# Linjära System – Föreläsning 13: Fouriertransformanalys

- Fouriertransformegenskaper
- Signalenergi
- Fouriertransformen av några viktiga signaler

## Sammanfattning – Fouriertransformen (från föregående föreläsning)

- Fouriertransformen till $x(t)$:

$$\boxed{\mathcal{F}\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

Jfr. fourierserie:

$$\left( D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x_{T_0}(t) e^{-jn\omega_0 t} dt \right)$$

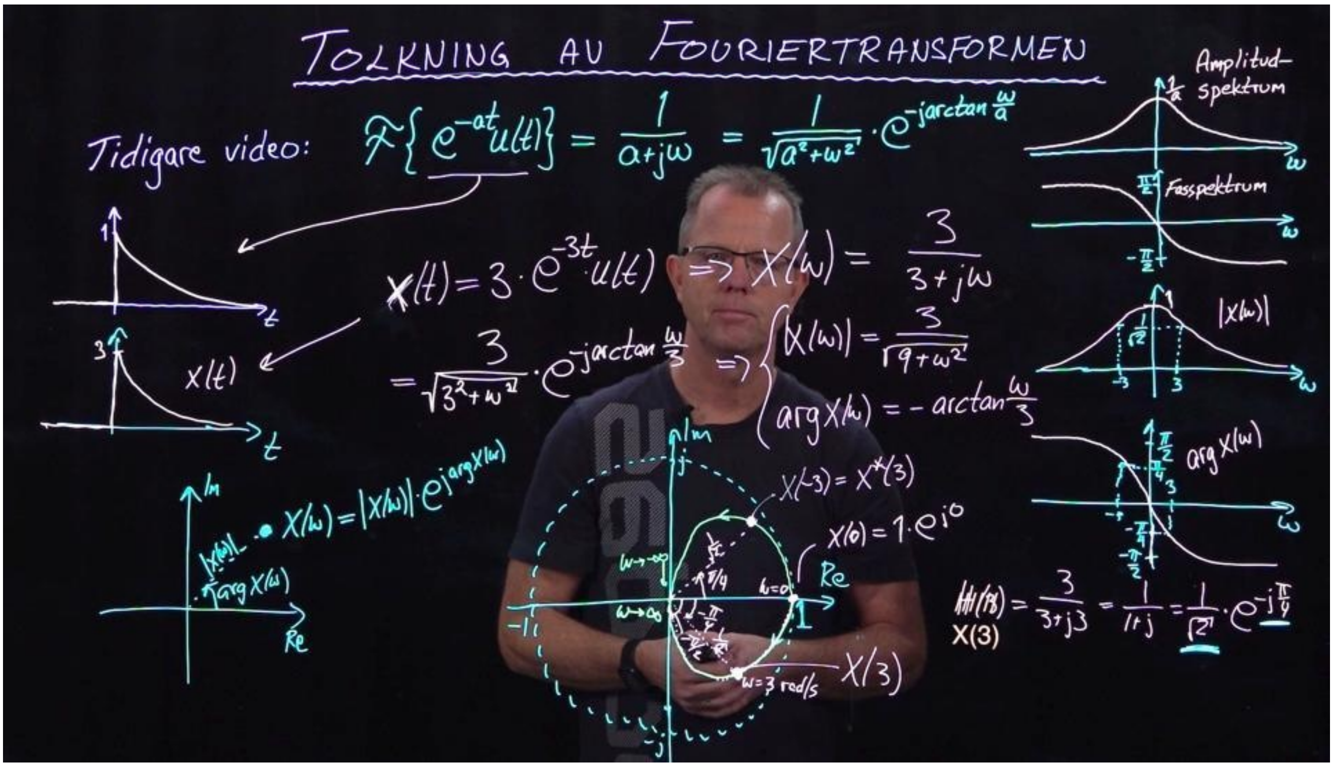
- Inversa fouriertransformen till $X(\omega)$:

$$\boxed{\mathcal{F}^{-1}\{X(\omega)\} = x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega}$$

Jfr. fourierserie:

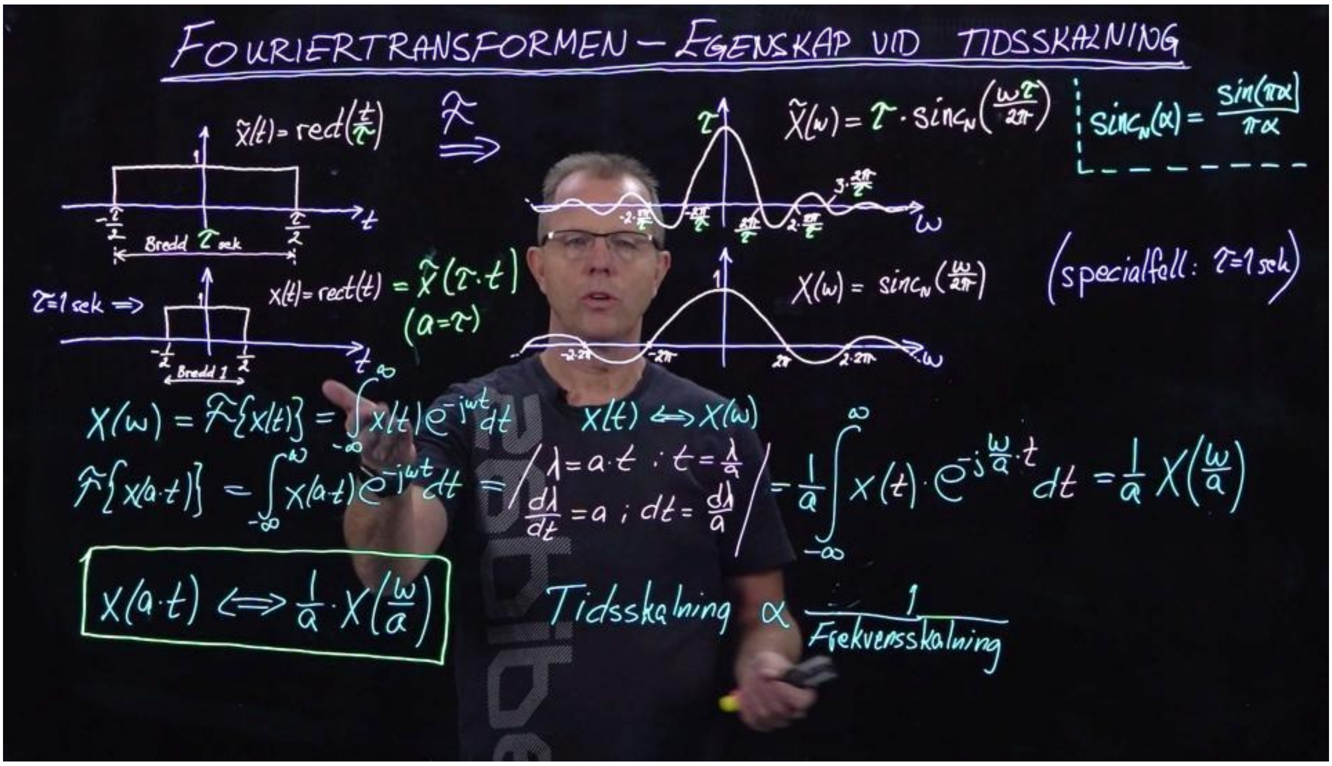
$$\left( x_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \right)$$

Existensvillkor: $\boxed{\mathcal{F}\{x(t)\} \,\exists \text{ om } \int_{-\infty}^{\infty} |x(t)|\, dt < \infty}$




VIDEO 1

(Det här pratade jag om redan på förra föreläsningen)

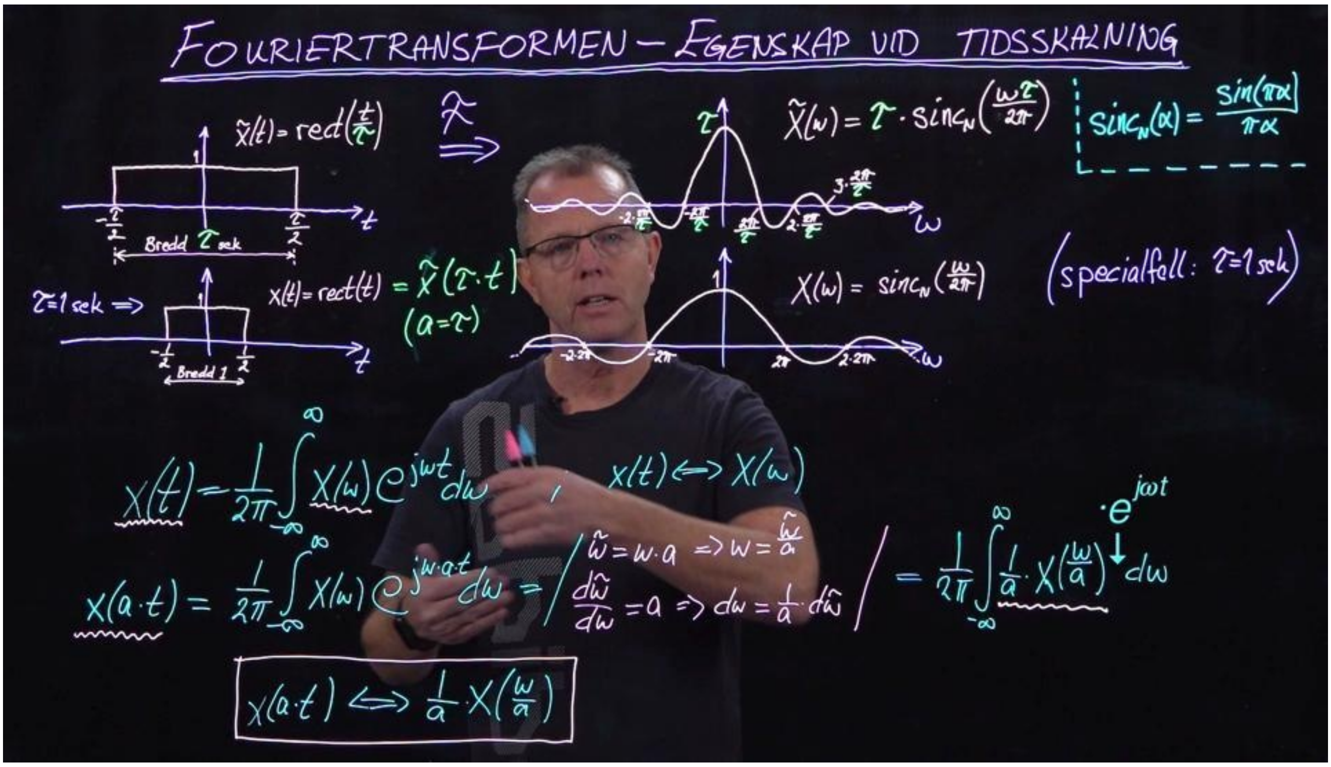



VIDEO 2.1

(Härledning utgående från fouriertransformen)

(På förra föreläsningen demonstrerade jag konsekvensen hos fouriertransformen vid tidsskalning av rect-funktionen. Denna video tar upp konsekvenser av tidsskalning för godtyckliga signaler)

| Tidsutbredning | Frekvensutbredning |
|---|---|
| | |
| | |

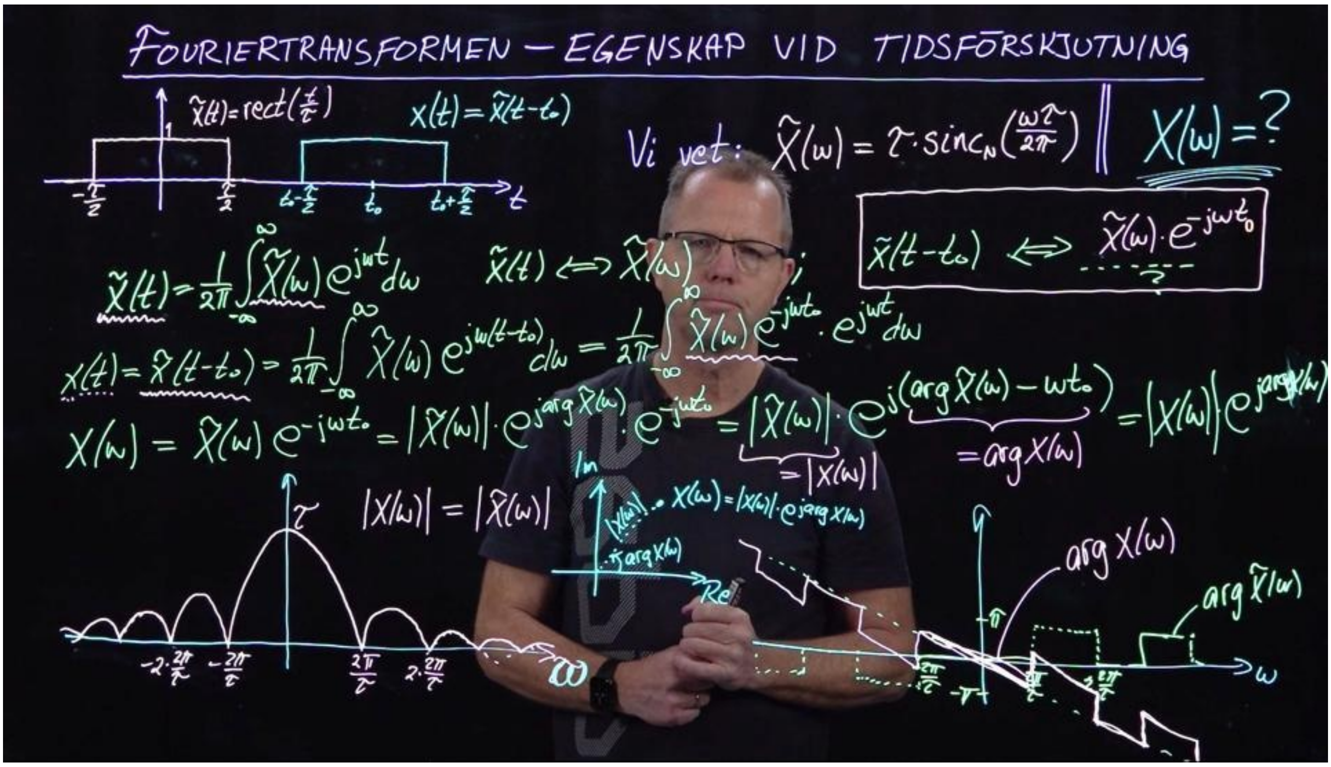


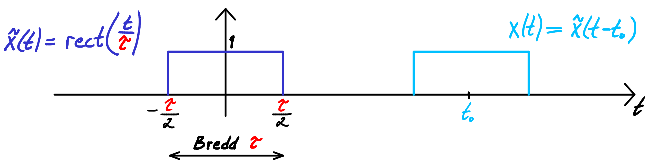

VIDEO 2.2

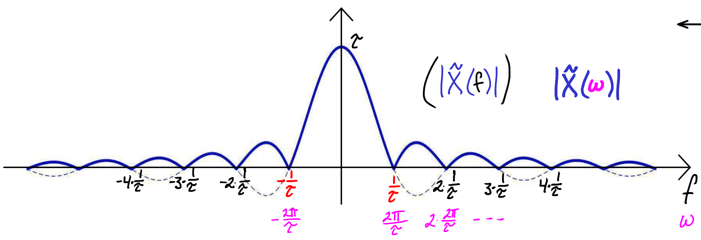
(Härledning utgående från inversa fouriertransformen)

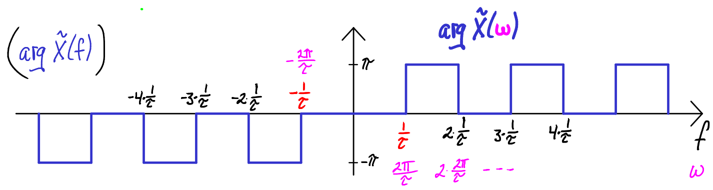
VIDEO 3

(Härledning utgående från inversa fouriertransformen)



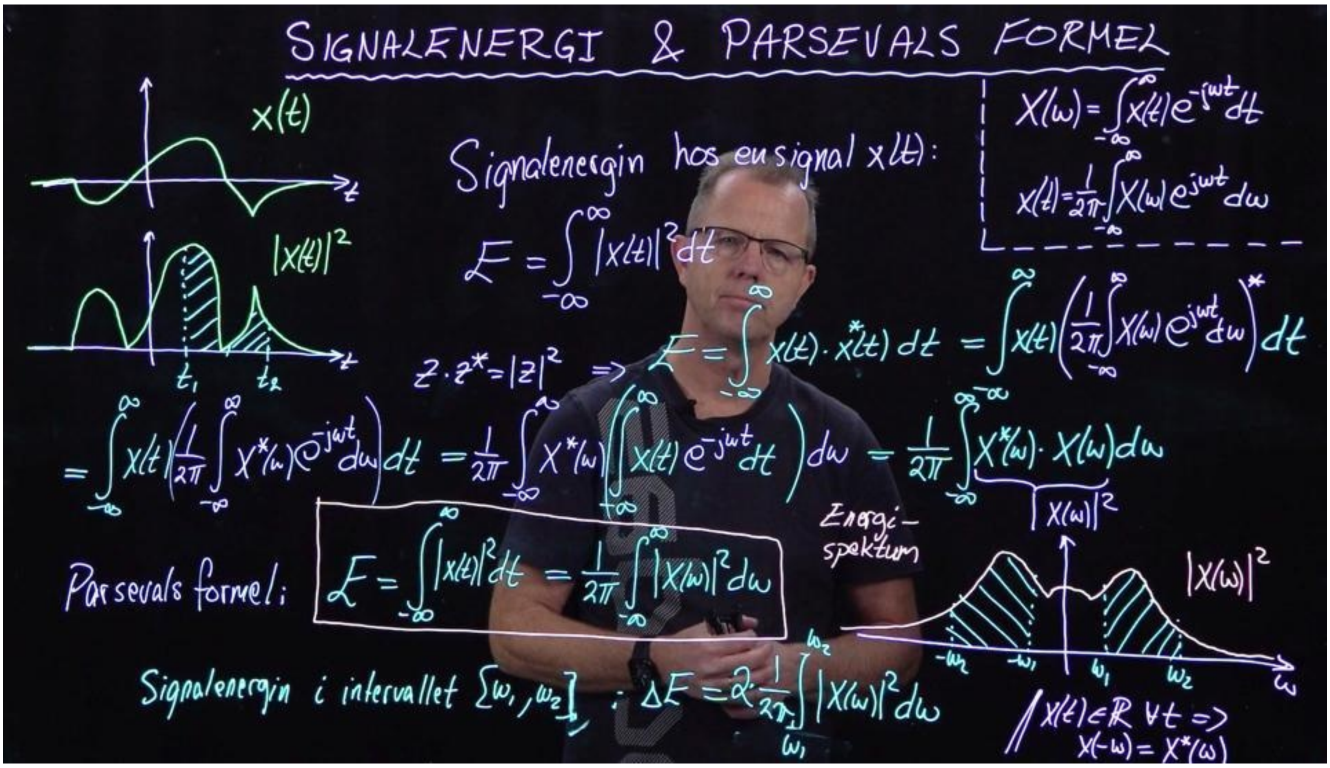



**Se formelsamlingens Tabell 2 för egenskaper hos fouriertransformen:**

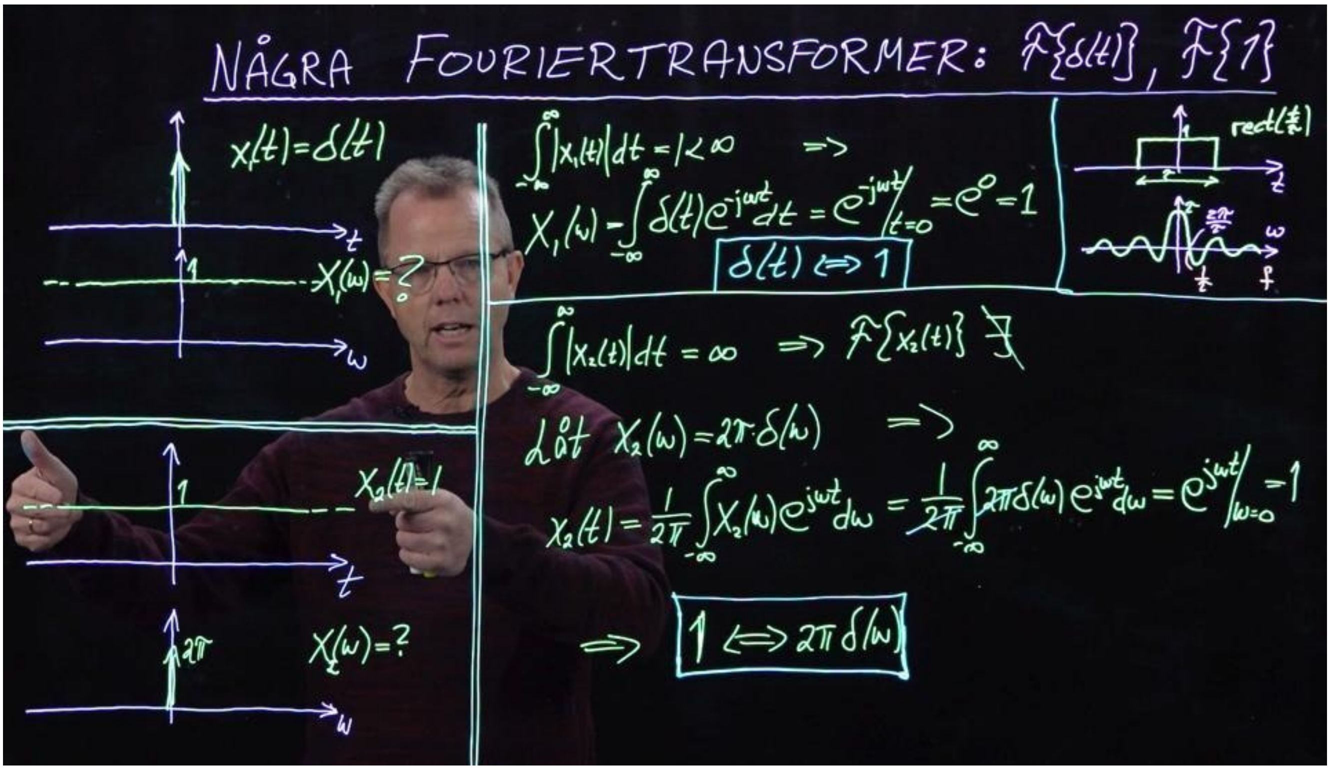
## 8.2 Egenskaper – fouriertransformen, $x(t) \Leftrightarrow X(\omega)$

| Operation | Tidsuttryck, $x(t)$ | Transformuttryck, $X(\omega)$ |
|---|---|---|
| 1. Transform | $x(t)$ | $X(\omega)$ |
| 2. Linjäritet, $a,b \in \mathbb{C}$ | $ax_1(t)+bx_2(t)$ | $aX_1(\omega)+bX_2(\omega)$ |
| 3. Komplexkonjugering | $x^*(t)$ | $X^*(-\omega)$ |
| 4. Symmetri | $x(t)$ reell | $X(-\omega)=X^*(\omega)$ |
| 5. Dualitet | $X(t)$ | $2\pi x(-\omega)$ |
| 6. Tidsskalning | $x(at)$ | $\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$ |
| 7. Spegling (*tidsskalning med* $a=-1$) | $x(-t)$ | $X(-\omega)$ |
| 8. Tidsförskjutning | $x(t-t_0)$ | $X(\omega)e^{-j\omega t_0}$ |
| 9. Frekvensskift, $\omega_0 \in \mathbb{R}$ | $x(t)e^{j\omega_0 t}$ | $X(\omega-\omega_0)$ |
| 10. Derivering | $\frac{d^n x(t)}{dt^n}$ | $(j\omega)^n X(\omega)$ |
| 11. Multiplikation med $t$ | $t^n x(t)$ | $j^n \frac{d^n X(\omega)}{d\omega^n}$ |

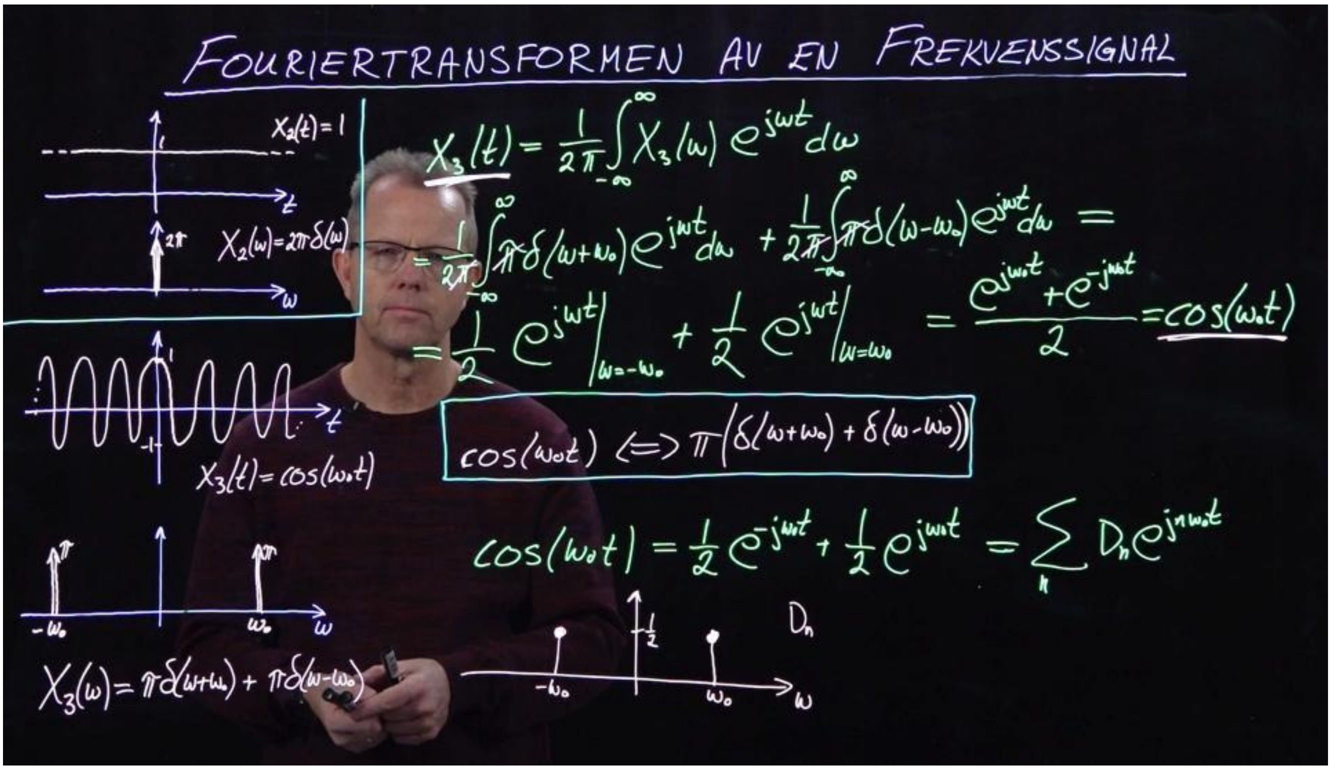
(Plus ytterligare några egenskaper...)



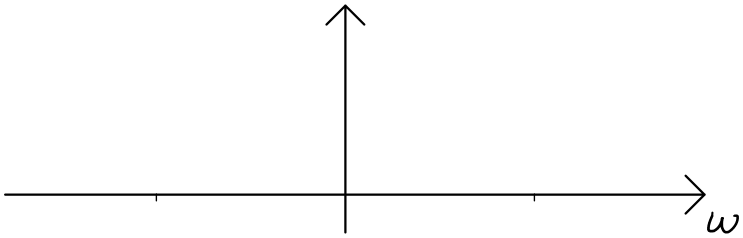

VIDEO 4

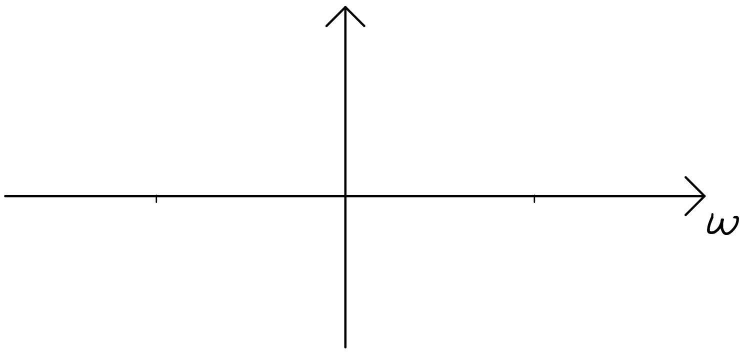


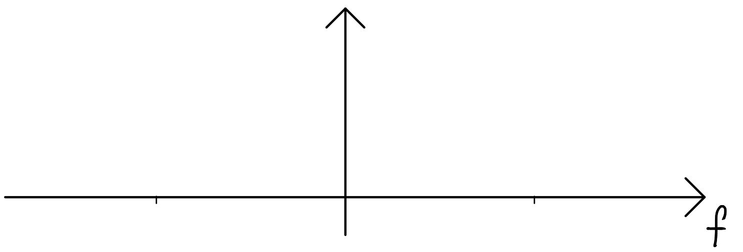

VIDEO 5


FOURIERTRANSFORMEN AV EN FREKVENSSIGNAL
$x_2(t)=1$
$X_2(\omega)=2\pi\delta(\omega)$
$x_3(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X_3(\omega)e^{j\omega t}d\omega$
$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\pi\delta(\omega+\omega_0)e^{j\omega t}d\omega+\frac{1}{2\pi}\int_{-\infty}^{\infty}\pi\delta(\omega-\omega_0)e^{j\omega t}d\omega=$
$=\frac{1}{2}e^{j\omega t}\Big|_{\omega=-\omega_0}+\frac{1}{2}e^{j\omega t}\Big|_{\omega=\omega_0}=\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}=\cos(\omega_0 t)$
$x_3(t)=\cos(\omega_0 t)$
$\cos(\omega_0 t)\Longleftrightarrow\pi\left(\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right)$
$\cos(\omega_0 t)=\frac{1}{2}e^{-j\omega_0 t}+\frac{1}{2}e^{j\omega_0 t}=\sum_n D_n e^{jn\omega_0 t}$
$D_n$
$X_3(\omega)=\pi\delta(\omega+\omega_0)+\pi\delta(\omega-\omega_0)$


VIDEO 6

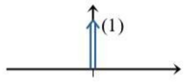
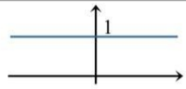
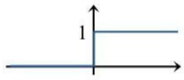
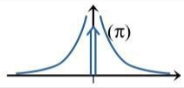
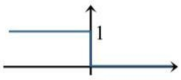
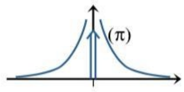
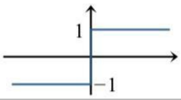
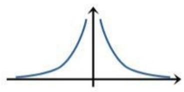
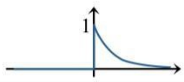
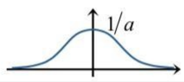
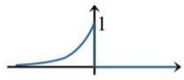
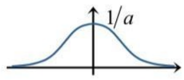
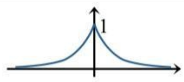
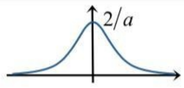
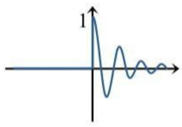
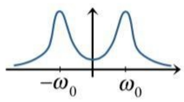
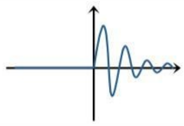
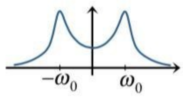
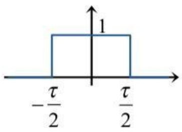
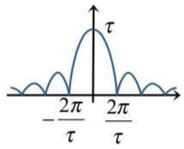
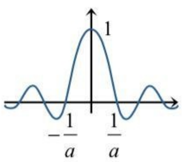
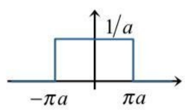
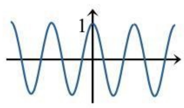
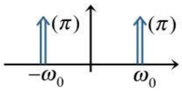
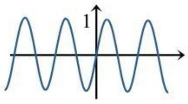
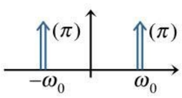


ω


ω


f

# Se formelsamlingens Tabell 3 för olika fouriertransformpar x(t) <=> X(w) :

## 8.3 Fouriertransformer, $x(t) \Leftrightarrow X(\omega)$

| Tidsuttryck, $x(t)$ | | Transformuttryck, $X(\omega)$ | $\lvert X(\omega)\rvert$ |
|---|---|---|---|
| 1. $\delta(t)$ |   | $1$ |   |
| 2. $u(t),\ u_0(t)$ |   | $\mathrm{vp}\left\{\frac{1}{j\omega}\right\}+\pi\delta(\omega)$ |   |
| 3. $u(-t),\ u_0(-t)$ |   | $\mathrm{vp}\left\{\frac{-1}{j\omega}\right\}+\pi\delta(\omega)$ |   |
| 4. $\mathrm{sgn}(t)=\frac{t}{\lvert t\rvert}$ |   | $\mathrm{vp}\left\{\frac{2}{j\omega}\right\}$ |  |
| 5. $e^{-at}u(t)$ |   | $\frac{1}{a+j\omega}$ |   |
| 6. $e^{at}u_0(-t)$ |   | $\frac{1}{a-j\omega}$ |   |
| 7. $e^{-a\lvert t\rvert}$ |   | $\frac{2a}{a^2+\omega^2}$ |   |
| 10. $e^{-at}\cos(\omega_0 t)u(t)$ |   | $\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$ |   |
| 11. $e^{-at}\sin(\omega_0 t)u(t)$ |  | $\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$ |   |
| 12. $\Pi\left(\frac{t}{\tau}\right)=\mathrm{rect}\left(\frac{t}{\tau}\right)=\begin{cases}1; & \lvert t\rvert\le\frac{\tau}{2}\\ 0; & \lvert t\rvert>\frac{\tau}{2}\end{cases}$ |   | $\tau\,\mathrm{sinc}_{\mathrm{N}}\left(\frac{\omega\tau}{2\pi}\right)=\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$ |   |
| 13. $\mathrm{sinc}_{\mathrm{N}}(at)=\mathrm{sinc}(a\pi t)$ |   | $\frac{1}{a}\Pi\left(\frac{\omega}{2\pi a}\right)=\frac{1}{a}\mathrm{rect}\left(\frac{\omega}{2\pi a}\right)=\begin{cases}\frac{1}{a}; & \lvert\omega\rvert\le\pi a\\ 0; & \lvert\omega\rvert>\pi a\end{cases}$ |   |
| 22. $\cos(\omega_0 t)=\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}$ |   | $\pi\left(\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right)$ |   |
| 23. $\sin(\omega_0 t)=\frac{e^{j\omega_0 t}-e^{-j\omega_0 t}}{2j}$ |   | $j\pi\left(\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\right)$ |   |

(Plus ytterligare ett antal transformpar)