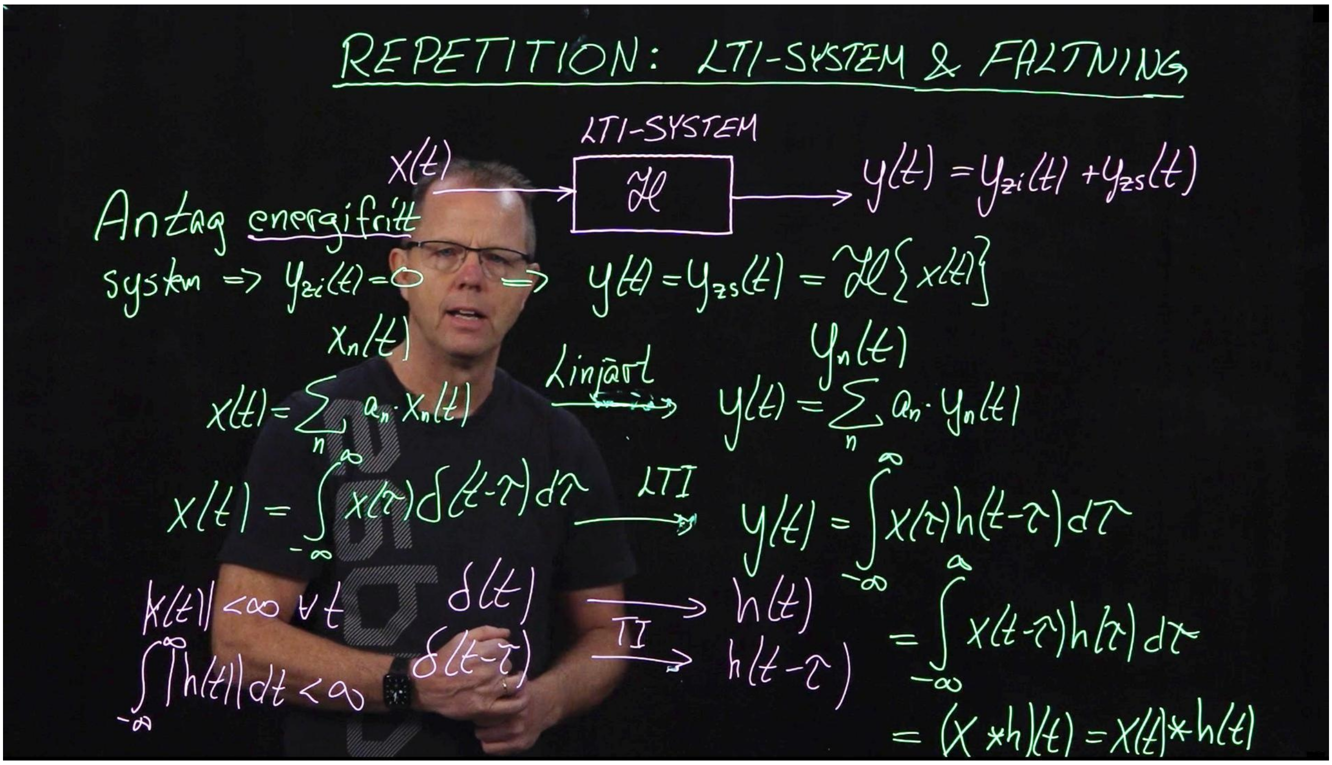
Linjära System – Föreläsning 12: Fouriertransformanalys – Inledning

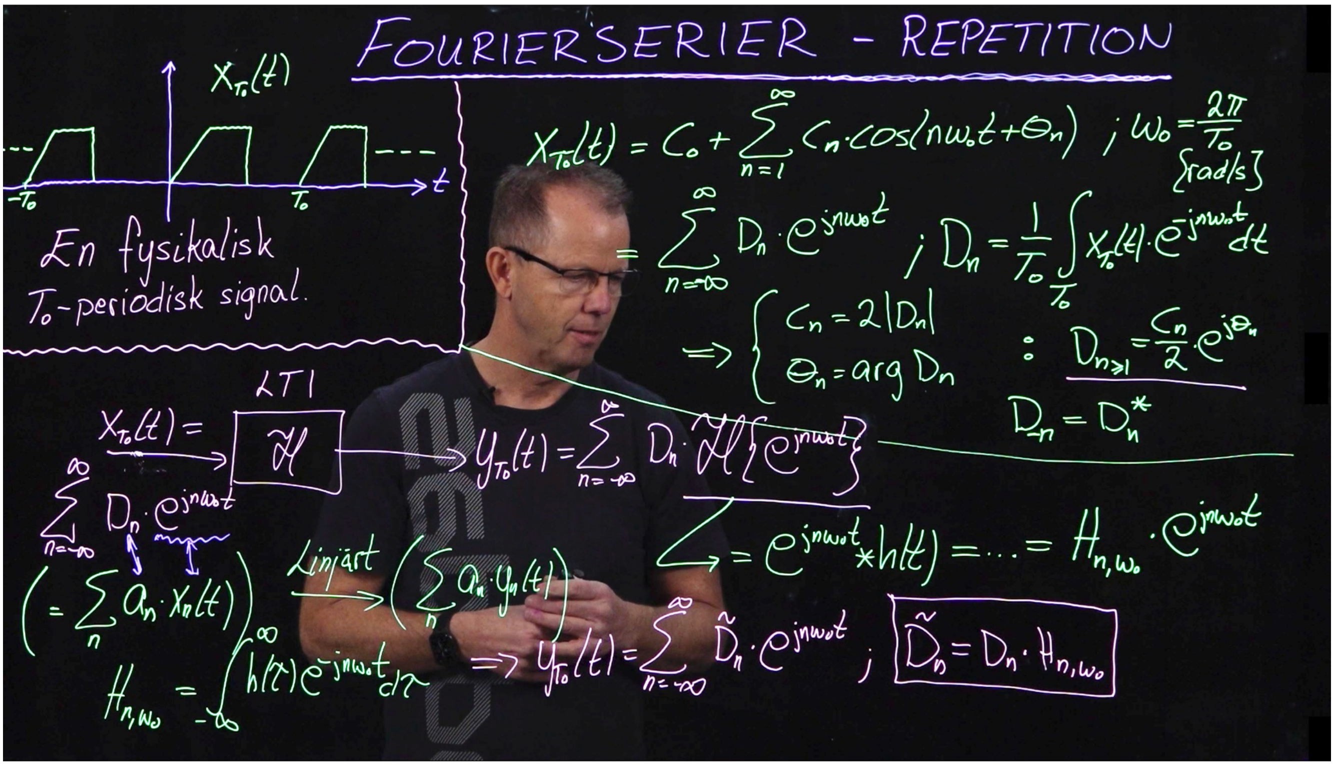
- Repetition, faltning
- Repetition, fourierserier

- Fouriertransformen – inledning

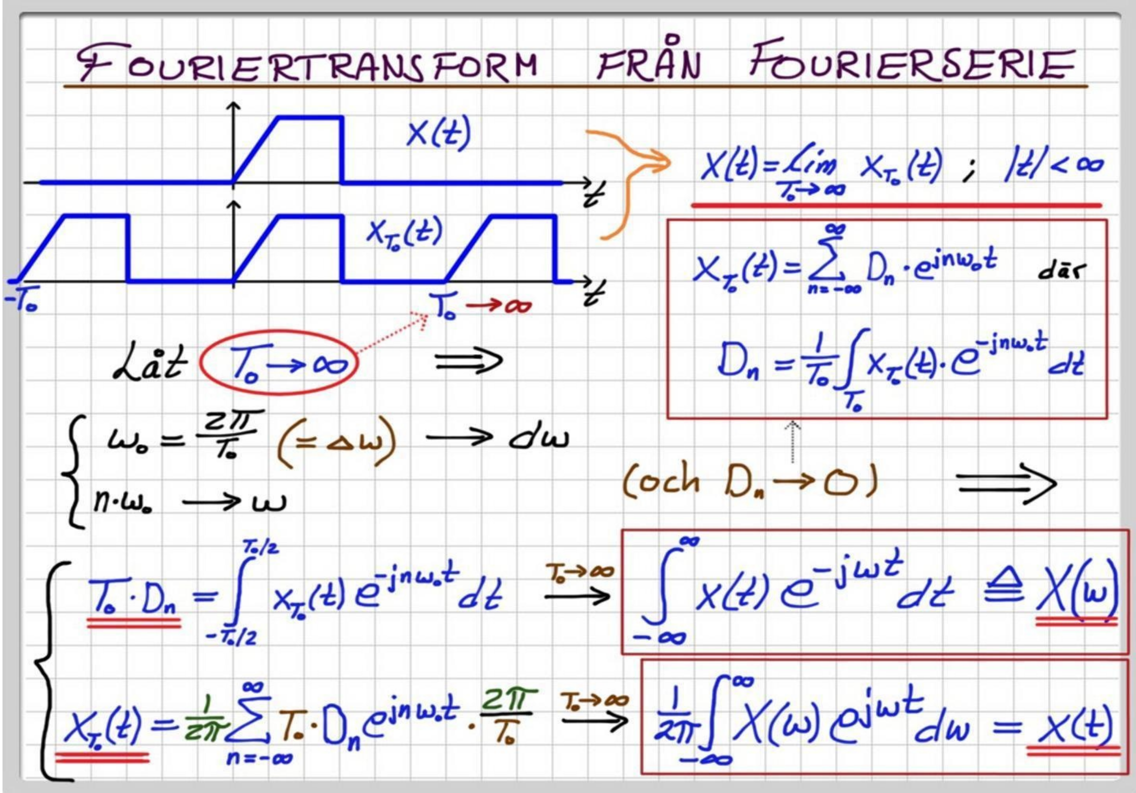


VIDEO 1

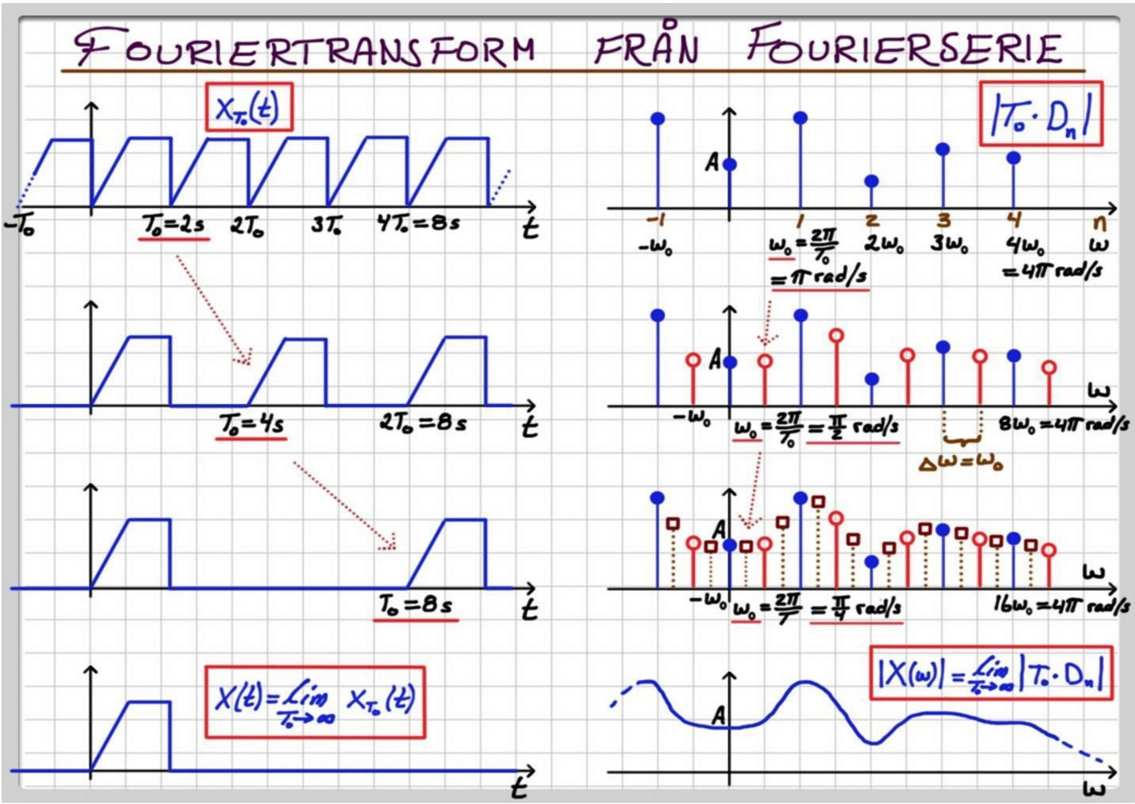
VIDEO 2


FOURIERSERIER - REPETITION
En fysikalisk
T0-periodisk signal.
LTI
Linjärt

Fouriertransformen från fourierserien



VIDEO 3



VIDEO 4

Konsekvens, om $x_{T_0}(t)$ erhålls genom en periodisering av $x(t)$:

$$D_n = \frac{1}{T_0} X(\omega)\Big|_{\omega = n\omega_0} = \frac{1}{T_0} X(n\omega_0)$$

Sammanfattning – Fouriertransformen

(*härledning – se videoklipp*)

- Fouriertransformen till $x(t)$:

$$\boxed{\mathcal{F}\left\{x(t)\right\} = X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

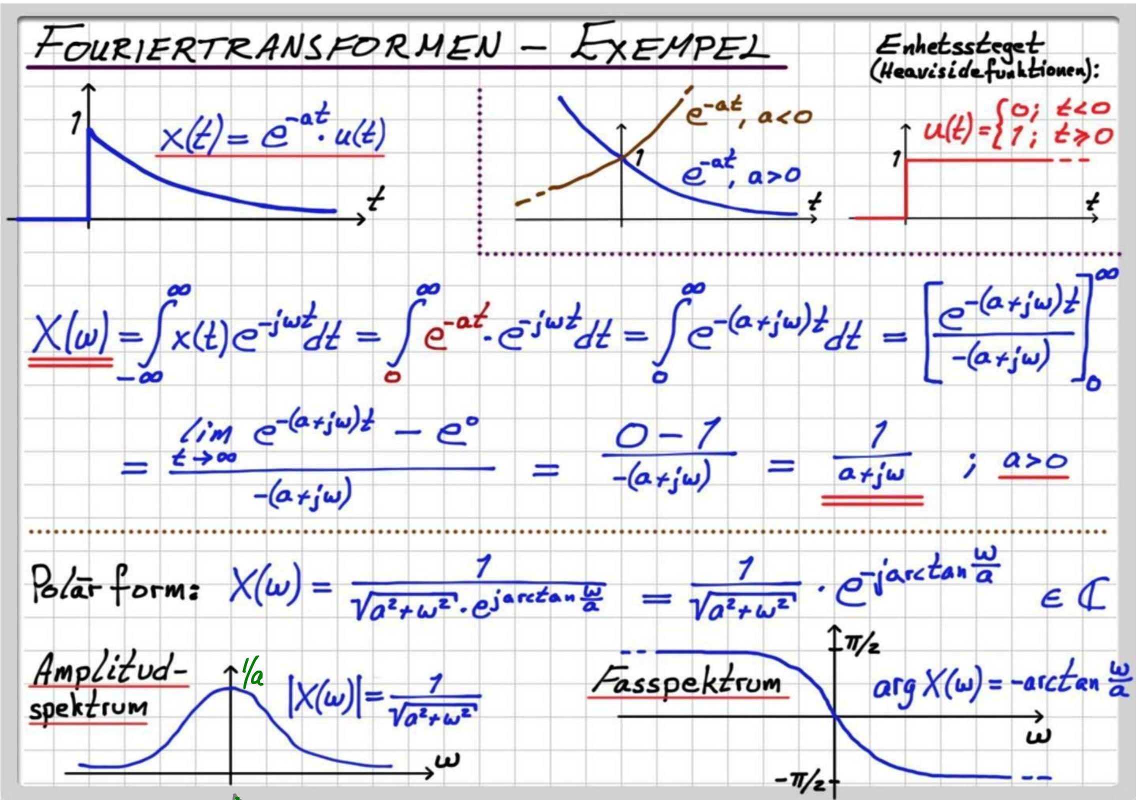
$\left(\text{Jfr. fourierserie: } D_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x_{T_0}(t) e^{-jn\omega_0 t} dt \right)$

- Inversa fouriertransformen till $X(\omega)$:

$$\boxed{\mathcal{F}^{-1}\left\{X(\omega)\right\} = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega}$$

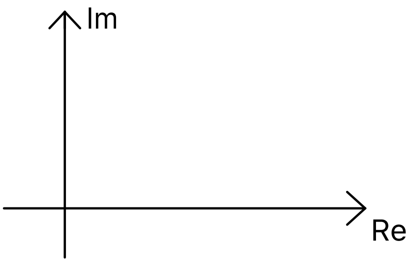
$\left(\text{Jfr. fourierserie: } x_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \right)$

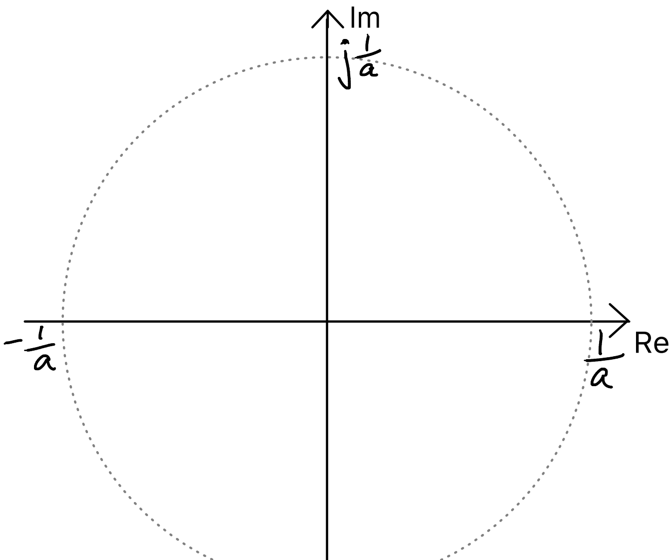
Existensvillkor: $\boxed{\mathcal{F}\left\{x(t)\right\} \exists \text{ om } \int_{-\infty}^{\infty} \left|x(t)\right| dt < \infty}$

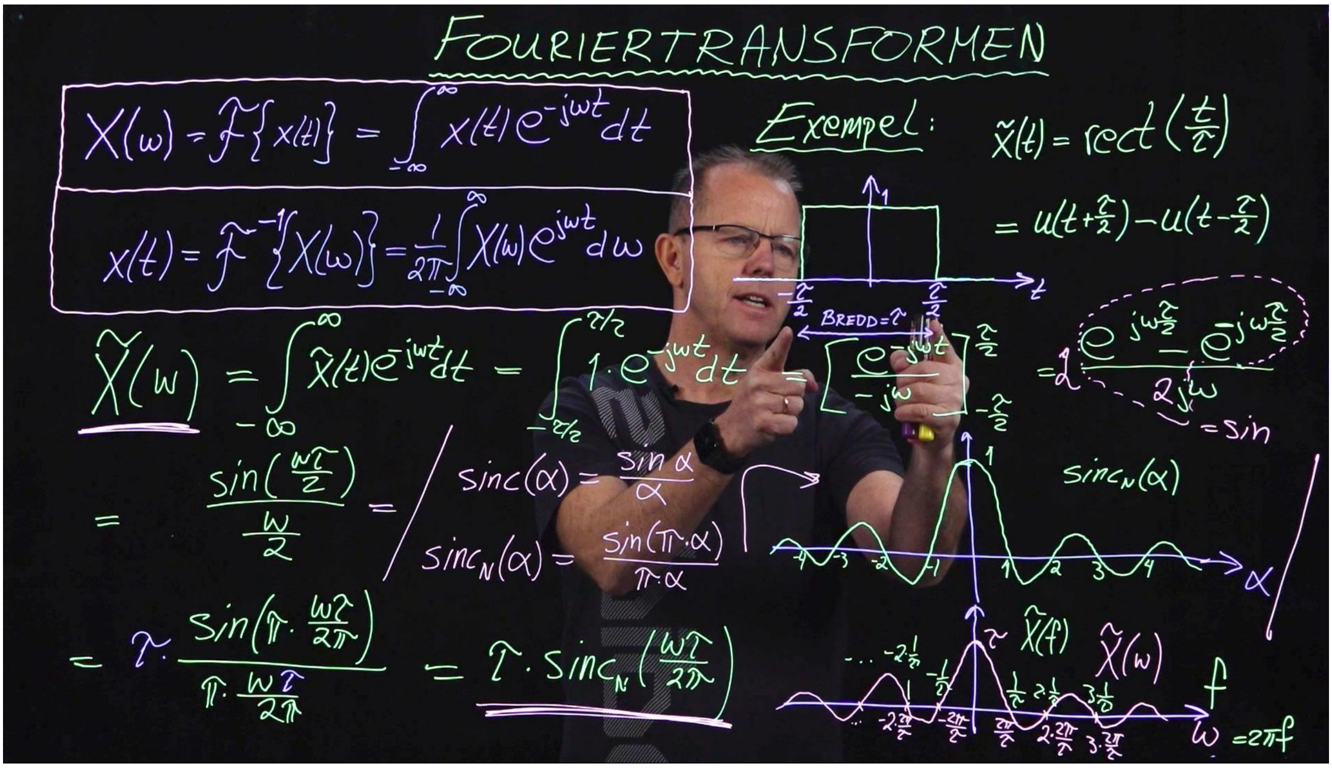


VIDEO 5

$\frac{1}{a}$ saknas i videon!

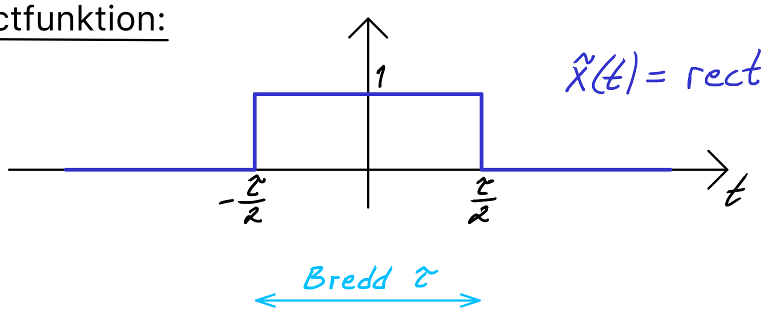


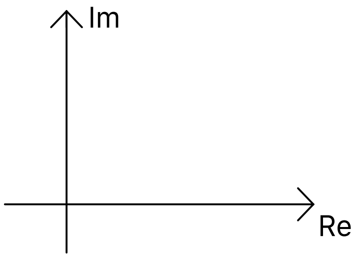




Tidssignal – rectfunktion:

$\tilde{x}(t) = \text{rect}\left(\frac{t}{\tau}\right)$

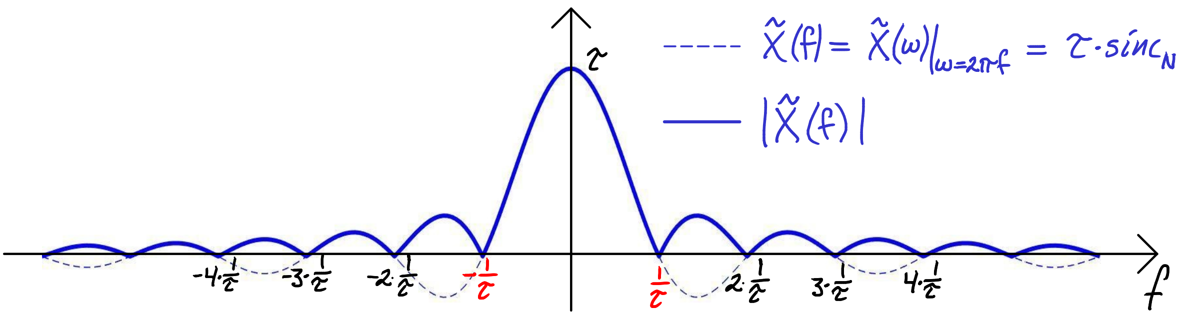




Amplitudspektrum: $|\tilde{X}(f)|$

$$\text{-----}\ \tilde{X}(f) = \tilde{X}(\omega)\big|_{\omega=2\pi f} = \tau\cdot\text{sinc}_N\left(\frac{2\pi f\tau}{2\pi}\right) = \tau\cdot\text{sinc}_N(f\tau)$$

$$\text{—}\ |\tilde{X}(f)|$$

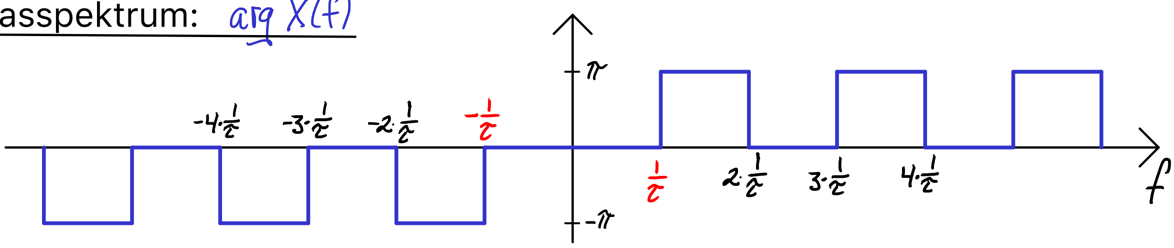


Sinc-funktionen är noll i multiplar av

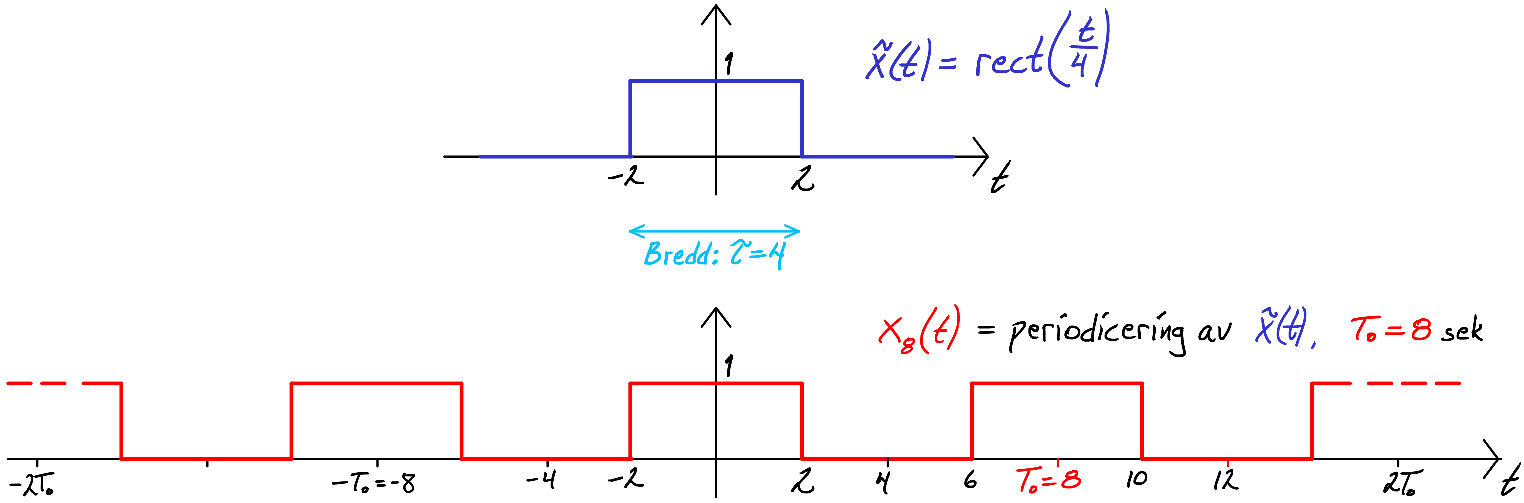
$f_z = \frac{1}{\text{pulsbredden}}$

$\left(\omega_z = \frac{2\pi}{\text{pulsbredden}}\right)$

Fasspektrum: $\arg \tilde{X}(f)$



Exempel – relationen mellan spektrum för rect-funktionen och för en fyrkantvåg



Konsekvens, om $x_{T_0}(t)$ erhålls genom en periodisering av $\tilde{x}(t)$:

$$D_n = \frac{1}{T_0}\tilde{X}(\omega)\Big|_{\omega=n\omega_0} = \frac{1}{T_0}\tilde{X}(n\omega_0)$$