TSBB15
Computer Vision

Lecture 8
Local invariant features

Today’s lecture

- What are invariant features used for?
- Local invariant features paradigm
- Invariances: Geometric, Photometric
- Examples: SIFT, MSER/MSCR…
- Matching

Image Matching

- KLT tracking and block matching are useful when matching between consecutive frames in a **video** sequence.

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- small changes in **scale**, **rotation** and **illumination**

Image Matching

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- Images are from **the same camera**
- small changes in **scale**, **rotation** and **illumination**
- Local invariant features work when these conditions are violated.

Wide-baseline stereo

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  - Matching images of the same scene, captured at different positions.

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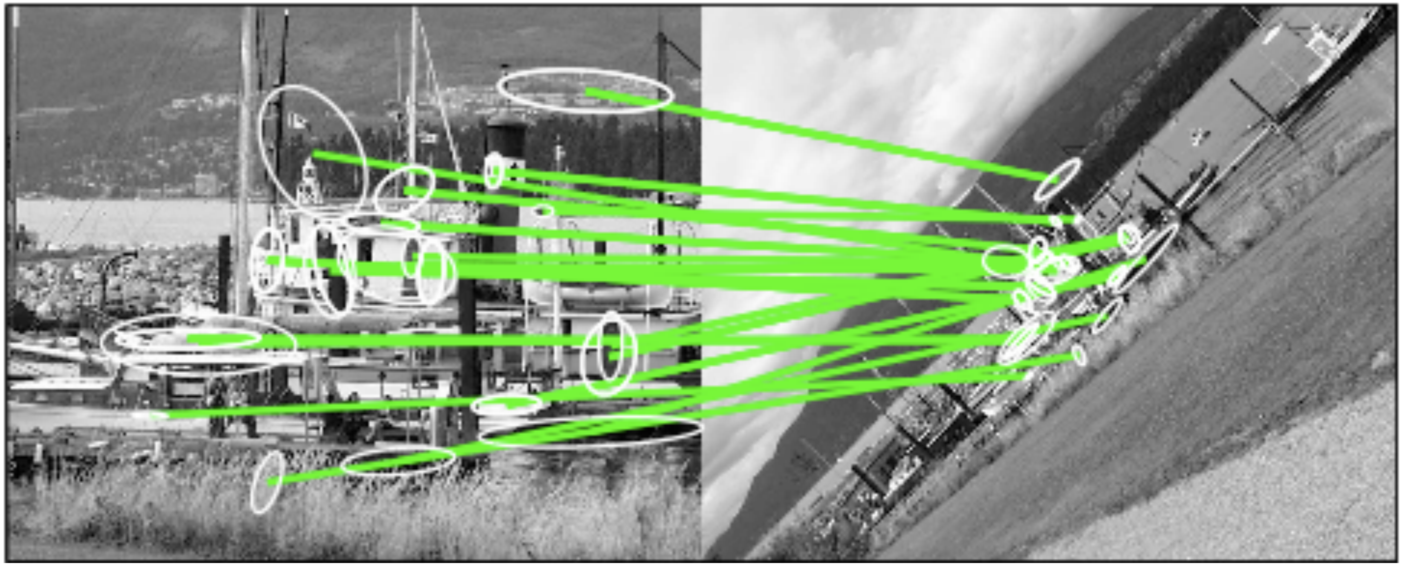


Object instance recognition and pose estimation

- Problem 2: bin picking
  - identity and pose estimation under partial occlusion
  - training set
  - test set
  - 6dof pose

Object recognition

- Example: Eddie the embodied

- See webpage for details

  http://www.cvl.isy.liu.se/research/objrec/EVOR/

Local invariant features

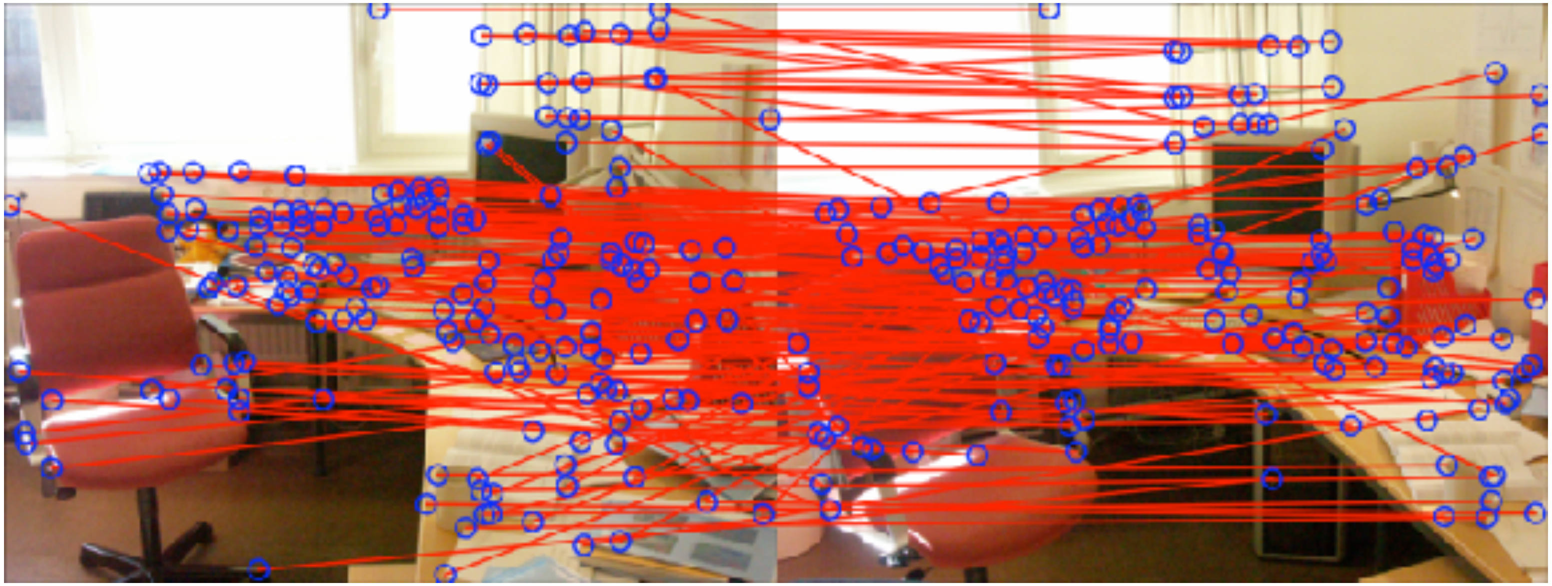
- In lecture 2 we discussed how to match across scale and translation. How?

Local invariant features

- In lecture 2 we discussed how to match across scale and translation. How?
- Another option is to use **interest points** e.g. Harris points [Z. Zhang et al. 95].
  - A. Detect interest points
  - B. Cut out image patches around each point
  - C. Matches can now be found by comparing patches+epipolar geometry constraints.

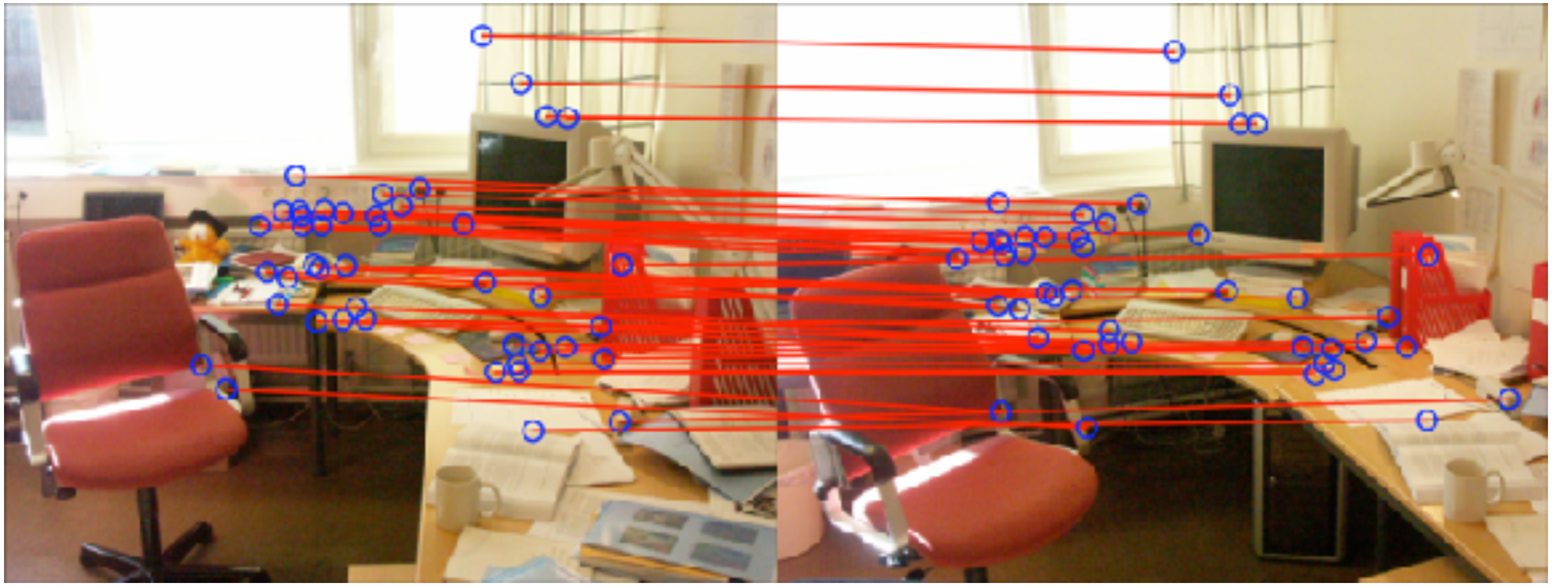
Local invariant features

- Correspondences from block matching at Harris points.



Local invariant features

- After applying the Epipolar constraint (You will test this in lab 3).



Epipolar constraint (recap)

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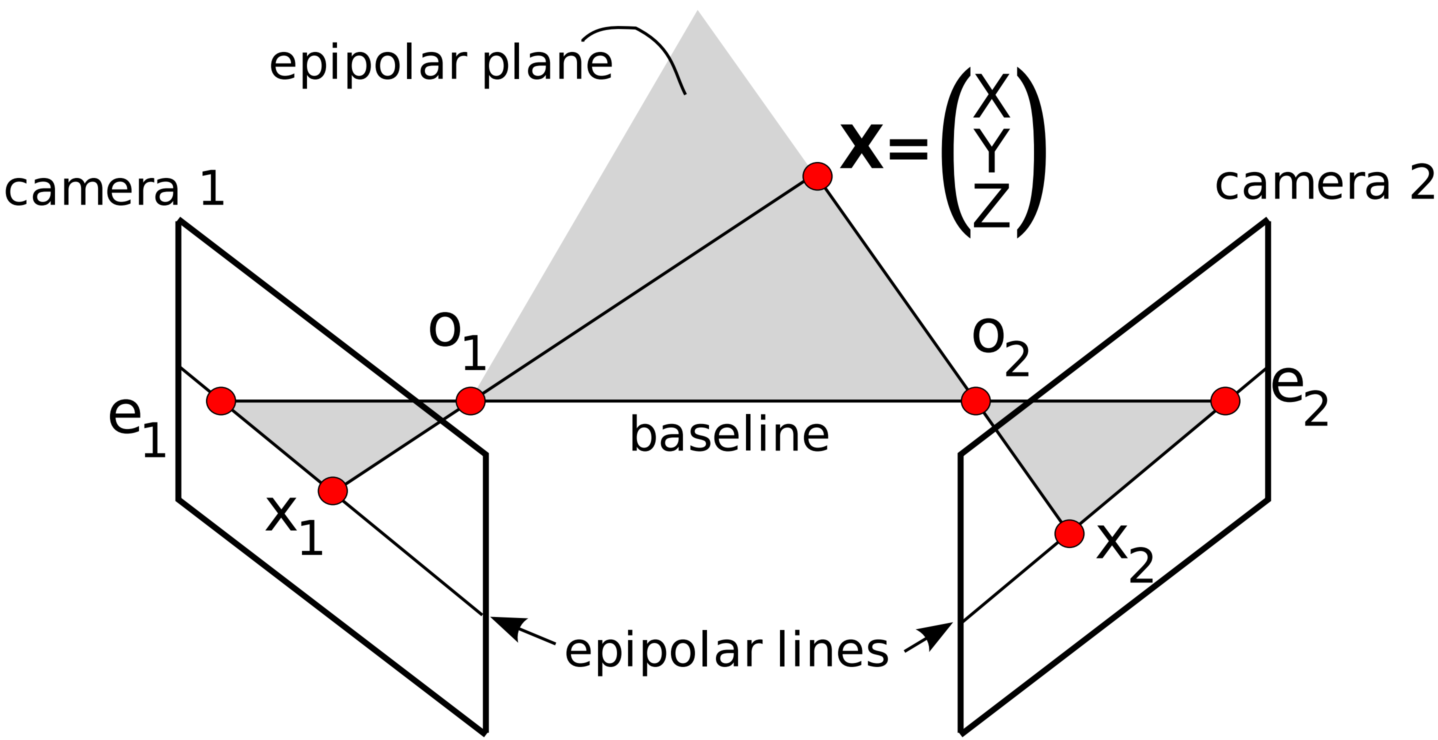

Epipolar constraint (recap)

- The epipolar constraint: $\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$
- $\mathbf{x}_1$ and $\mathbf{x}_2$ are projections of the same 3D point in two views.
- Scene is static, i.e. no motion has taken place (except the change of camera position).
- **F** can be estimated from 7 or more correspondences. E.g. 8-pt algorithm.

Epipolar constraint (recap)

- The epipolar constraint: $\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$
- See the compendium, *Introduction to Estimation, Representation and Geometry (IREG)*, Klas Nordberg

Local invariant features

- Zhang's **interest point** method. (repeat)
  - A. Detect interest points
  - B. Cut out image patches around each point
  - C. Find matches, by comparing patch **descriptors** and epipolar geometry constraints.

Local invariant features

- Zhang's method is invariant to translation (and partially to scale).

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = s \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \mathbf{t}$$

  – 2 degrees-of-freedom (DOF) of invariance (transl. only) (3 if scale is also counted)

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- We will now add invariance to image rotations and view changes.

Local invariant features

- In general, the *local invariant feature approach* can be described as three steps:
  - **Detection**: Use a *detector* to find a local, canonical frame (coordinate system)

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Local invariant features

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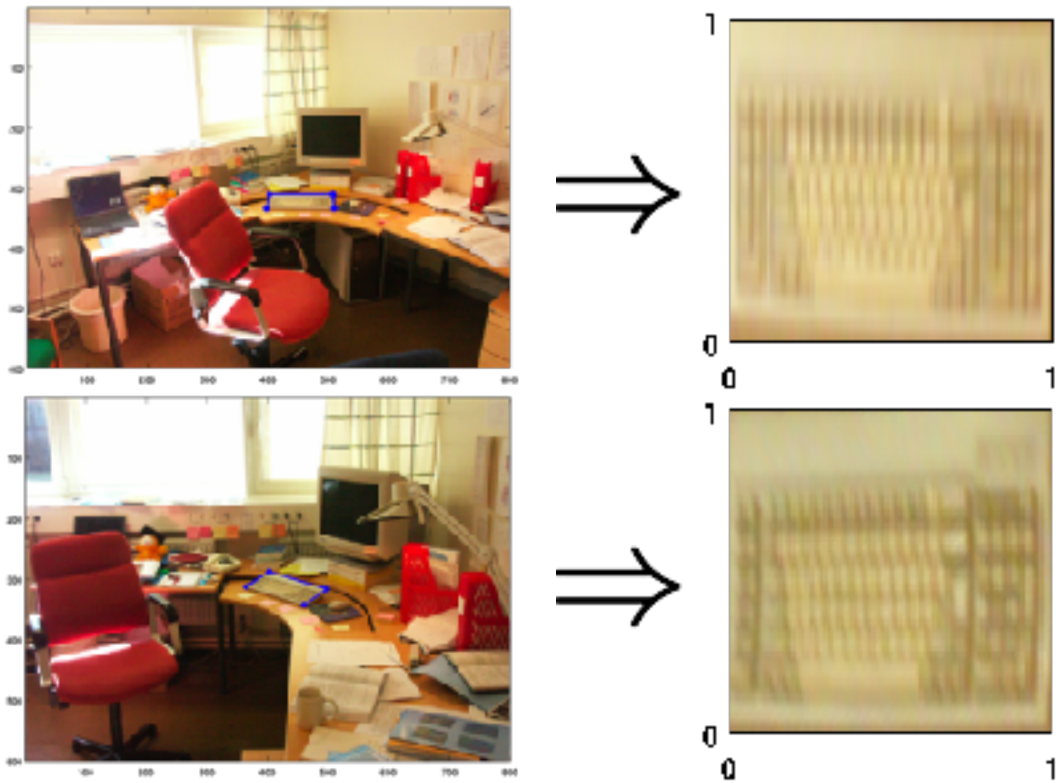
  - **Detection**: Use a *detector* to find a local, canonical frame (coordinate system)
  - **Description**: Compute a *descriptor*, by sampling the image in the canonical frame
  - **Matching**: Find correspondences, by comparing descriptors from two images

Canonical frame example

- Resampling to canonical frame



Local invariant features

- ***Geometric invariances***
  Robustness to view changes


- ***Photometric invariances***
  Robustness to illumination changes


Local invariant features

- ***Geometric invariances*** can be obtained by choosing a frame that is *equivariant* to rotations, scalings, and image skews
- ***Photometric invariances*** can be obtained by computing the descriptor in a more advanced way than direct sampling.

Geometric Invariance

- The geometric invariances used in local features make a *locally planar assumption.*
- They can thus be described using *homographies (See IREG, TSBB06)*.

Geometric Invariance

- Recap: A **Homography** is a transformation between points ***x*** on one plane, and points ***y*** on another.

$$\lambda \begin{pmatrix} y_1 \\ y_2 \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}$$

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- **Degrees of freedom**: number of unique elements in **H**.
  - at most 8dof (for plane projective case), as **H** and $k\mathbf{H}$, $k \in \mathbb{R} \setminus 0$ give the same output

Geometric Invariance

- A hierarchy of transformations:
  - scale+translation (3dof)
  - similarity (4dof) (scale+translation+rotation)
  - affine (6dof) (similarity+skew)
  - plane projective (8dof) (affine+forshortening)

$$\begin{bmatrix} s & 0 & t_1 \\ 0 & s & t_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} s_1 & s_2 & t_1 \\ -s_2 & s_1 & t_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & t_1 \\ a_{21} & a_{22} & t_2 \\ 0 & 0 & 1 \end{bmatrix}$$

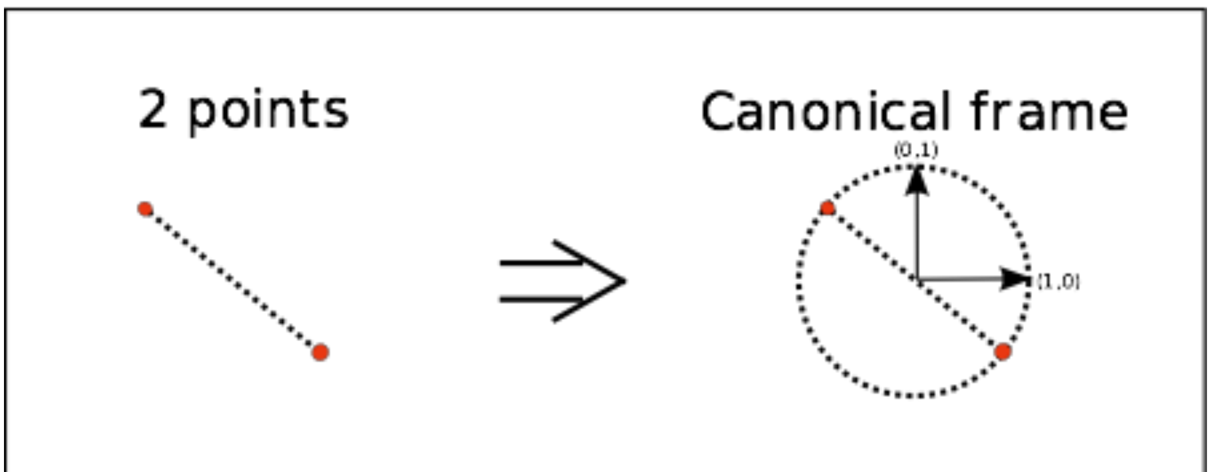
$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

Geometric Invariance

- We can find the canonical frame by using more than one point [Brown&Lowe 02] aka. *interest-point groups*
- We will now give some examples...

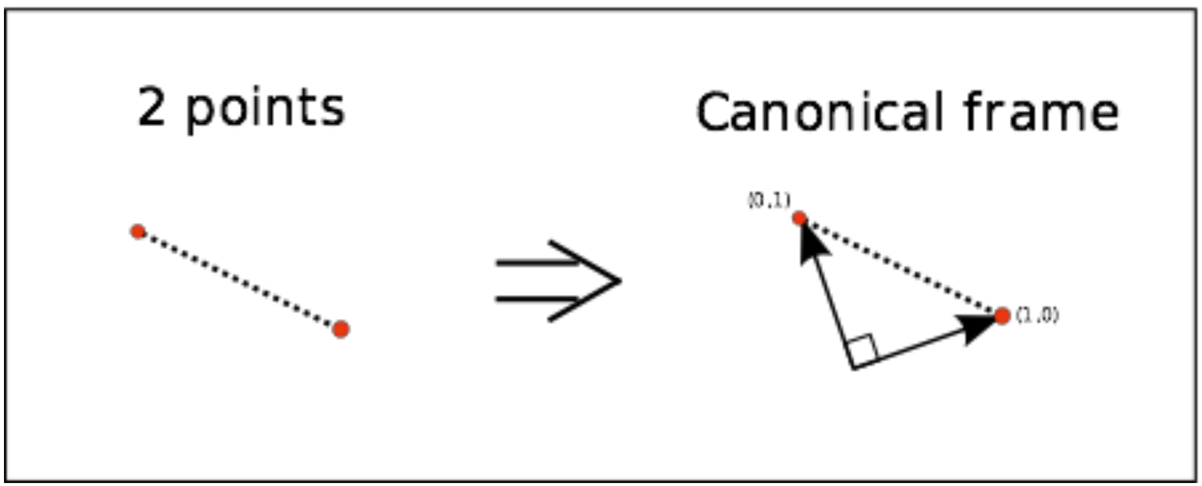
Geometric Invariance

– Scale+translation: Useful if we know that there is no rotation. E.g. for a camera mounted in a car, looking at upright pedestrians.



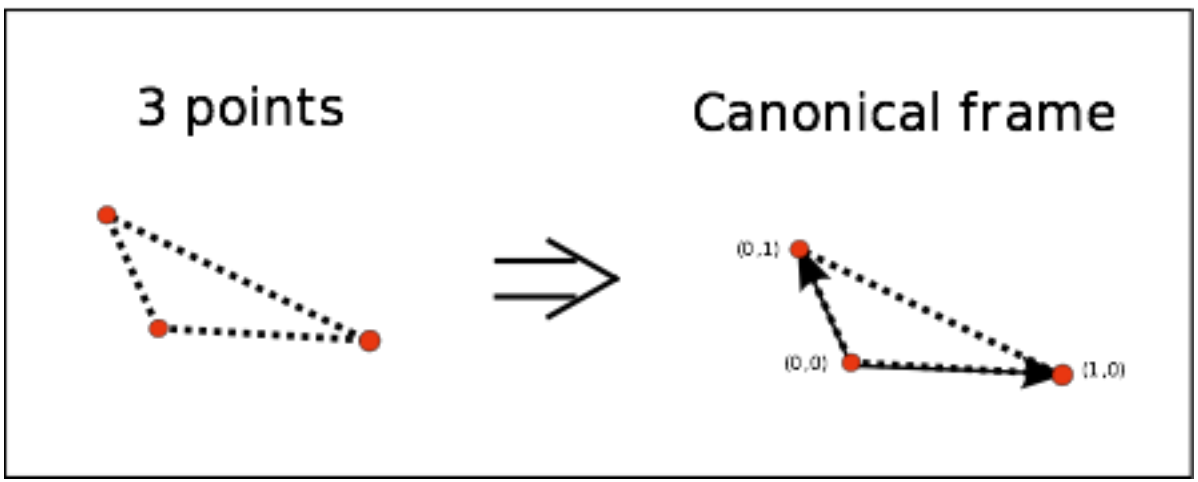
Geometric Invariance

– Similarity: Full invariance in image plane, none outside image plane.
Useful e.g. for pose estimation.



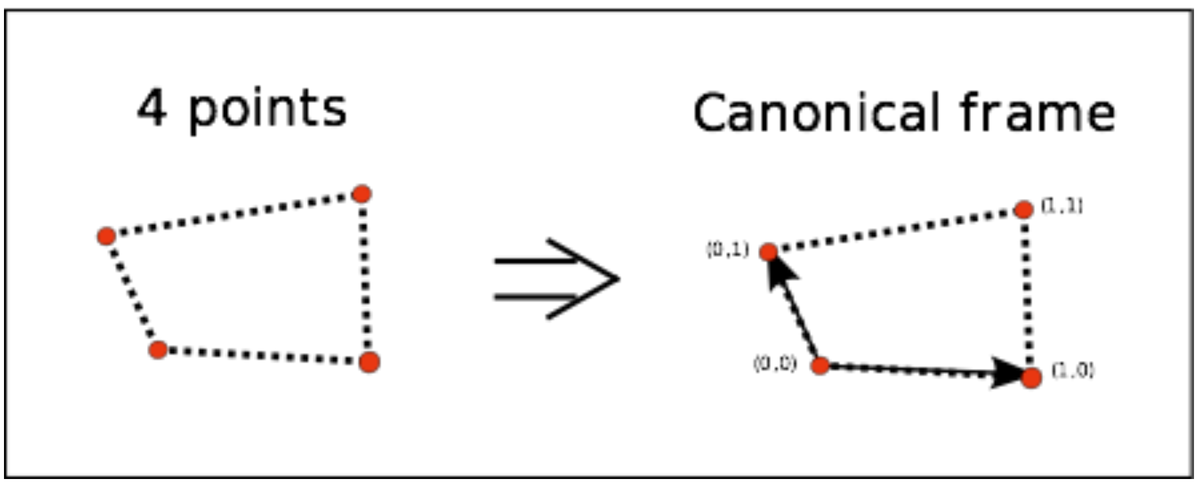
Geometric Invariance

– Affine: Deals with most common projective distortions. Good if patch size is small relative to distance to patch.



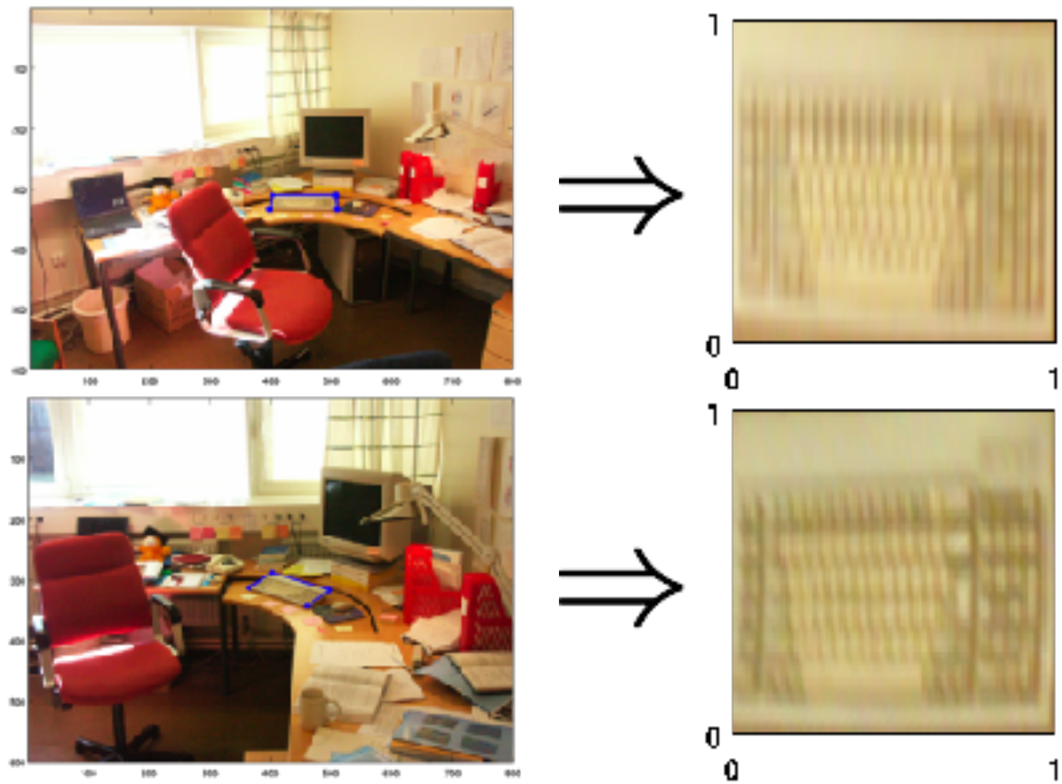
Geometric Invariance

– Plane projective: Full modelling of a plane in 3D. Requires more image measurements, but is better for extreme view angles.



Geometric Invariance

- Resampling to canonical frame results in geometric invariance:



Geometric Invariance

- Problems with interest-point groups:
  - Sensitive to missing points: If e=P(point-detected|present) then P(frame-is-detected|present)=e^N where N is number of points in frame.
  - Combinatorics: if K points in image, we have $\binom{N}{K}$ possible canonical frames.
  - We will introduce other ways to find the frame soon.

Photometric Invariance

- Image intensity is approx. linear in radiance (at least before gamma correction)
- E.g. adding a second, identical light source will double the sensor activation, *a*(**x**).

$$a(\mathbf{x}) = \int s(\lambda) r(\lambda, \mathbf{x}) e(\lambda) d\lambda$$

- s-sensor absorption spectrum, r-reflectance spectrum of object, e-emission spectrum of light source (attenuated by the atmosphere)

Photometric Invariance

- If illumination changes, image matching fails:

$$\begin{aligned} I(\mathbf{x}) &= I_0(\mathbf{x})k_1 \\ J(\mathbf{x}) &= I_0(\mathbf{x})k_2 \end{aligned} \Rightarrow \sum_{x\in\Omega}(I(\mathbf{x}) - J(\mathbf{x}))^2 = \text{non-zero}$$

- We want a function that is invariant to scalings:

$$\sum_{x\in\Omega}(f(I(\mathbf{x})) - f(J(\mathbf{x})))^2 = \text{small number}$$

- How should we choose the invariant *f()*?

Photometric Invariance

- For cameras with non non-linear radiometric response (and e.g. gamma correction), or if two different cameras are used we may use the **affine model**:

$$I(\mathbf{x}) = I_0(\mathbf{x})k_1 + k_2$$

- How should we choose *f ()*? we want:

$$\sum_{x \in \Omega} (f(I(\mathbf{x})) - f(J(\mathbf{x})))^2 = \text{small number}$$

Photometric Invariance

– Invariance to intensity offsets:
  Mean subtraction, and any DC free linear filters, e.g. derivatives.
– Scaling invariance:
  Normalising a patch by an L_p-norm, e.g. the L_2-norm or the standard deviation
– Affine invariance by combining both:

$$\hat{I}(\mathbf{x}) = (I(\mathbf{x}) - \mu_I)/\sigma_I$$

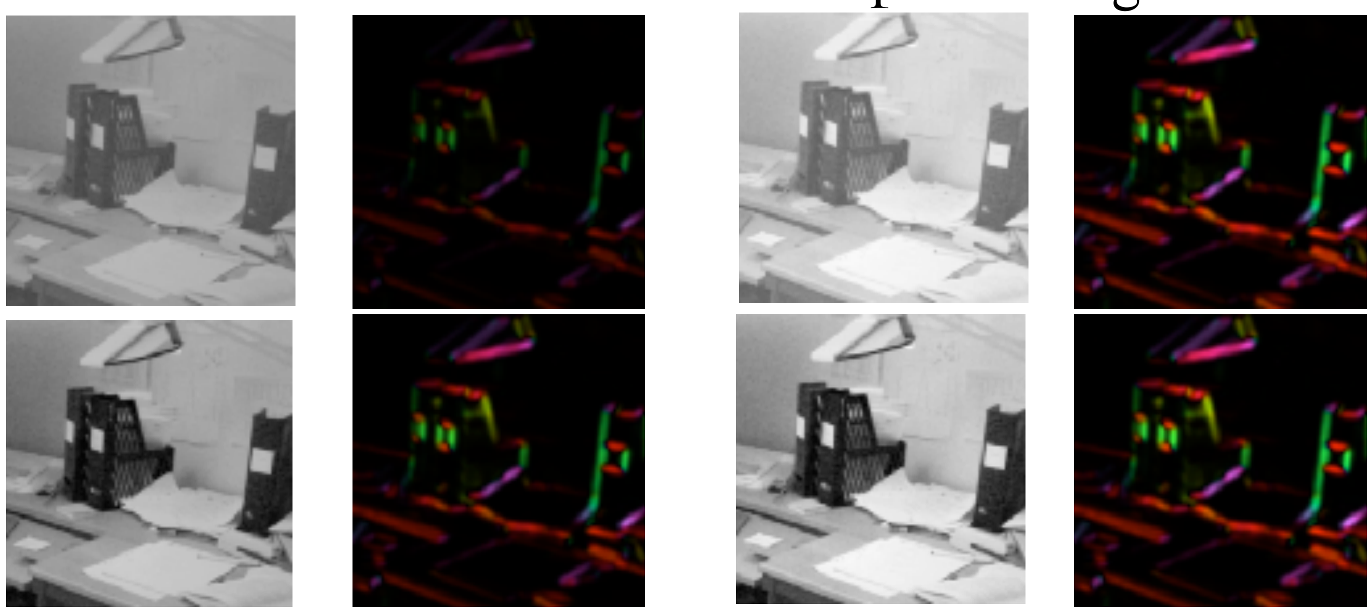
Photometric Invariance

- Illustration:

Input | Gradient | St.dev-Normalised input | St.dev-Normalised gradient

I(x)

J(x)



Local Invariant Features

- There are many examples of features that fit the descriptor+detector paradigm.
- The two most widely used are:
  - **SIFT** Scale Invariant Feature Transform (Lowe 99)
  - **MSER** Maximally Stable Extremal Regions (Matas et al. 02)
- We will look at these two in more detail.

SIFT

- Scale Invariant Feature Transform [Lowe'99]. In brief:
  - The **SIFT detector** finds points using Difference-of-Gaussians in a pyramid Gives: position x,y and scale s
  - Rotation is found from a gradient histogram
  - This gives a frame for the **SIFT descriptor**, which is computed from gradient orientation histograms.

SIFT detector

- Scale Space (recap.)
  - The image is extended with an extra dimension for scale/blur:
    $$f(x, y, s) = (f_0 * g(s))(x, y)$$
  - The blurring kernel $g(s)$ is typically a Gaussian:
    $$g(\mathbf{x}, s) = \frac{1}{2\pi s} e^{-\mathbf{x}^T \mathbf{x}/2s^2}$$

SIFT detector

- Scale Selection [Lindeberg'93]
  - Find a characteristic point (e.g. local max) on a function of position and scale:

$$(\hat{\mathbf{x}}, \hat{s}) = \arg\max h(f(\mathbf{x}, s))$$

  - Example: Maximum of normalised Laplacian:

$$h(f(\mathbf{x}, s)) = s^2 (f * \nabla^2 g(s))(\mathbf{x})$$

SIFT detector

$$(\hat{\mathbf{x}}, \hat{s}) = \arg\max h(f(\mathbf{x}, s))$$

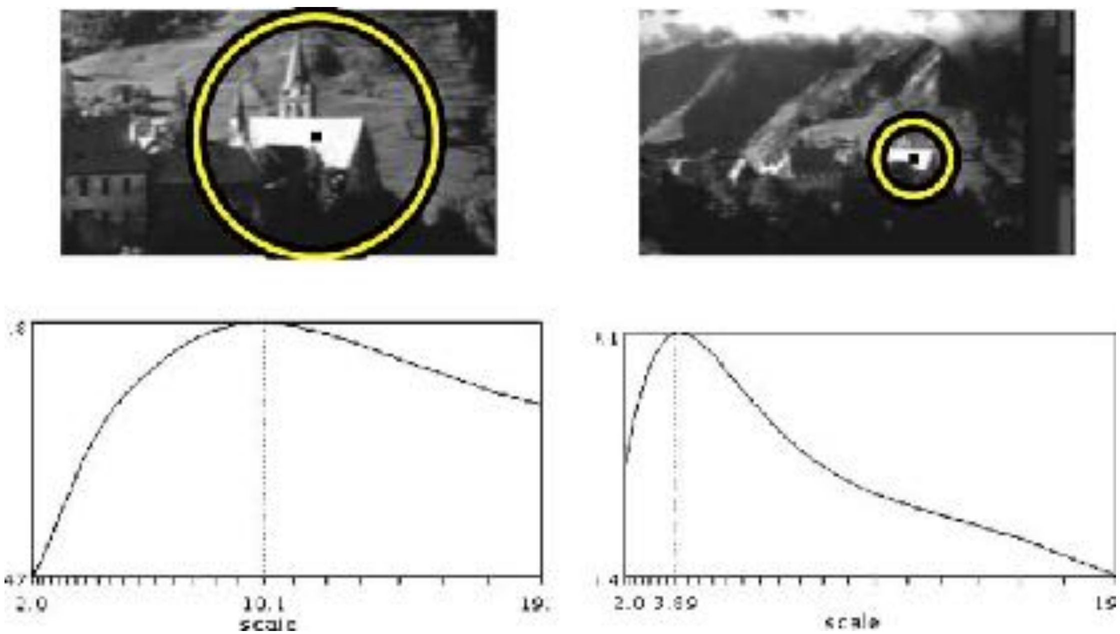


Illustration by (Mikolajczyk et al. 2005)

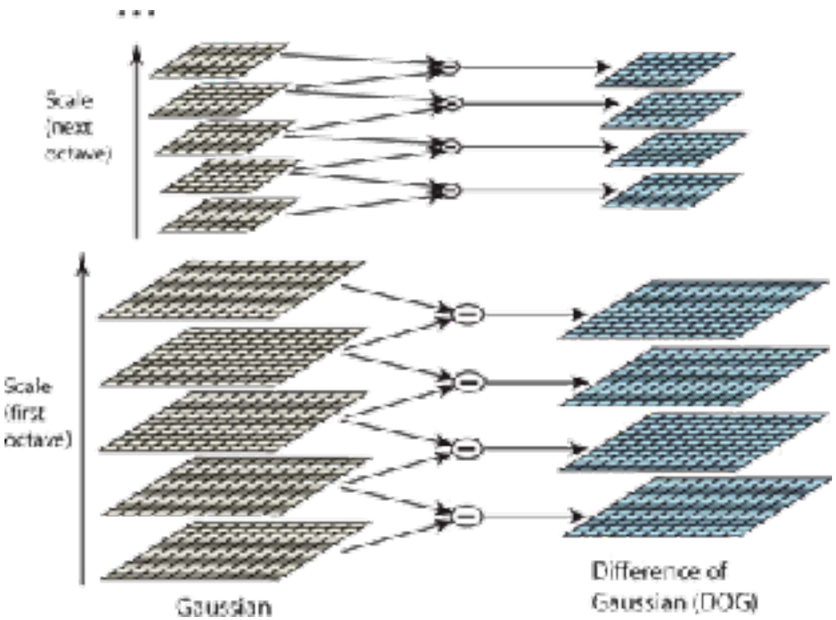
SIFT detector

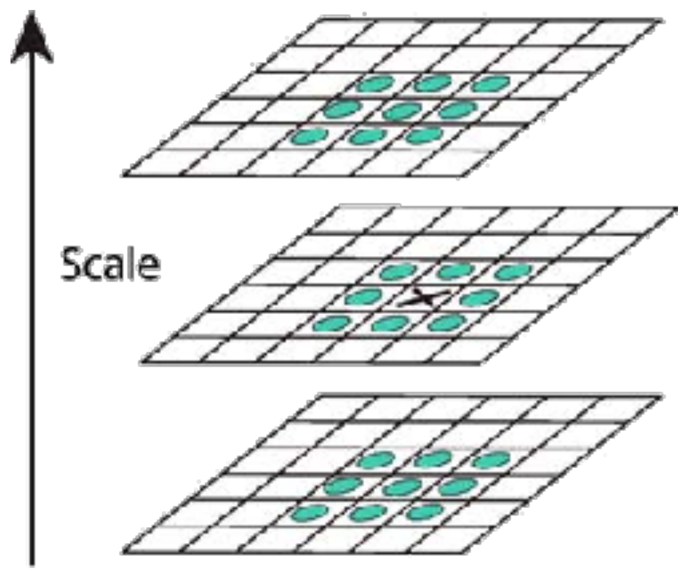
- In SIFT, scale selection is done using difference-of-Gaussians:

$$h_{\mathrm{SIFT}}(f(\mathbf{x},\sigma)) = (f * (g(\sigma) - g(k\sigma)))(\mathbf{x})$$

- Efficient implementation using pyramids [Lowe'99]
- Sampling in scale space with $\Delta\sigma = 1/\sqrt{2}$

SIFT detector



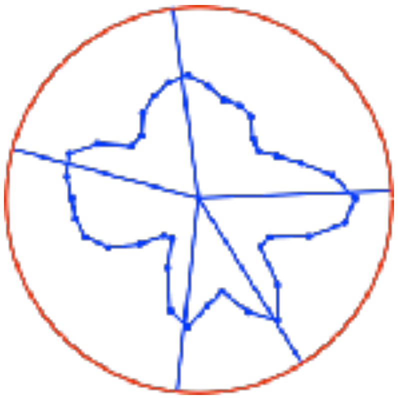


$$g(\sigma_1) * g(\sigma_2) = g\left(\sqrt{\sigma_1^2 + \sigma_2^2}\right)$$

Non-max suppression in (x,y,s)

SIFT detector

– Finally we find one or more reference directions using a gradient orientation histogram ***h*** at the found location in scale space.



$$h_k = \sum_{\text{patch}} |\nabla \mathbf{f}(\mathbf{x})| B_k(\tan^{-1} \nabla \mathbf{f}(\mathbf{x}))$$

SIFT descriptor

- The SIFT detector gives us a similarity frame. What is this?
  - We now want to convert the image patch at the frame to a 128-byte *descriptor vector*.
  - The purpose of this is to add photometric invariance, and some extra translation and scale robustness.

SIFT descriptor

– Compute x- and y-gradients through convolution:

$$\nabla \mathbf{f}(\mathbf{x}) = \begin{bmatrix} (d_x * f)(\mathbf{x}) \\ (d_y * f)(\mathbf{x}) \end{bmatrix}$$

– Rotate gradient map to direction from orient-hist:

$$\nabla \hat{\mathbf{f}}(\mathbf{x}) = \mathbf{R} \nabla \mathbf{f}(\mathbf{R}^{\mathbf{T}} \mathbf{x})$$

– Compute gradient orientation histograms in 4x4 spatial regions:

$$h_{kl} = \sum_{\mathbf{x} \in \text{patch}_l} |\nabla \hat{\mathbf{f}}(\mathbf{x})| w(\mathbf{x} + \mathbf{d}_l) B_k(\tan^{-1} \nabla \hat{\mathbf{f}}(\mathbf{x}))$$

SIFT descriptor

- Compute gradient orientation histograms in 4x4 spatial regions :

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- $B_k(x)$ linear interpolation kernel
  Quadratic is better (Jonsson&Felsberg)
- Subwindows $l \in [1 \ldots 16]$ directions $k \in [1 \ldots 8]$
- Spatial weight $w(\mathbf{x} + \mathbf{d}_l)$ (Gaussian decay)

SIFT descriptor

– Implementation with source code in both VLfeat and OpenCV.

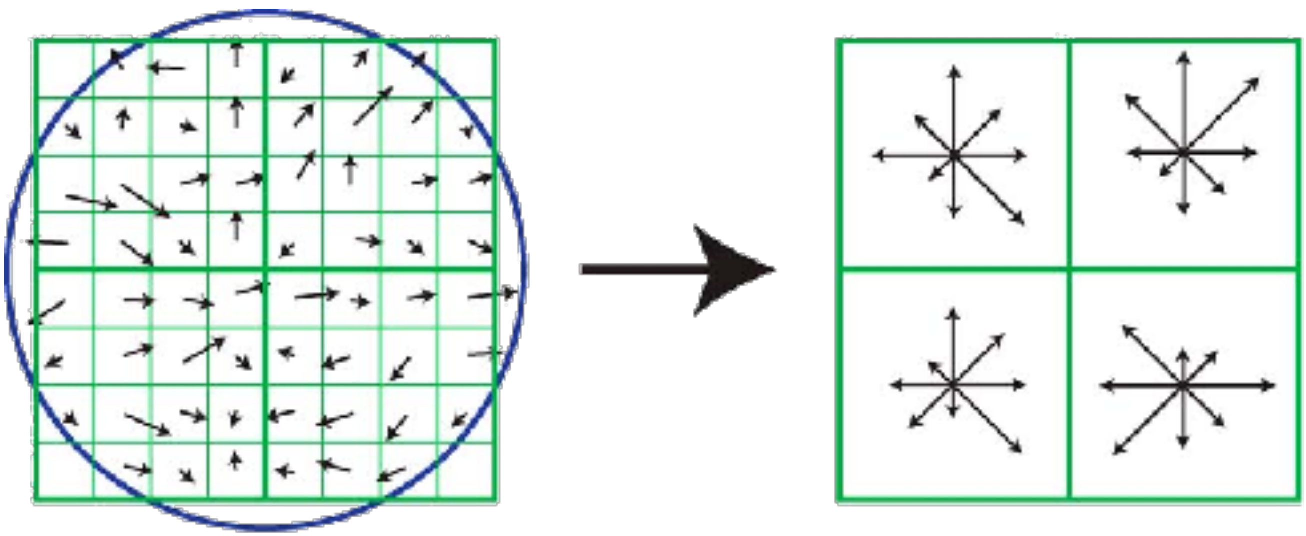

Image gradients Keypoint descriptor

Note that 4x4 regions are actually used, with 8 orientations -> 128 elements

SIFT descriptor

- Affine illumination invariance by using gradients and normalising descriptor $\hat{\mathbf{h}} = \mathbf{h}/||\mathbf{h}||$
- Some robustness by truncating and normalising again $\hat{\hat{\mathbf{h}}} = \min(\mathbf{t}, \hat{\mathbf{h}})/\|\min(\mathbf{t}, \hat{\mathbf{h}})\|$
- The spatial histogramming gives robustness to scale/rotation/translation errors.

SIFT descriptor

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- Some robustness by truncating and normalising again $\hat{\hat{\mathbf{h}}} = \min(t, \hat{\mathbf{h}})/||\hat{\mathbf{h}}||$
- The spatial histogramming gives robustness to scale/rotation/translation errors.
- SIFT is used commercially in many places. (The Sony AIBO anno 1999, was an early example.)

MSER

- Maximally Stable Extremal Regions [Matas et al.'02]
- Consider the set of all possible thresholdings of an image...

[Movie clip]

MSER

MSER

- Connected regions form segments.
  - Cf. Watershed algorithm (similar idea but different output)
  - Look at stability of a function of segment across image evolution. e.g.

$$\mathtt{area}(\mathtt{component}(t))$$

  - MSERs are components that are **maximally stable**, i.e., have a local minimum of the rate of change: $\frac{\partial \mathtt{area}(\mathtt{component}(\mathtt{t}))}{\partial t}$

MSER

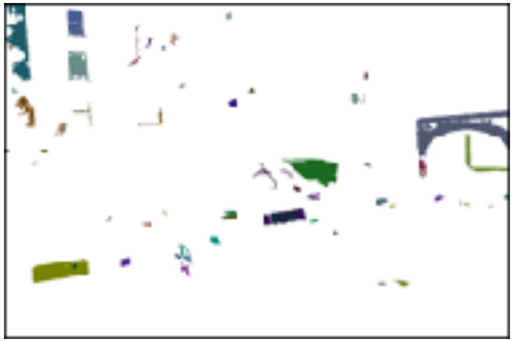
  - compare: Maximal Stability, Scale Selection
- Stability measure: Range of stable thresholds t_2-t_1 around min is called the *margin* of the region.

MSER

– Two possible thresholdings: $I(\mathbf{x}) < t$, $I(\mathbf{x}) > t$

Input image

64 MSER- (total 272)



64 MSER+ (total 294)

– Very fast (using union/find+path compression).

– MSER type (+/-) is useful for matching **How?**

MSER

- MSER is invariant to monotonic changes of intensity.
  i.e. I(x) and f(I(x)) have the same output if

$$f(x+k) > f(x) \; \forall \; k > 0$$

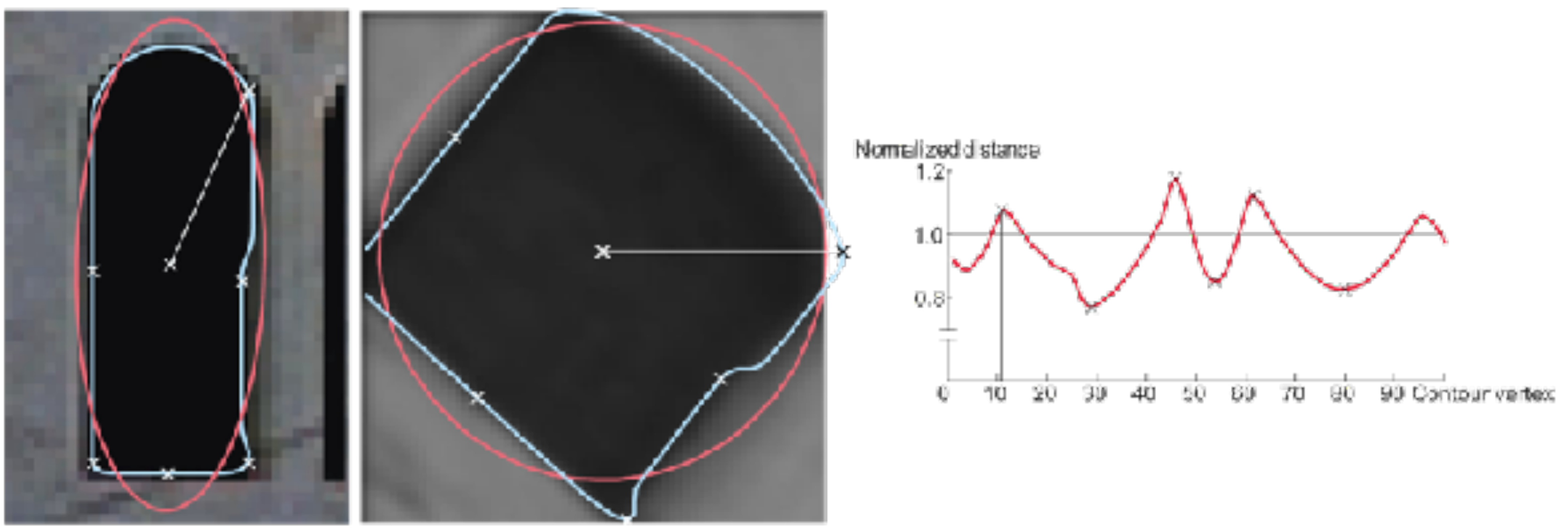
- Wide range of sizes obtained without a scale pyramid.
  Better still with a pyramid (Forssén&Lowe ICCV'07)
- Colour objects can be tracked by computing MSERs
  on the Mahalanobis distance to a colour distribution.
  (Donoser&Bischof CVPR'06)
- Colour regions by looking at gradients.
  Called MSCR (Forssén CVPR'07)

MSCR

MSCR

MSER

- Reference directions from extremal points along ellipse-normalized contour.



Matas et al. ICPR'02

MSER

- Approximating ellipse
  - from moments of binary mask $v : \Omega \mapsto \{0, 1\}$

$$\mu_{k,l}(v) = \sum_x \sum_y x^k y^l v(x, y)$$

$$\mathbf{m} = \frac{1}{\mu_{0,0}} \begin{bmatrix} \mu_{1,0} \\ \mu_{0,1} \end{bmatrix} \qquad \mathbf{C} = \frac{1}{\mu_{0,0}} \begin{bmatrix} \mu_{2,0} & \mu_{1,1} \\ \mu_{1,1} & \mu_{0,2} \end{bmatrix} - \mathbf{m}\mathbf{m}^T$$

$$\mathcal{R}(\mathbf{m}, \mathbf{C}) = \{\mathbf{x} : (\mathbf{x} - \mathbf{m})^T \mathbf{C}^{-1} (\mathbf{x} - \mathbf{m}) \leq 4\}$$

MSER

– Normalisation to a circle (axis aligned)
Compute the eigenfactorisation:

$$\mathbf{C} = \mathbf{R}\mathbf{D}\mathbf{R}^T, \quad \det \mathbf{R} > 0$$

The circle normalisation can now be performed as:

$$\mathbf{x} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{m}, \quad \text{for } \mathbf{A} = 2\mathbf{R}\mathbf{D}^{1/2}$$

$\hat{\mathbf{x}}$ - canonical coordinates
$\mathbf{x}$ - image coordinates

MSER

- Ellipse+extrema of distance to centre is just one frame construction option.
- Other (affine covariant) choices:
  - Points of maximum curvature.
  - Bi-tangens.
  - See Obdrzalek&Matas BMVC’02
- Implementation w. source: in both VLfeat and OpenCV

MSER descriptor

- The MSER detector originally used normalized colour patches as descriptor vectors:

$$
\begin{aligned}
\hat{I}_r(\mathbf{x}) &= (I_r(\mathbf{x}) - \mu_r)/\sigma_r \\
\hat{I}_g(\mathbf{x}) &= (I_g(\mathbf{x}) - \mu_g)/\sigma_g \\
\hat{I}_b(\mathbf{x}) &= (I_b(\mathbf{x}) - \mu_b)/\sigma_b
\end{aligned}
$$

- Nowadays other descriptors, e.g. the SIFT descriptor are used.

Other local invariant features

- **SFOP**
  http://www.ipb.uni-bonn.de/sfop/
- **BRISK**
  Source Code+description
  http://www.asl.ethz.ch/people/lestefan/personal/BRISK
- **FREAK and ORB**
  In OpenCV
- **SURF and SIFT**
  in OpenCV nonfree

Binary descriptors

- To save memory and time, many descriptors use **local binary patterns**:

Image from Alexandre et al. CVPR 2012

- sign of intensity difference has monotonic illumination invariance

Binary descriptors

- To save memory and time, many descriptors use **local binary patterns**:

Image from Alexandre et al. CVPR 2012

- E.g. **BRIEF** (ECCV'10), **BRISK** (ICCV'11), **ORB** (ICCV'11), **FREAK** (CVPR'12)

Deep learning descriptors

Examples:

- DeCAF (ArXiv'13) descriptors
- **LIFT** (ECCV'16) detector and descriptor

Better matching performance at the price of more expensive computations.

A note on invariance

- Always strive to limit amount of invariance
- Use knowledge on imaging situation,
  - e.g. A car mounted camera may not need rotation invariance for pedestrians.
  - e.g. in a video with smooth illumination changes, affine illumination invariance is not necessary

Descriptor Matching

- The ***Local Invariant Feature*** method:
- Detection
- Description
- **Matching**

Descriptor Matching

– For a descriptor $\boldsymbol{q}$ in a query image. Which prototype in memory ($\boldsymbol{p}_1,\boldsymbol{p}_2,...,\boldsymbol{p}_N$) is *most likely* to correspond to the same world object?

Descriptor Matching

- For a descriptor ***q*** in a query image. Which prototype in memory (***p***1,***p***2,...,***p***N) is *most likely* to correspond to the same world object?
- Assuming additive i.i.d. Gaussian noise on all elements:

$$p(\mathbf{q}|\mathbf{p}_k) \propto \prod_{l=1}^{D} e^{-.5(p_{kl}-q_l)^2/\sigma^2}$$

$$\max(J) \Leftrightarrow \min(-\log(J))$$

$$-\log(p(\mathbf{q}|\mathbf{p}_k)) \propto \sum_{l=1}^{D} (p_{kl}-q_l)^2$$

Descriptor Matching

– So, the match with smallest distance is most likely correct, assuming i.i.d. Gaussian noise.
– What about the scalar product for normalised vectors/NCC?

Descriptor Matching

– So, the match with smallest distance is most likely correct, assuming i.i.d. Gaussian noise.

– What about the scalar product for normalised vectors/NCC?

$$||\mathbf{p} - \mathbf{q}||^2 = \mathbf{p}^T\mathbf{p} + \mathbf{q}^T\mathbf{q} - 2\mathbf{p}^T\mathbf{q} = 2(1 - \mathbf{p}^T\mathbf{q})$$

– But are all values identically distributed?

– ...are they all independent?

Descriptor Matching

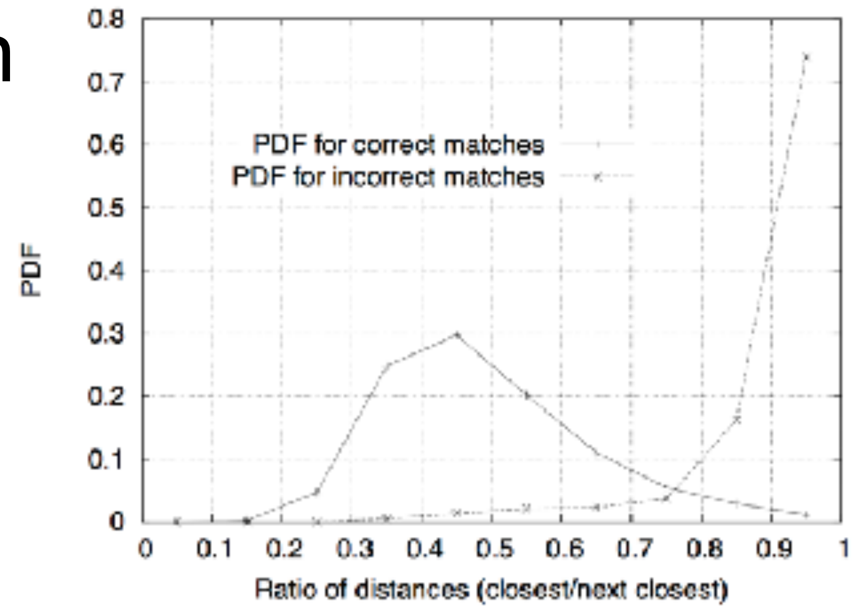
- For binary descriptors (e.g. **BRIEF**) the Hamming distance is used:

$$s = \text{bitcnt}(\text{XOR}(P,Q))$$

- Also makes i.i.d. assumption.

Ratio score

Risk of mismatch can also be taken into account by looking at the ratio of the best and second best match.



$$p(r|\text{correct}) \;\; \text{and} \;\; p(r|\text{incorrect})$$

$$r = d_{\text{min}} / d_{\text{second\_smallest}}$$

Dense invariant features

- (semi-)dense flow for wide baseline problems can be obtained by matching invariant features
- located **at every pixel** and
- also at several scales
- e.g. **SIFTflow**, **DSIFT**, **PHOW**, **DAISY**
- Much more expensive to compute. GPGPU etc. is helpful here.

Summary

- Use local invariant features:
  when KLT fails
- But use no more invariance than needed
- Two types of invariance: **Photometric** and **Geometric** invariance
- Recognition in three steps: **Detection**, **Description** and **Matching**

Upcoming course events

- Lab 1: Checkup tomorrow 13.00-14.30 in Olympen.
- Next Lecture (20/2 10-12)
  Biological vision. Voluntary.
  Based on PhD course on Biological Vision Systems.