The Horn & Schunck method

- At each point we seek the motion vector **v** = (v_1, v_2) that satisfies the BCCE:

$$\frac{\partial I}{\partial u} v_1 + \frac{\partial I}{\partial v} v_2 + \frac{\partial I}{\partial t} = 0$$

- Problem: one equation but two unknowns
- Previously, we dealt with this problem by considering a *local* set of equations, assuming **v** constant in a *local* region Ω
- Finding **v** can also be dealt with by means of a *global* approach (with respect to the image)

The Horn & Schunck method

- Let **v**(*u*, *v*) denote the velocity vector field in an image, as a function of image position (*u*, *v*)
- BCCE suggests that we should find **v**(*u*, *v*) that minimizes

$$\varepsilon = \int \left[\mathbf{v}(u,v) \cdot \nabla I + \frac{\partial I}{\partial t} \right]^2 d\mathbf{x}$$

Image gradient at (*u*, *v*)

Time derivative at (*u*, *v*)

Integration is now made over an entire image!

The Horn & Schunck method

- We can (in principle) always find **v**(*u*, *v*) that gives ε = 0:

$$\mathbf{v}(u,v) = -\frac{\partial I}{\partial t} \frac{\nabla I}{\|\nabla I\|^2} + \alpha(u,v) \begin{pmatrix} \frac{\partial I}{\partial v} \\ -\frac{\partial I}{\partial u} \end{pmatrix}$$

(why?)

Arbitrary function of (*u*, *v*)

The Horn & Schunck method

- Problem I:
  Singularities when ∇*I*= **0**
- Problem II:
  Does not provide a unique solution since α(*u*, *v*) can be arbitrary chosen
- Problem III:
  Strong variations in ∇*I* may not correspond to strong variations in **v**(*u*, *v*)

The Horn & Schunck method

- H&S 1981: Let's make **v**(*u*, *v*) unique by adding a smoothness term to ε
- This term should assure that **v**(*u*, *v*) is as smooth at possible, seen as a function of (*u*, *v*)
- Smoothness =
  "as little variation in **v** as possible"

The Horn & Schunck method

- H&S used a smoothness term:

$$\|\nabla v_1\|^2 + \|\nabla v_2\|^2$$

- Other types of smoothness terms are appear in the literature

The Horn & Schunck method

- New cost function

$$\varepsilon = \int \left[\mathbf{v}(u,v) \cdot \nabla I + \frac{\partial I}{\partial t} \right]^2 d\mathbf{x} +$$

$$+ \lambda \int \left[\|\nabla v_1\|^2 + \|\nabla v_2\|^2 \right] d\mathbf{x}$$

- The integrals are taken *over the entire image*
- λ is a "smoothness weight"
- Our goal: find **v**(*u*, *v*) that minimizes ε

The Horn & Schunck method

- This was one of the first established methods for motion estimation
- Often referred to as a "global" method
- Can (to some extent) deal with the aperture problem
- In practice: **v** cannot be determined by solving a linear equation, instead iterative methods are required
  - Efficient algorithms exist
  - See e.g. D. Sun, et al, Secrets of Optical Flow Estimation and Their Principles, CVPR 2010.
- Not obvious how to choose λ
  - constant or dependent on **x**?
- The smoothness constraint is not always valid
  - Sharp motion boundaries exist in practice
- More "sophisticated" methods use other types of smoothness terms

The Horn & Schunck method

NOTE!!

- Horn & Schunck's method is not correctly described in the book by R. Szeliski
  - In the printed book and e-book: on page 360, equation (8.70)
  - In the draft version on the web: on page 410, equation (8.70)
- The cost function E_{HS} lacks the regularization term