

- Repetition, faltning
- Repetition, fourierserier
- Fouriertransformen – inledning

REPETITION: LTI-SYSTEM & FALTNING

VIDEO 1

$x(t)$ → $\mathcal{L}$ → $y(t) = y_{zi}(t) + y_{zs}(t)$
 LTI-SYSTEM
 Antag energifritt system $\Rightarrow y_{zi}(t) = 0 \Rightarrow y(t) = y_{zs}(t) = \mathcal{L}\{x(t)\}$

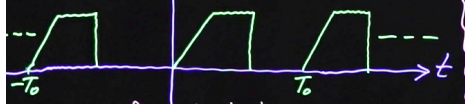
$x(t) = \sum_n a_n x_n(t)$ $\xrightarrow{\text{linjärt}}$ $y(t) = \sum_n a_n y_n(t)$

$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t-\tau) d\tau \xrightarrow{\text{LTI}} y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$

$k(t) < \infty \forall t$ $\delta(t)$ $\xrightarrow{\text{TI}} h(t)$
 $\int_{-\infty}^{\infty} |h(t)| dt < \infty$ $\delta(t-\tau)$ $\xrightarrow{\text{TI}} h(t-\tau)$

$= \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$
 $= (x * h)(t) = x(t) * h(t)$

FOURIERSERIER - REPETITION

 $x_{T_0}(t)$ 

En fysikalisk
 T_0 -periodisk signal. $\omega_0 = \frac{2\pi}{T_0}$

$$x_{T_0}(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot \cos(n\omega_0 t + \theta_n) ; \omega_0 = \frac{2\pi}{T_0} \text{ [rad/s]}$$

$$= \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t} ; D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x_{T_0}(t) \cdot e^{-jn\omega_0 t} dt$$

$$\Rightarrow \begin{cases} C_n = 2|D_n| \\ \theta_n = \arg D_n \end{cases} ; D_{n \geq 1} = \frac{C_n}{2} e^{j\theta_n}$$

$$D_{-n} = D_n^*$$

LT I

$$x_{T_0}(t) \rightarrow \mathcal{H} \rightarrow y_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n \cdot \mathcal{H}\{e^{jn\omega_0 t}\}$$

Linjärt $(\sum_n a_n x_n(t)) \rightarrow (\sum_n a_n y_n(t))$

$$\mathcal{H}\{e^{jn\omega_0 t}\} = e^{jn\omega_0 t} * h(t) = \dots = H_{n,\omega_0} \cdot e^{jn\omega_0 t}$$

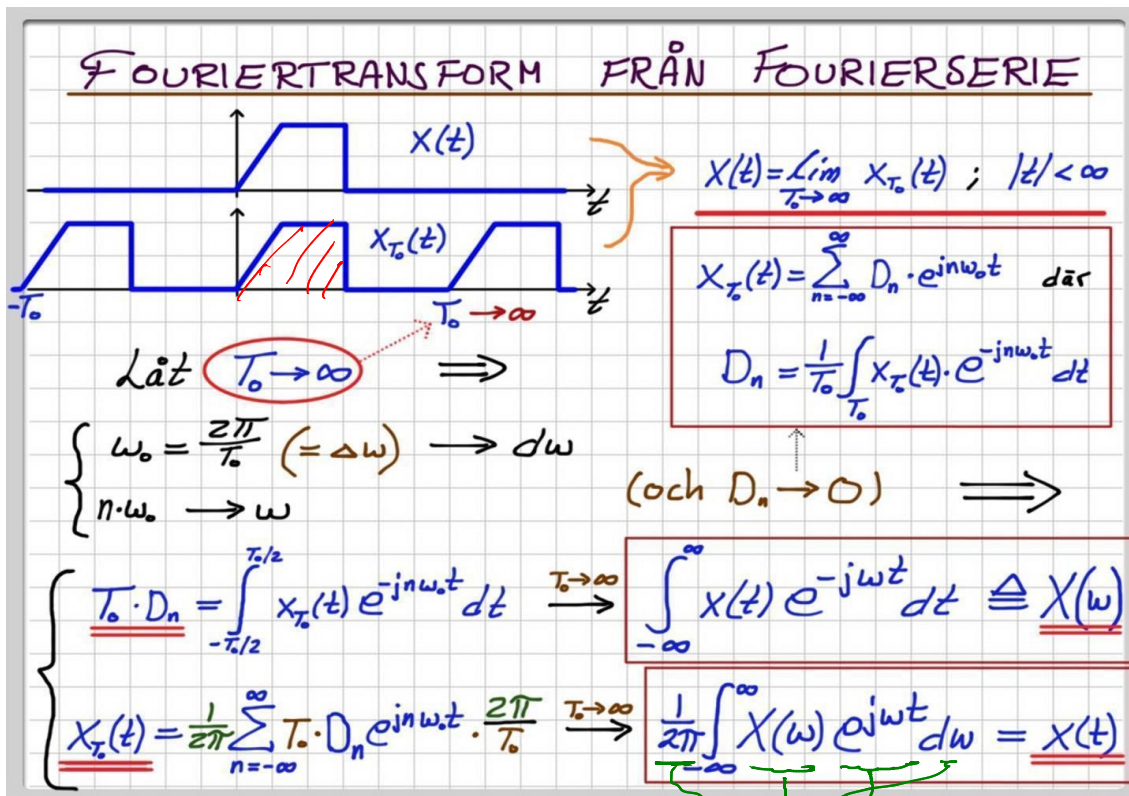
$$H_{n,\omega_0} = \int_{-\infty}^{\infty} h(\tau) e^{-jn\omega_0 \tau} d\tau \Rightarrow y_{T_0}(t) = \sum_{n=-\infty}^{\infty} \tilde{D}_n \cdot e^{jn\omega_0 t} ; \tilde{D}_n = D_n \cdot H_{n,\omega_0}$$

τ (inte t)

$n \rightarrow$ fr.komp. $\omega = n \cdot \omega_0$

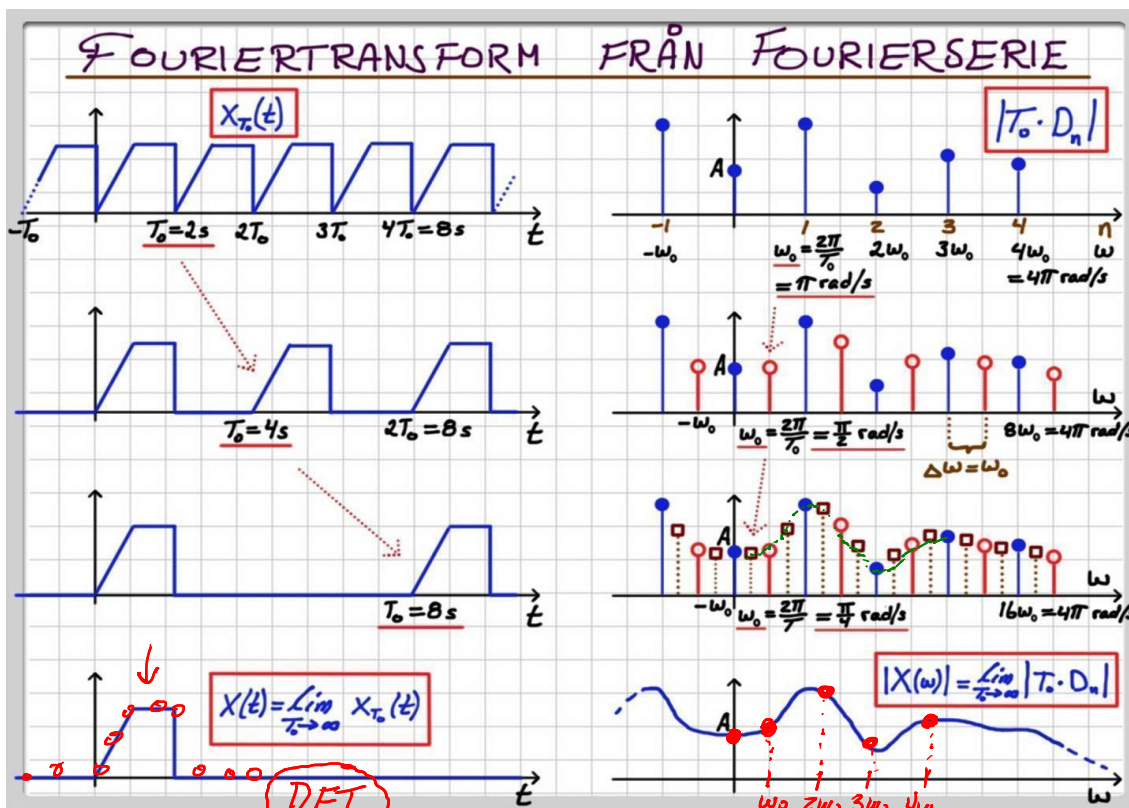
$$\omega_0 = \frac{2\pi}{T_0}$$

VIDEO 3



$$x(t) = \sum_n a_n \cdot x_n(t)$$

VIDEO 4



Konsekvens, om $x_{T_0}(t)$ erhålls genom en periodisering av $x(t)$:

$$D_n = \frac{1}{T_0} X(\omega) \Big|_{\omega=n\omega_0} = \frac{1}{T_0} X(n\omega_0)$$

Skapa $x_{T_0}(t)$ från $x(t) \Rightarrow D_n = \frac{1}{T_0} X(\omega = n \cdot \omega_0)$

Sammanfattning – Fouriertransformen

(härlledning – se videoklipp)

- Fouriertransformen till $x(t)$:

$$\mathcal{F}\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Jfr. fourierserie:

$$D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x_{T_0}(t) e^{-jn\omega_0 t} dt$$

- Inversa fouriertransformen till $X(\omega)$:

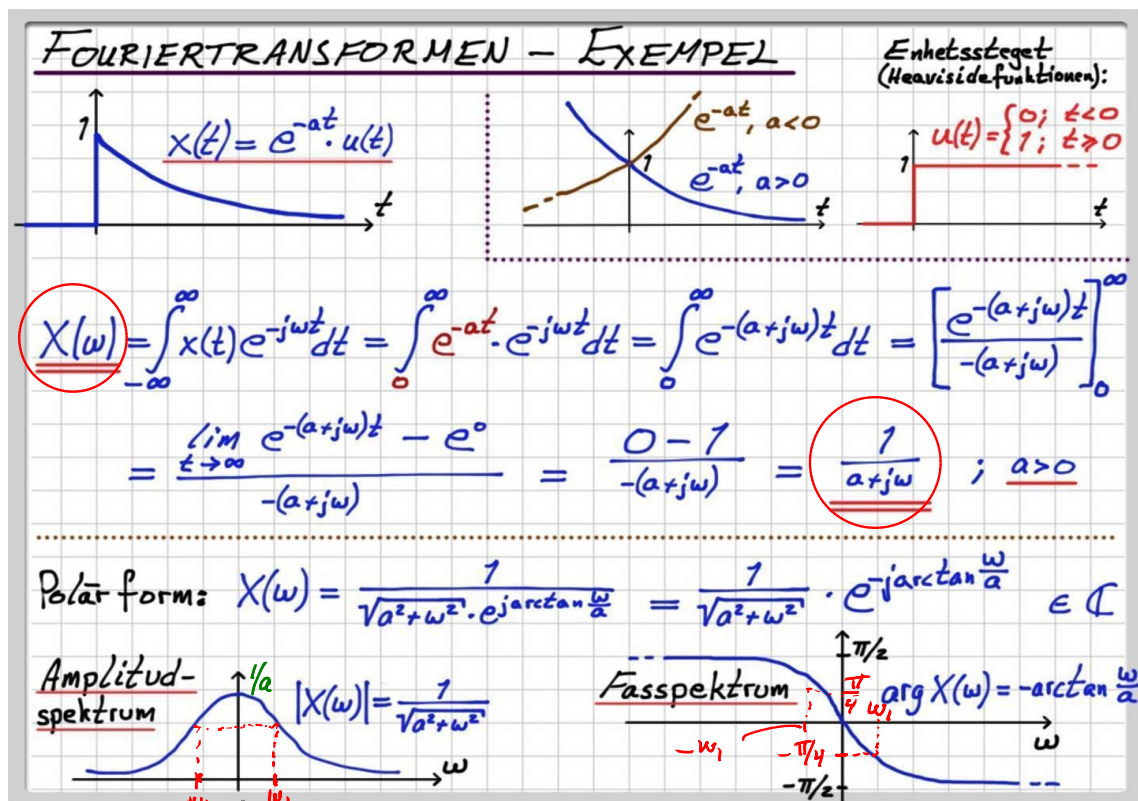
$$\mathcal{F}^{-1}\{X(\omega)\} = x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Jfr. fourierserie:

$$x_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

Existensvillkor:

$$\mathcal{F}\{x(t)\} \exists \text{ om } \int_{-\infty}^{\infty} |x(t)| dt < \infty$$



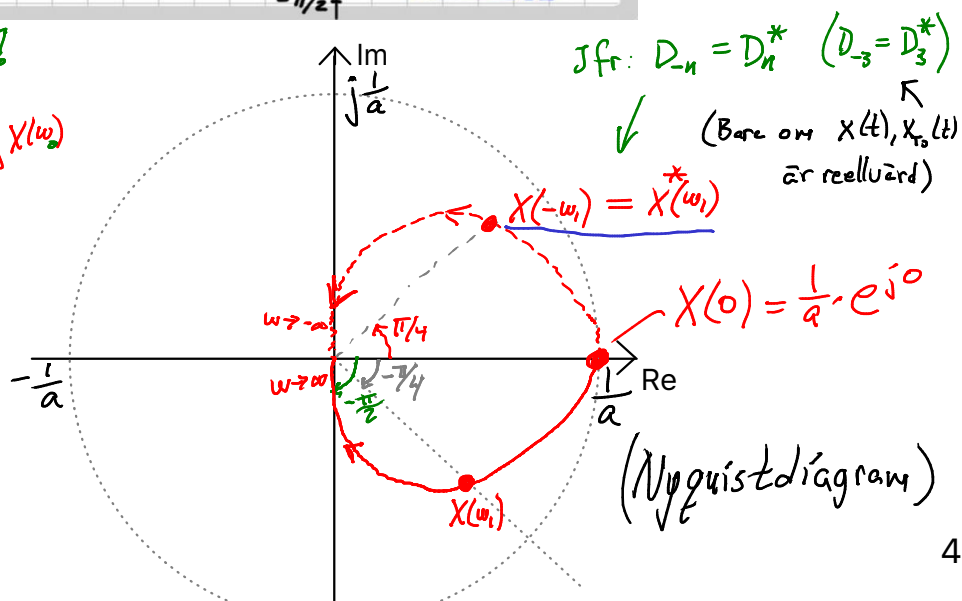
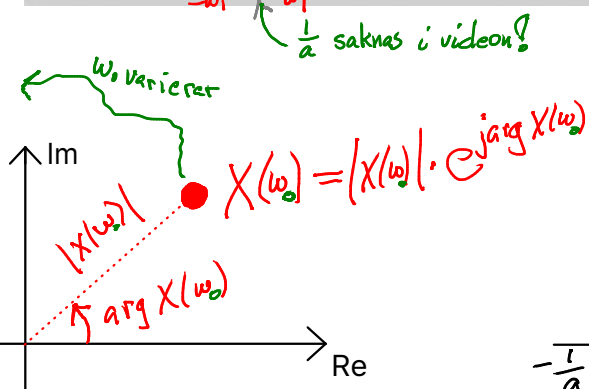
$C_n \cdot \cos(n\omega_0 t + \Theta_n)$
VIDEO 5

$D_n \in \mathbb{C} \forall n$
 $|D_n|, \arg D_n$

$C_n = 2|D_n| \leftarrow \text{Ampl.sp}$
 $\Theta_n = \arg D_n \leftarrow \text{Fassp.}$

$T_0 \cdot D_n \xrightarrow{T_0 \rightarrow \infty} X(\omega)$

$D_n = |D_n| \cdot e^{j \arg D_n}$



FOURIERTRANSFORMEN

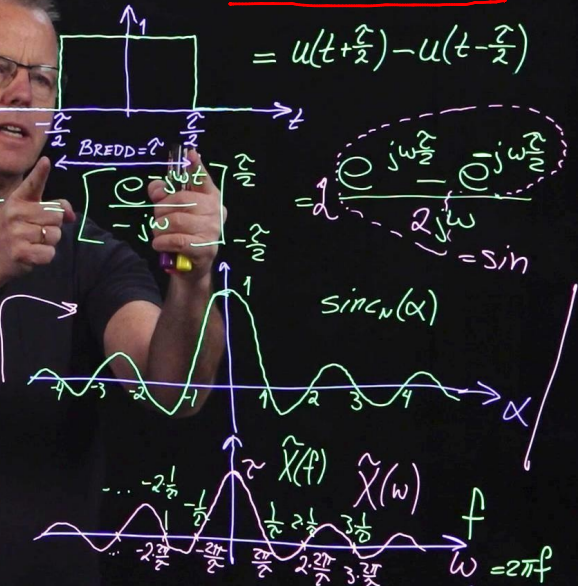
VIDEO 6

$$X(\omega) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

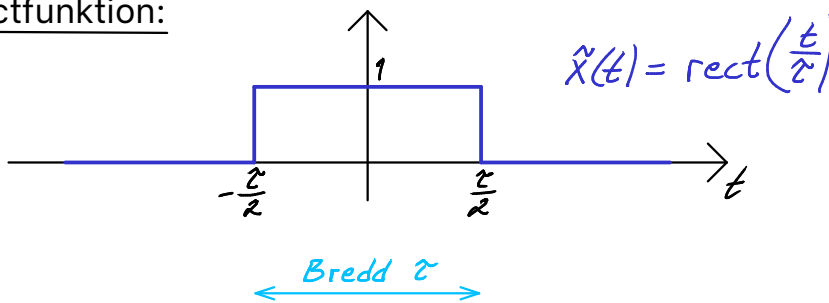
$$x(t) = \mathcal{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Exempel: $\tilde{x}(t) = \text{rect}\left(\frac{t}{\tau}\right)$
 $= u(t + \frac{\tau}{2}) - u(t - \frac{\tau}{2})$

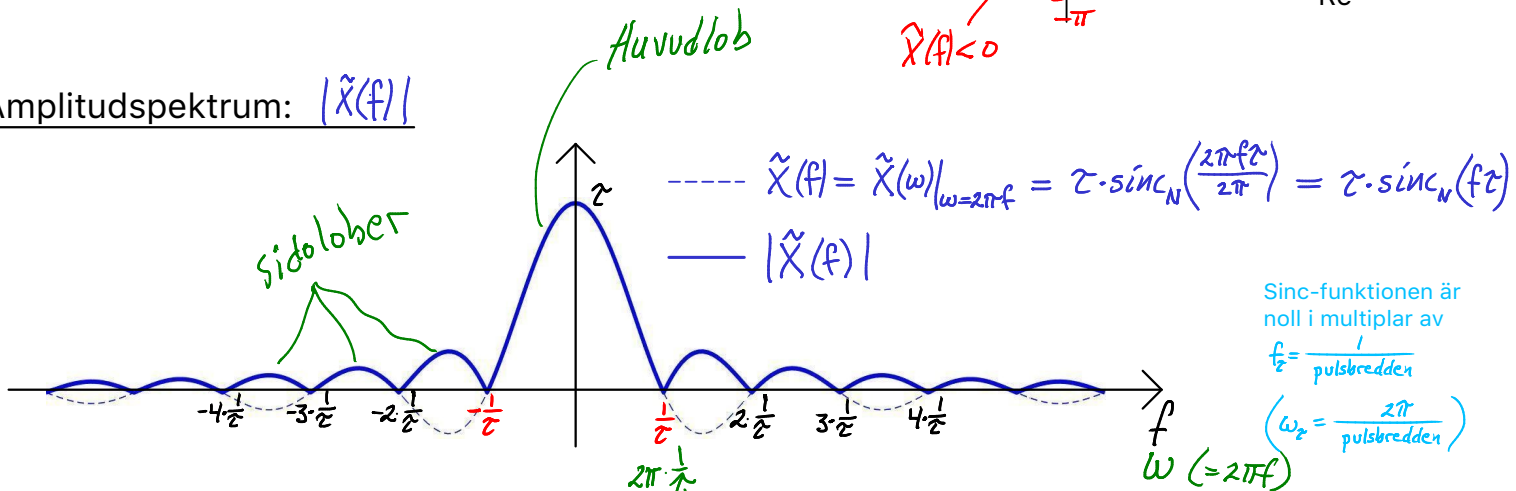
$$\begin{aligned} \tilde{X}(\omega) &= \int_{-\infty}^{\infty} \tilde{x}(t) e^{j\omega t} dt = \int_{-\tau/2}^{\tau/2} 1 \cdot e^{j\omega t} dt = \left[\frac{e^{j\omega t}}{j\omega} \right]_{-\tau/2}^{\tau/2} = \frac{e^{j\omega \tau/2} - e^{-j\omega \tau/2}}{j\omega} = \frac{2 \sin(\omega \tau/2)}{\omega} \\ &= \frac{\sin(\frac{\omega \tau}{2})}{\frac{\omega}{2}} = \text{sinc}(\alpha) = \frac{\sin \alpha}{\alpha} \\ &= \tau \cdot \frac{\sin(\pi \cdot \frac{\omega \tau}{2\pi})}{\pi \cdot \frac{\omega \tau}{2\pi}} = \tau \cdot \text{sinc}_N\left(\frac{\omega \tau}{2\pi}\right) \end{aligned}$$



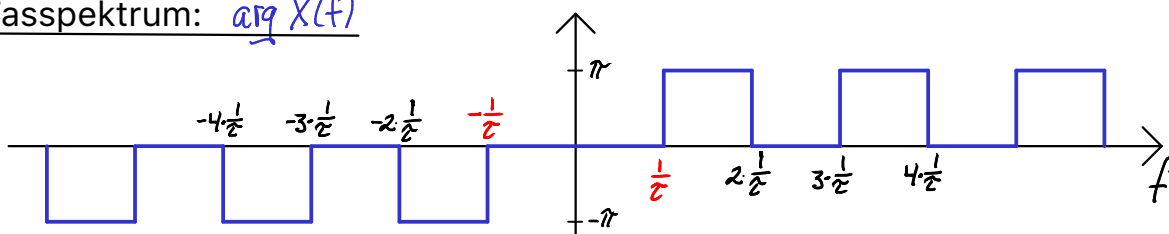
Tidssignal – rectfunktion:



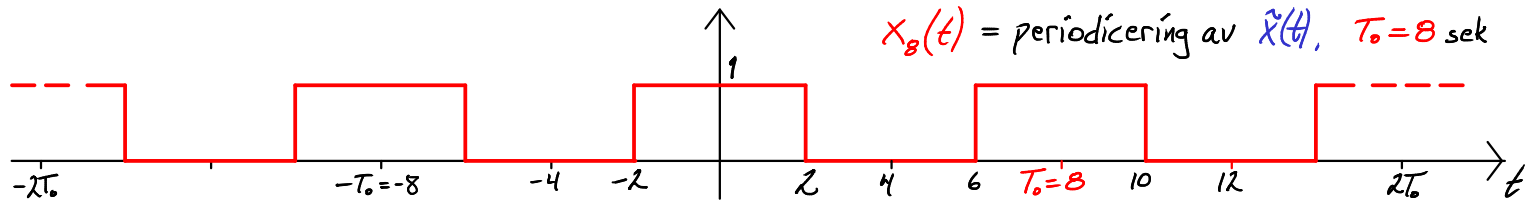
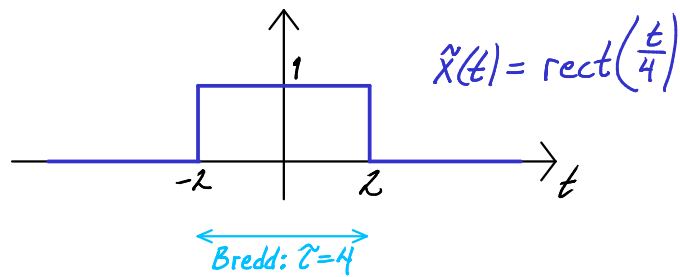
Amplitudspektrum: $|\tilde{X}(f)|$



Fasspektrum: $\arg \tilde{X}(f)$



Exempel – relationen mellan spektrum för rect-funktionen och för en fyrkantvåg



Konsekvens, om $x_{T_0}(t)$ erhålls genom en periodisering av $\tilde{x}(t)$:

$$D_n = \frac{1}{T_0} \tilde{X}(\omega) \Big|_{\omega=n\omega_0} = \frac{1}{T_0} \tilde{X}(n\omega_0)$$

