Geometry for Computer Vision

Lecture 6b

Sample Consensus Strategies

Per-Erik Forssén

Overview

- Issues with standard RANSAC
- Maximum-likelihood scoring
- LO-RANSAC
- Preemptive RANSAC
- DEGENSAC

Not covered here: All the other variants RANDOMIZED RANSAC, G-SAC, NAPSAC etc.

Paper to read for next week: PROSAC

RANSAC issues

In lecture 3 we introduced RANSAC (Fischler&Bolles 81).

It finds a **model with maximal support** in the presence of **outliers**

Approach: randomly **generate hypotheses** and **score** them.

Most novelties since 1981 covered in thesis by: **Ondrej Chum, *Two-View Geometry Estimation by Random Sample and Consensus*, July 2005**

RANSAC issues

```
S - correspondence set, K - number of trials
  for k=1:K
      s=sample_draw_minimal(S)
      m=model_estimate(s)
      [v,inliers]=model_score(m,S)
      if v>best_v
          best_inlier_set=inliers
          best_v=v
          best_m=m
      end
  end
```

RANSAC issue #1

Underlying optimization problem:

$$\varepsilon = \sum_k \sum_l v_k ||\mathbf{x}_{kl} - \mathrm{proj}(\mathbf{X}_k, \theta_l)||^2 \quad v_k \in \{0, 1\}$$

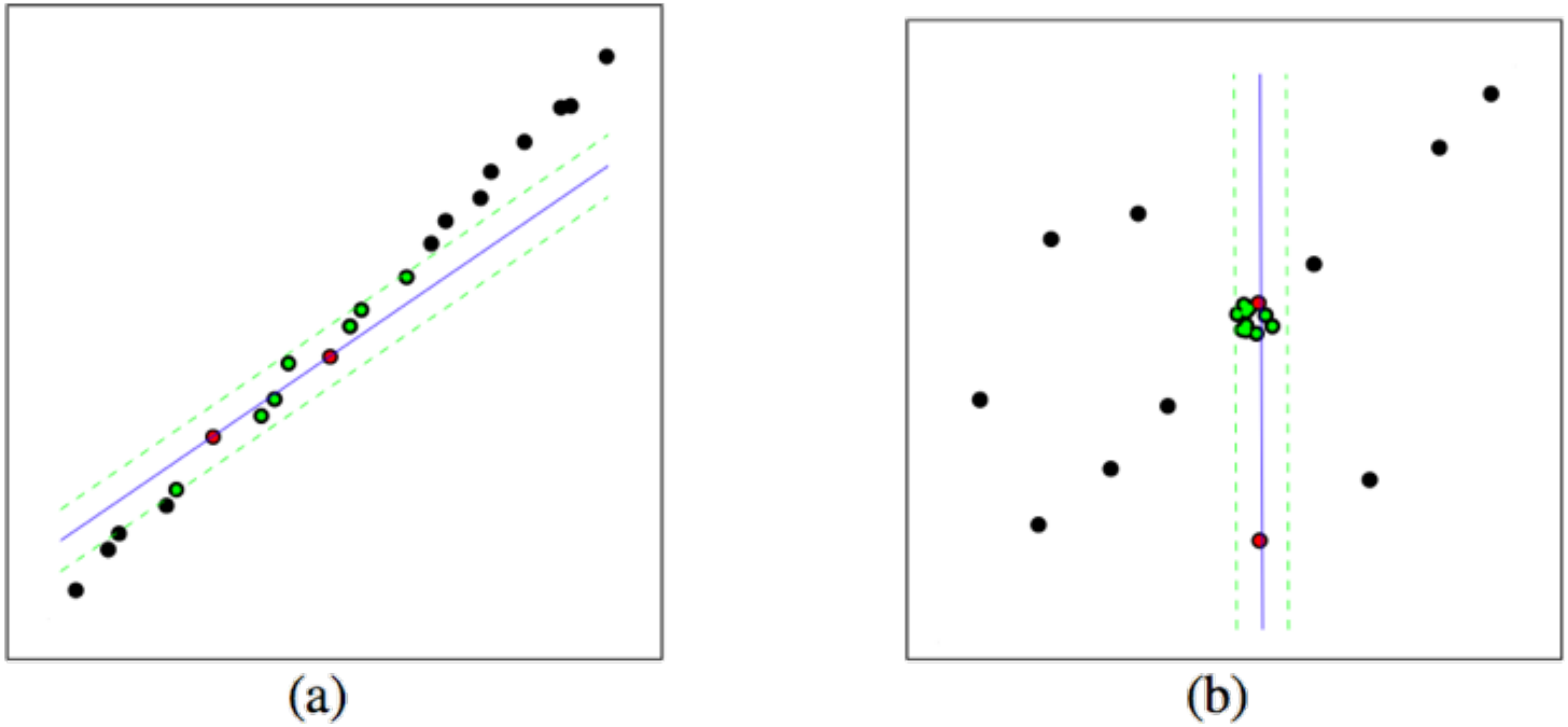
score is a discrete inlier count:

$$s = \sum_k v_k$$

there may be many equally good optima!

RANSAC issues #2 and #3

Two more problems with the original approach:



Inlier noise

Near degeneracies

RANSAC issue #3

Near degeneracies can be dealt with by sampling non-randomly, e.g.

- DEGENSAC, for **F** estimation in plane dominant scenes. **Chum et al., *Two-view Geometry estimation unaffected by a Dominant Plane*, CVPR05**
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- DEGENSAC, for **F** estimation in plane dominant scenes. **Chum et al., *Two-view Geometry estimation unaffected by a Dominant Plane*, CVPR05**
- Distance constraint for points used in **E** estimation. **Hedborg et al., *Fast and Accurate Structure and Motion Estimation*, ISVC09** Reduces #iterations by 50% in forward motion.

Maximum likelihood scoring

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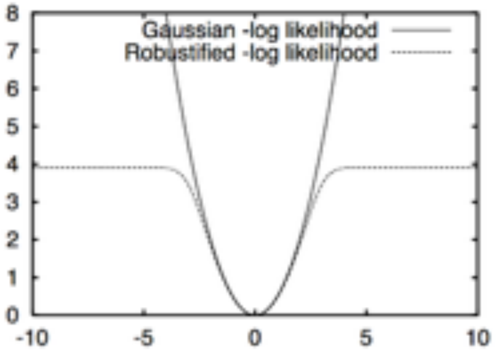
Maximum likelihood scoring

Better to use a robust error norm:

$$\varepsilon = \sum_k \sum_l \rho(\mathbf{x}_{kl} - \mathrm{proj}(\mathbf{X}_k, \theta_l))$$



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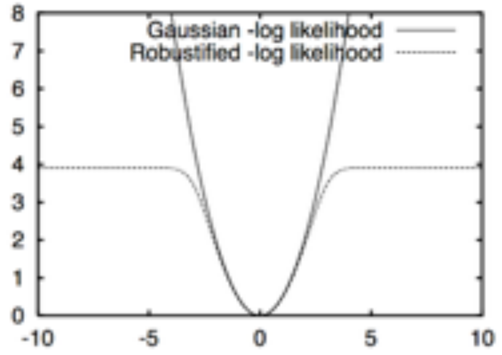

and as score, the minimum error (MLESAC):

$$s = -\varepsilon$$

Maximum likelihood scoring

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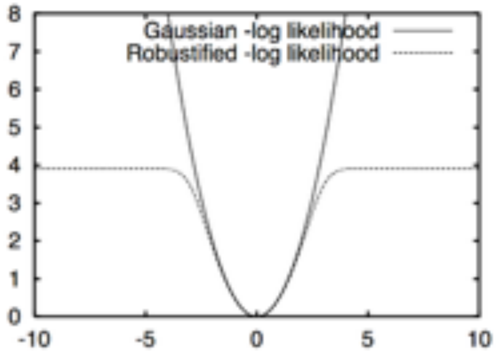

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and as score, the minimum error (MLESAC):

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called **maximum likelihood scoring**

See: Torr & Zisserman, *MLESAC: A new robust estimator with application to estimating image geometry*, CVIU’00

LO-RANSAC

Inlier noise means that the heuristic for number of samples to draw:

$$N = \log(1-p)/\log(1-w^s)$$

is overly optimistic.

A small modification makes the heuristic work again: **Chum et al., *Locally Optimized RANSAC*, DAGM03**

LO-RANSAC

Small modification:

```
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   for k=1:K
       s=sample_draw_minimal(S)
       m=model_estimate(s)
       [v,inliers]=model_score(m,S)
       if v>best_v
           [inliers,v,m]=local_optimization(inliers,v,m)
           best_inlier_set=inliers
           best_v=v
           best_m=m
       end
   end
```

LO-RANSAC

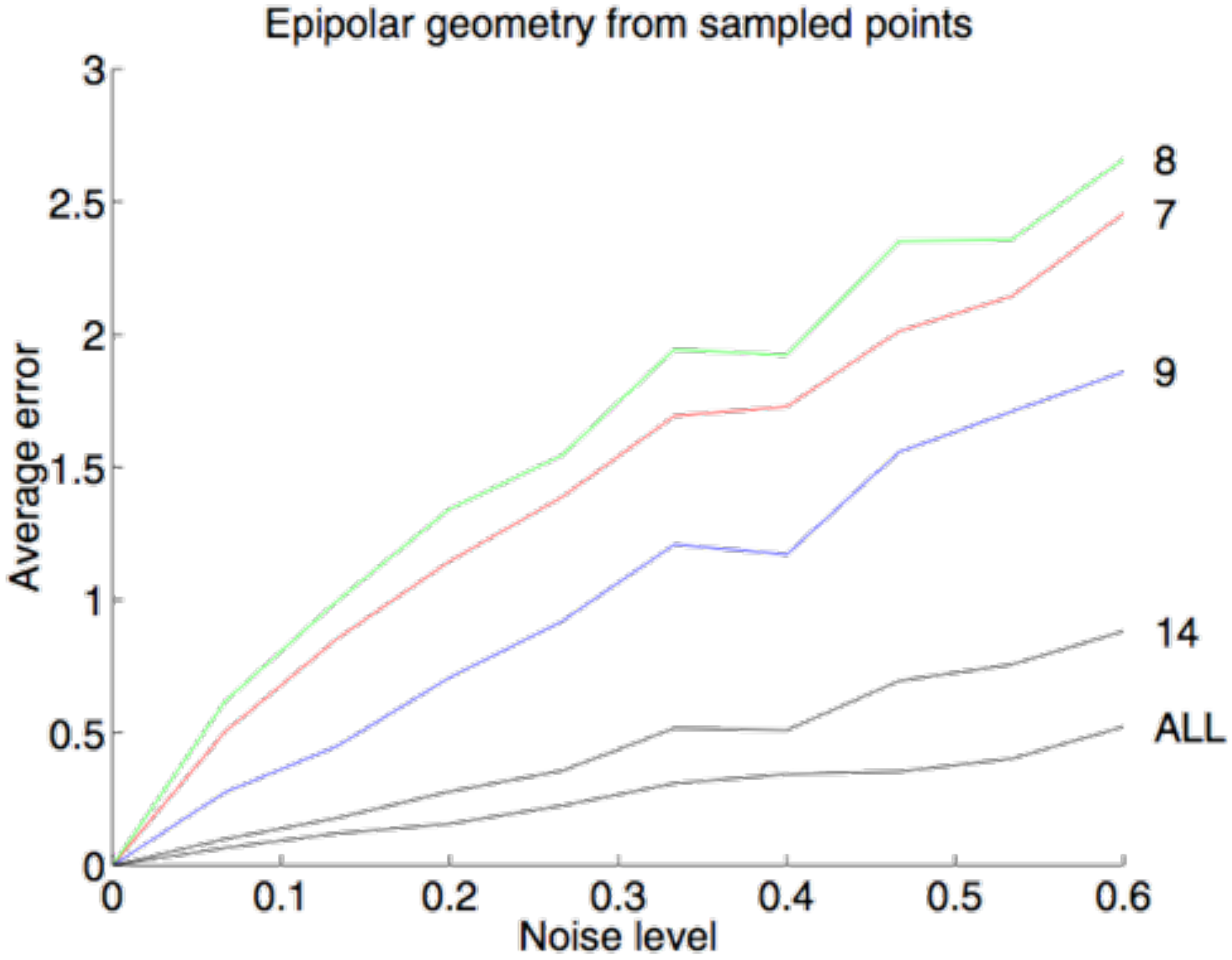
Chum tries four variants of local optimisation:

1. Linear estimation from all inliers
2. Iterative linear estimation with decreasing inlier threshold.
3. Inner RANSAC
4. Inner RANSAC with #2.

#3 and #4 worked best, and #4 came close to the heuristically expected #samples.

LO-RANSAC

The inner RANSAC step uses non-minimal sample sets. Errors for linear **F** estimation:



Open source

Lebeda, Matas, Chum, *Fixing the Locally Optimized RANSAC*, BMVC’12

C/C++ library available on the web. Source code on request. The ”Fixes” are:

1. Maximum likelihood scoring (i.e. MLESAC)
2. Iterative Reweighted Least-Squares on random subsets of bounded size (7xmin_subset)

Preemptive Ransac

David Nister, *Preemptive RANSAC for live structure and motion estimation,* ICCV03

Total time for RANSAC is given by:

$$t = k(t_M + E[m_S]t_V)$$

k- #iterations t_M-model estimation time, t_V-verification time. m_S - #models/iteration

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If many correspondences, t_V will dominate.

Preemptive Ransac

Idea: Do a probabilistic verification instead.

$$t = k(t_M + E[m_S]t_V)$$

- In a **real-time system**, t is fixed, so if we reduce t_V we may increase k.
- Preemptive RANSAC does this by evaluating all hypotheses in parallel.
- In each step, a fixed number of most promising hypotheses are kept.

Preemptive Ransac

Preemptive RANSAC:

```
1. Generate f(1) hypotheses in parallel.
2. For n=1 to N
3.      Evaluate f(n) hypotheses on
        a random correspondence
4.      Keep the f(n+1) best hypotheses
        according to accumulated score.
```

f(1)=M and $f(n+1) \leq f(n)$

Preemptive Ransac

f(n) - the preemption function

$$f(n) = \lfloor M 2^{-\lfloor \frac{n}{B} \rfloor} \rfloor$$

B - block size (f only changes every B steps)

M - number of models

Accumulated scoring $L(m) = \sum_{n=1}^{N} \rho(n, m)$

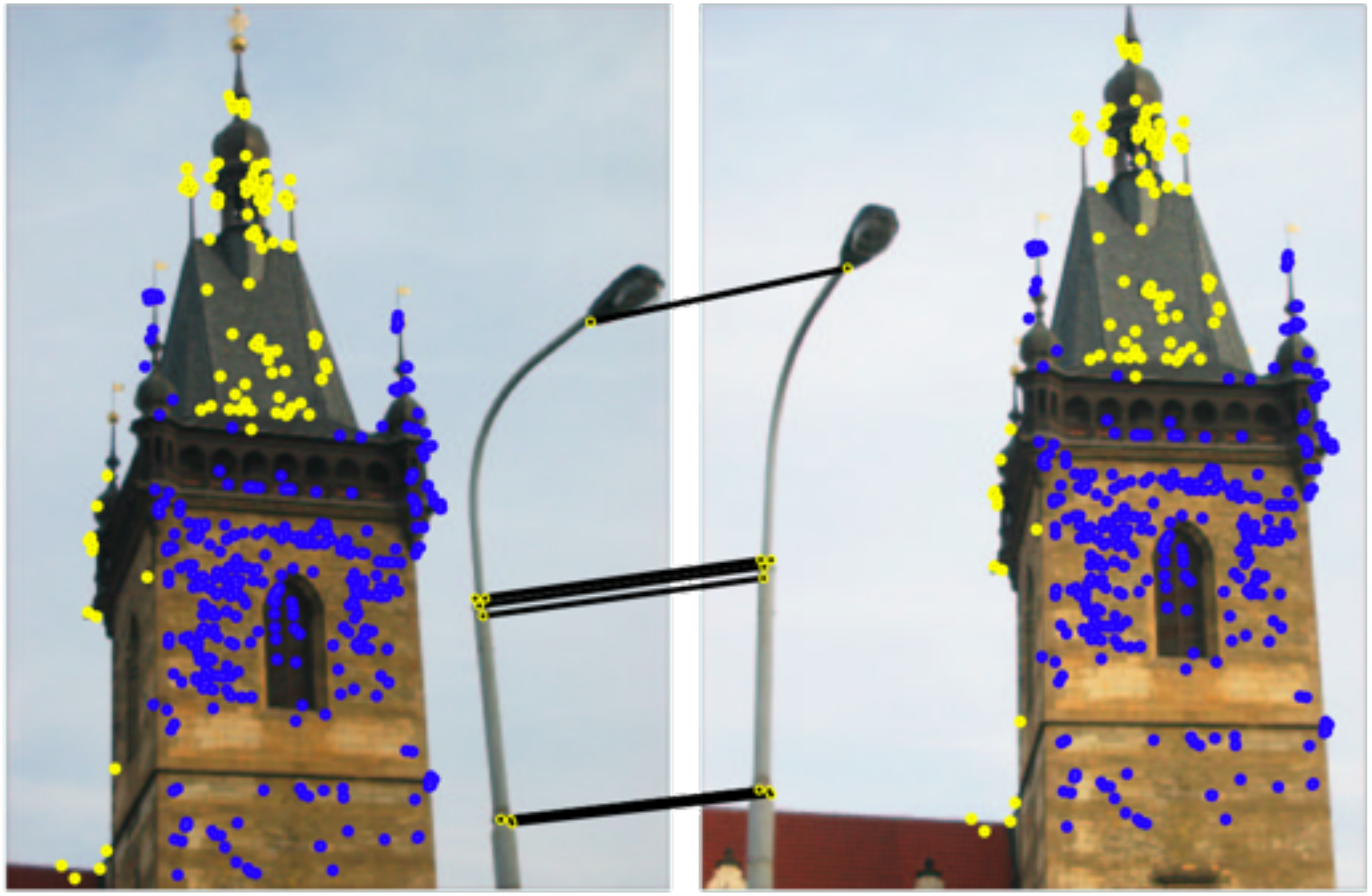
Log-likelihood of sample *n* given model *m*

$$\rho(n, m)$$

Degensac

Chum, et al., *Two-view Geometry Estimation Unaffected by a Dominant Plane*, CVPR’05

Planar dominant scenes are also problematic



Degensac

Actually, the **F** estimation problem is even worse than it might appear, as 5 points in a plane +2 **arbitrary** correspondences gives an **F** compatible with the plane.

Degensac

Actually, the **F** estimation problem is even worse than it might appear, as 5 points in a plane +2 ***arbitrary*** correspondences gives an **F** compatible with the plane.

In le5 we saw that if all seven points are in a plane, then

$$\mathbf{x}_k^T \mathbf{F} \mathbf{y}_k = 0\,,\ \mathbf{x}_k = \mathbf{H}\mathbf{y}_k\,, \quad k = 1 \ldots 7$$

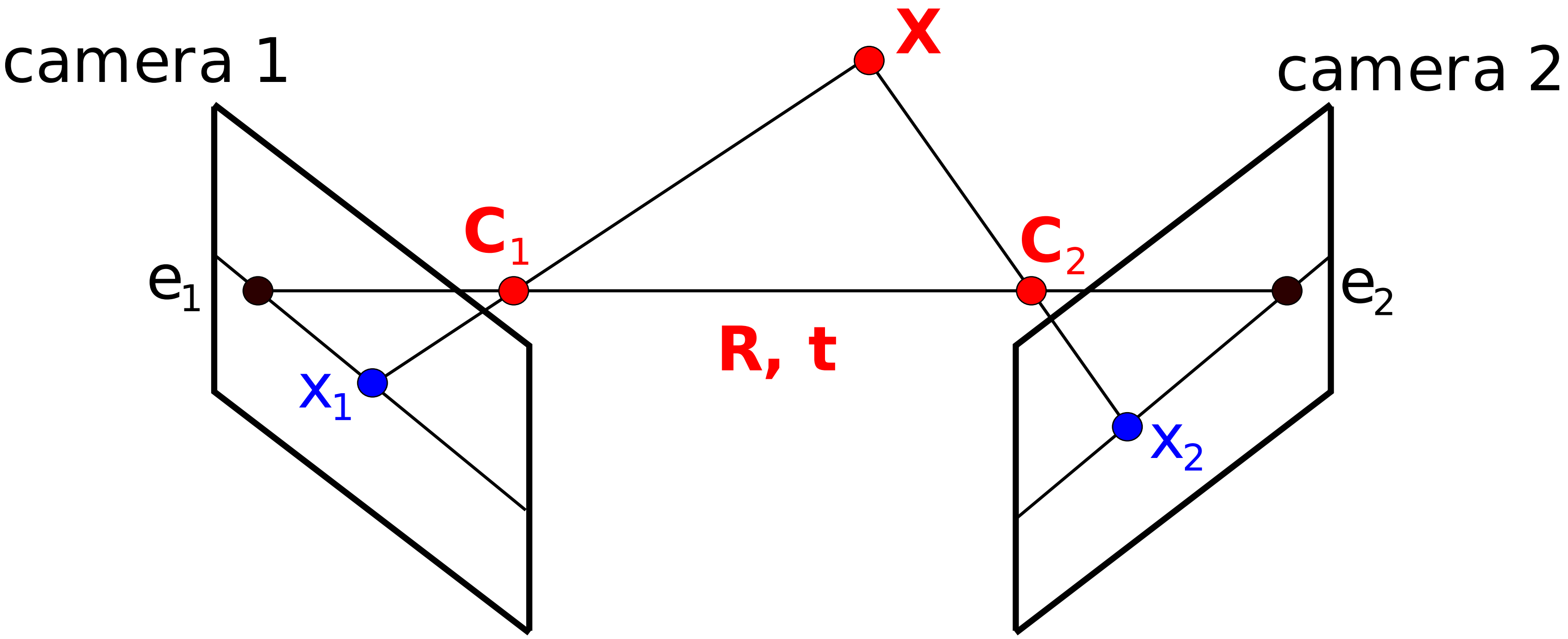
and $\mathbf{F} = [\mathrm{e}]_\times \mathbf{H}$ for any epipole **e**

Degensac

If ***six*** points are in a plane

$$\mathbf{x}_k^T \mathbf{F} \mathbf{y}_k = 0\,, \quad k = 1 \ldots 7 \quad \mathbf{x}_k = \mathbf{H}\mathbf{y}_k\,, \quad k = 1 \ldots 6$$

$$\mathbf{F} = [\mathbf{e}]_\times \mathbf{H} \ \text{ for } \ \mathbf{e} \in \mathbb{R}^3\,,\ \mathbf{e}^T(\mathbf{H}\mathbf{x}_7 \times \mathbf{y}_7) = 0$$



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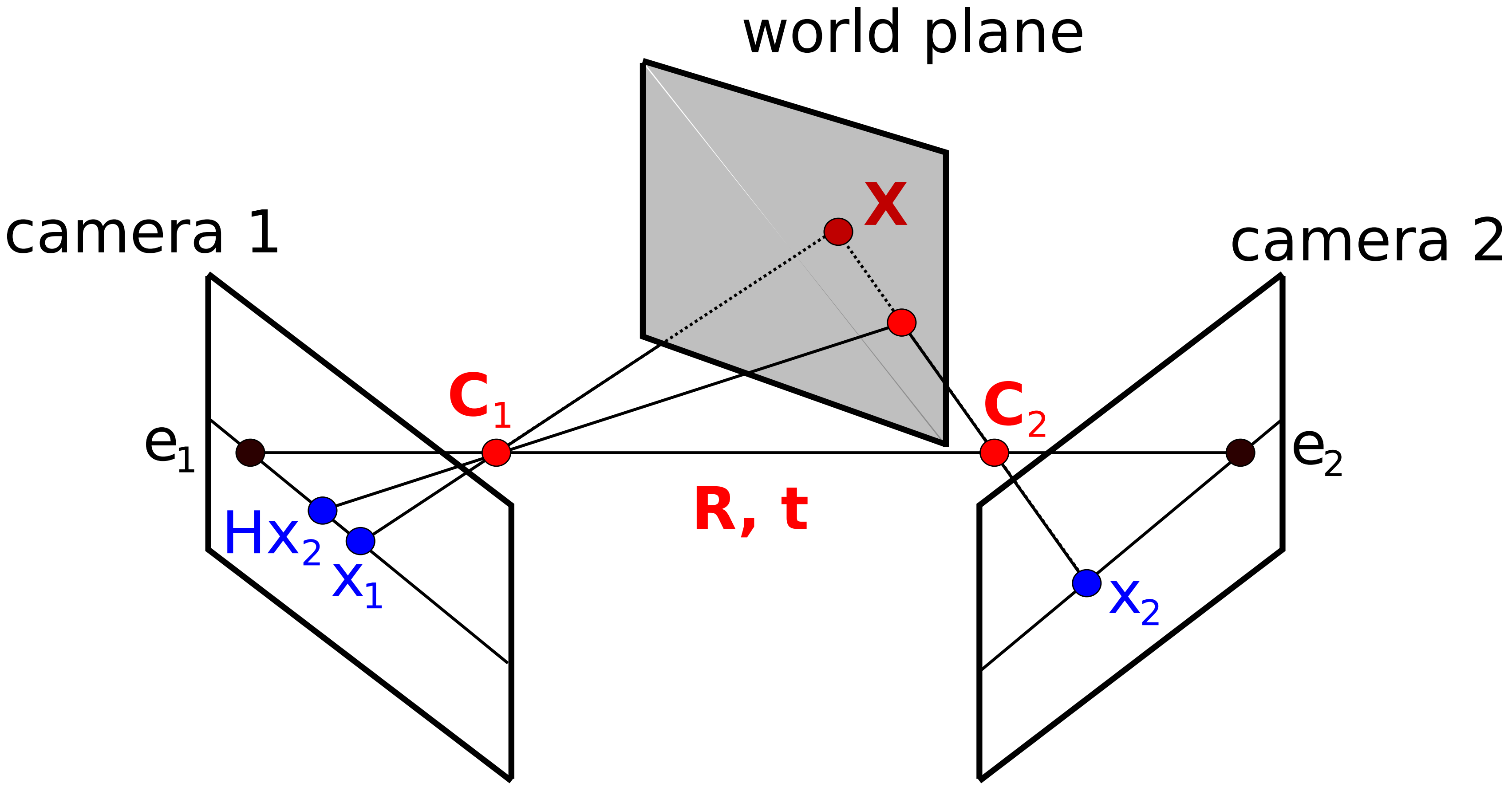

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For ***five*** points in the plane

$$\mathbf{x}_6 \times (\mathbf{H}\mathbf{y}_6) \quad \text{and} \quad \mathbf{x}_7 \times (\mathbf{H}\mathbf{y}_7)$$

define two lines that intersect in **e**. **F** will have all points consistent with **H** as inliers.

Degensac

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Also used in plane+parallax algorithm

Degensac - detection

From **F** and $\{\mathbf{x}_k \leftrightarrow \mathbf{y}_k\}_{k=1}^{3}$ we can compute a homography $\mathbf{H} = \mathbf{A} - \mathbf{e}_1(\mathbf{M}^{-1}\mathbf{b})^T$

where $\mathbf{A} = [\mathbf{e}_1]_\times \mathbf{F}$ $\mathbf{M} = [\mathbf{x}_1 \;\; \mathbf{x}_2 \;\; \mathbf{x}_3]^T$

and $b_k = (\mathbf{x}_k \times \mathbf{A}\mathbf{y}_k)^T(\mathbf{x}_k \times \mathbf{e}_1)||\mathbf{x}_k \times \mathbf{e}_1||^{-2}$

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This **H** is now checked for two additional inliers. If found, **F** is said to be **H**-degenerate

Degensac - detection

There are $\binom{7}{5} = 21$ ways to pick five points from 7.

But, if we pick the 3 points that define **H** as

$$\{1,2,3\},\{4,5,6\},\{1,2,7\},\{4,5,7\},\{3,6,7\}$$

We will have covered all 21 permutations.

Thus at most five **H** need to be computed and tested to find out if **F** is **H**-degenerate.

Degensac

DEGENSAC algorithm

```
1. Select 7 random correspondences and estimate $\mathbf{F}$
2. IF best support this far
3.     IF $\mathbf{H}$-degeneracy
4.         Do inner RANSAC and estimate $\mathbf{F}$
           from $\mathbf{H}$ and 2 correspondences
           that are inconsistent with $\mathbf{H}$
           (Plane+Parallax algorithm)
5.         IF new $\mathbf{F}$ has even bigger support, store $\mathbf{F}$
6.         ELSE store $\mathbf{H}$
```

Paper to discuss next week…

Ondrej Chum and Jiri Matas, *Matching with PROSAC -- Progressive Sample Consensus*, CVPR’05