s}{\omega_{\text{p}}}} = \left.\frac{1}{S^n + a_{n-1}S^{n-1} + \cdots + a_1 S + 1}\right|_{S=\varepsilon^{\frac{1}{n}}\cdot\frac{s}{\omega_{\text{p}}}}$$

$B_n(S)$: Butterworthpolynomet av ordning n

Filtrets A_{p} dB-gränsvinkelfrekvens är $\omega_{\text{p}} = \omega_{3\text{dB}} \cdot \varepsilon^{1/n}$

Butterworthfilter, forts



Butterworthfilter har maximalt flat amplitud-karaktäristik i passbandet!

(dvs ger bästa passbands-approximation)

Filterkraven uppfylls om $n \geq \dfrac{\log\left(\dfrac{10^{0.1A_s}-1}{10^{0.1A_p}-1}\right)}{2\cdot{}^{10}\log\left(\dfrac{\omega_s}{\omega_p}\right)}$ (n heltal)

Chebyshev I-filter

$$\left|H_{\text{norm}}(\omega)\right| = \left|H_C(\omega)\right| = \frac{1}{\sqrt{1+\varepsilon^2 T_n^2\left(\dfrac{\omega}{\omega_p}\right)}}$$

$$\left(L_n(\omega) = \varepsilon \cdot T_n\left(\frac{\omega}{\omega_p}\right)\right)$$

$$\varepsilon = \sqrt{10^{0.1A_p} - 1}$$

där $T_n(\omega)$ är chebyshevpolynomet av ordning n

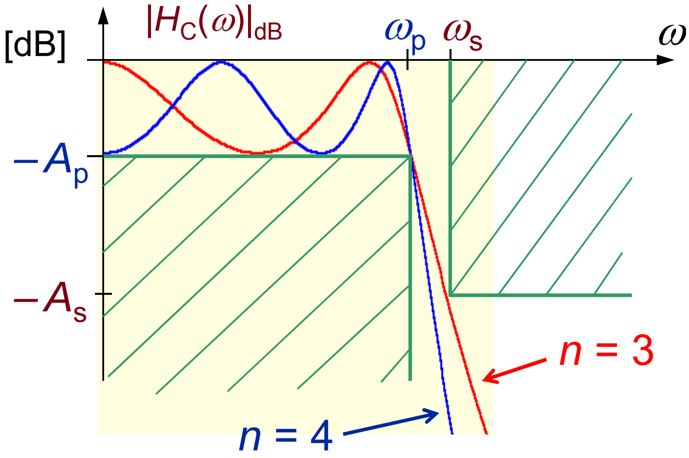
Motsvarande systemfunktion:

$$H_{\text{norm}}(s) = H_C(s) = \frac{K}{\left(\dfrac{s}{\omega_p}\right)^n + a_{n-1}\left(\dfrac{s}{\omega_p}\right)^{n-1} + \cdots + a_1\dfrac{s}{\omega_p} + a_0}$$

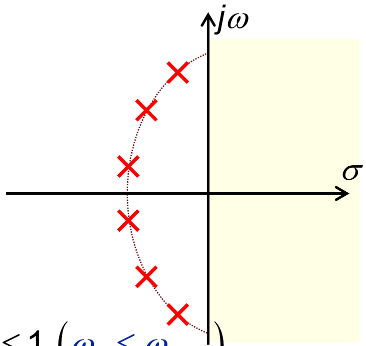
$$\left|H_C(0)\right| = \left|\frac{K}{a_0}\right| = \begin{cases} 1 & ; \quad n \text{ udda} \\ \dfrac{1}{\sqrt{1+\varepsilon^2}} & ; \quad n \text{ jämn} \end{cases}$$

$$\left|H_C(\omega_p)\right| = \frac{1}{\sqrt{1+\varepsilon^2}} \quad (\text{motsv. } -A_p \text{ dB})$$

Chebyshev I-filter, forts



- Rippel (A_p dB) i passbandet!
- Optimalt m.a.p. brantheten i övergångsbandet
- Polerna ligger längs en halv-ellips i vänster halvplan



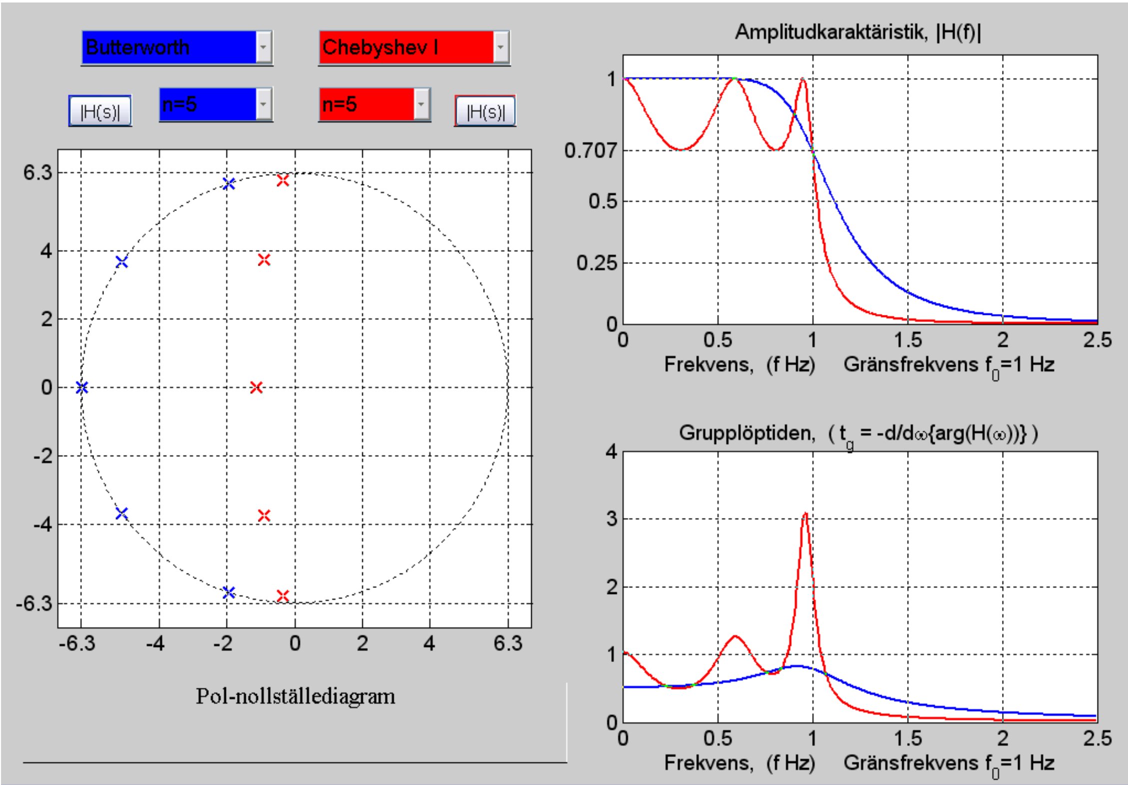
Filterkraven uppfylls om $n \geq \dfrac{\operatorname{arcosh}\left(\sqrt{\dfrac{10^{0.1A_s}-1}{\varepsilon^2}}\right)}{\operatorname{arcosh}\left(\dfrac{\omega_s}{\omega_p}\right)}$

(n heltal)

Om $\varepsilon \leq 1$ $\left(\omega_p \leq \omega_{3dB}\right)$

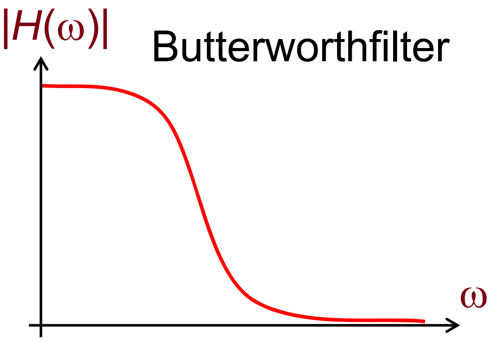
$\Rightarrow\ \omega_{3dB} = \omega_p \cosh\left(\frac{1}{n}\cosh^{-1}\frac{1}{\varepsilon}\right)$

Kretsdemo1 – ”Passiva LP-filter”:

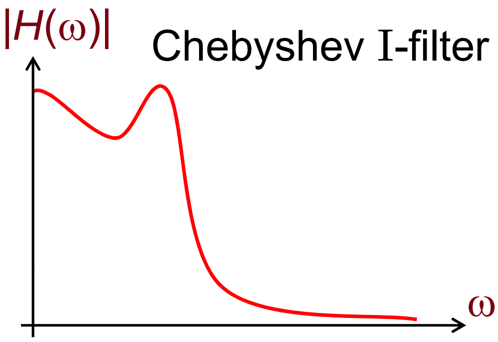


Klassiska ideala LP-approximationer

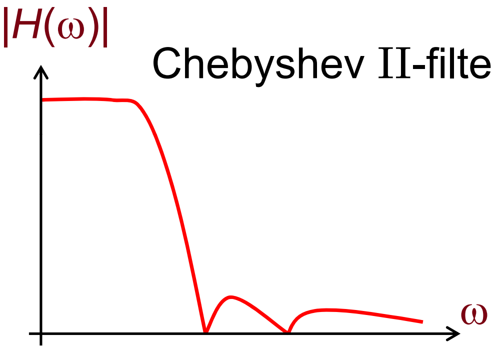
Butterworthfilter



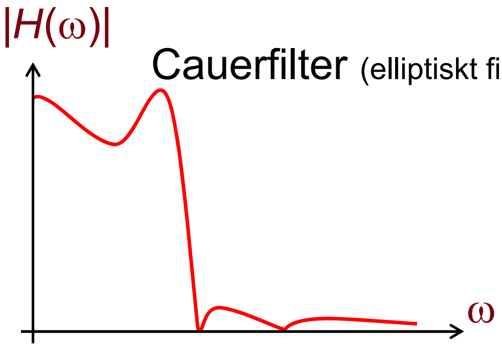
Chebyshev I-filter



Chebyshev II-filter



Cauerfilter (elliptiskt filter)

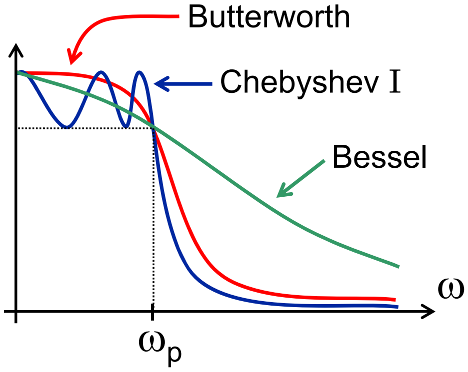


Besselfilter

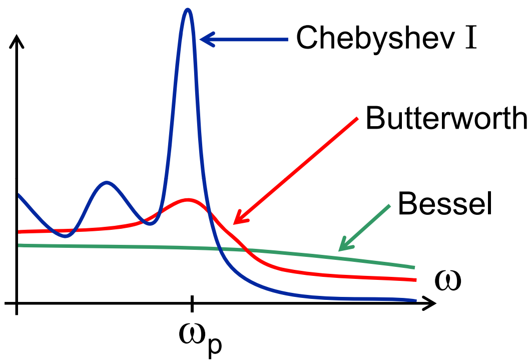
Butterworth, Chebyshev I & II och Cauerfilter ger bra approximationer till ideala LP-filtrets amplitudkaraktäristik.

Besselfiltret har istället goda faskaraktäristikegenskaper !

$|H_{norm}(\omega)|$

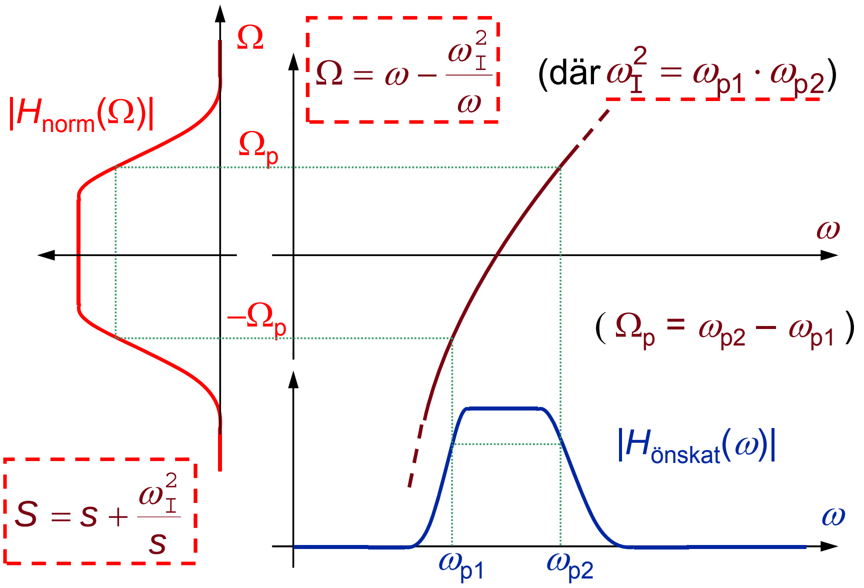


$t_g(\omega)$; grupplöptiden

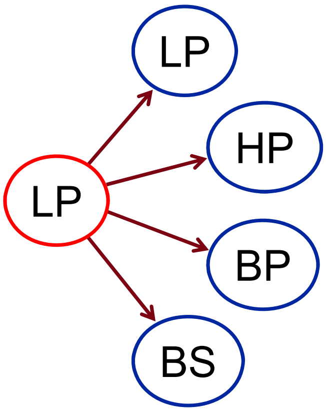


Frekvenstransformation (filtertransf.)

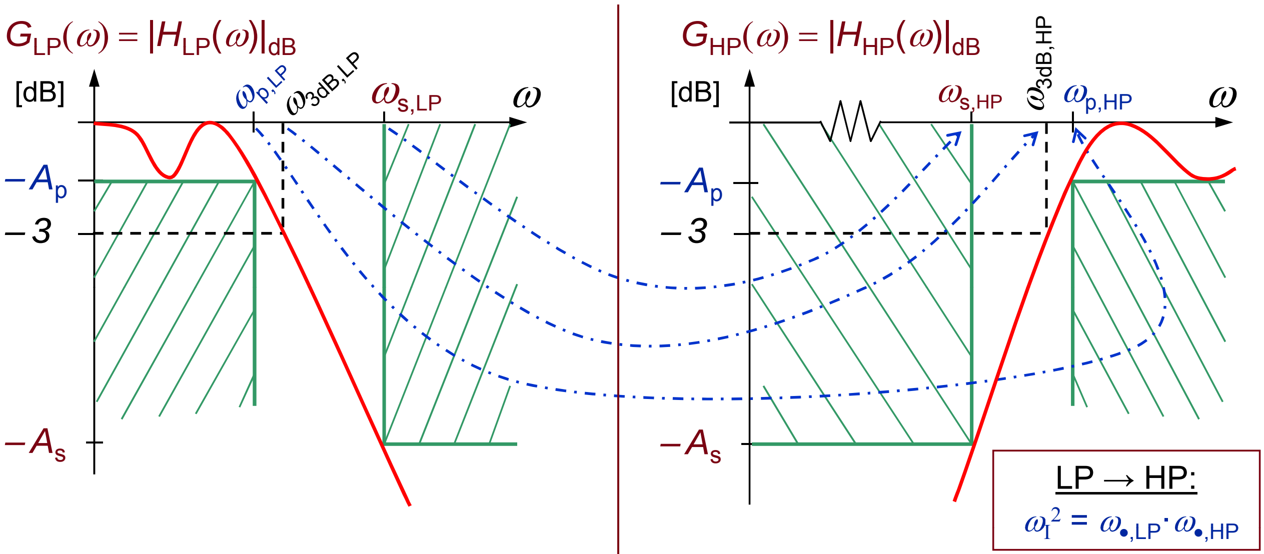
1. Syntes av normerat <u>referens(LP-)filter</u> som uppfyller ställda dämpningskrav (tag fram $H_{\text{norm}}(S)$)
2. Filtertransformera till önskat filter ($H_{\text{norm}}(S) \rightarrow H_{\text{önskat}}(s)$)



Filtertransformationer (kap 6.6):



Räkneuppgift, Chebyshev I-filter



<u>Referensfilter, Chebyshev, LP:</u>

A_p = 1 dB, $\omega_{p,LP}$ = 1000 rad/s

$A_s \geq$ 20 dB vid $\omega_{s,LP}$ = 2000 rad/s

<u>Önskat Chebyshevfilter, HP:</u>

$f_{3dB,HP}$ = 2 kHz

$\Rightarrow \omega_{3dB,HP} = 2\pi f_{3dB,HP} = 4\pi$ krad/s