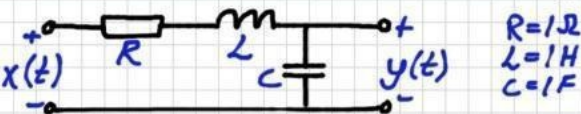


- Forts. på förra föreläsningens stora räkneexempel – fouriertransformanalys av ett LTI-system, deluppgift c) & d) samt extrauppgift e)
- Video inför deluppgift d): LTI-system och frekvenssignaler
- Tillämpningsexempel, fouriertransformanalys: Amplitudmodulering (AM)

Från förra föreläsningen – då gick vi igenom deluppgift a) & b):

(Större) Räkneexempel – Systemanalys med fouriertransformen



$R=1\Omega$
 $L=1H$
 $C=1F$

a) Beräkna/bestämt LTI-systemets

- frekvensfunktion $H(\omega)$
- impulssvar $h(t)$, samt skissera detta
- kausalitets- & stabilitets-egenskap
- systembeskrivande differentialekvation
- ordning
- amplitudkaraktäristik $|H(\omega)|$

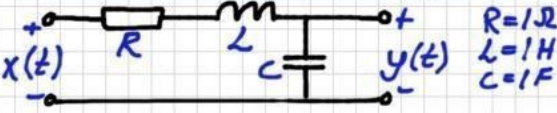
b) Skissera amplitudkaraktäristiken och ange/motivera vilken typ av frekvensselektivt filter (LP, BP, HP osv.) det elektriska systemet utgör.

c) Beräkna utsignalen $y_1(t)$ då insignalen är $x_1(t) = 3 \cdot e^{-2t} u(t)$ [V] och systemet är energifritt.

d) Beräkna utsignalen $y_2(t)$ då insignalen är $x_2(t) = 4 + 3 \cos(2t + \frac{3\pi}{4})$ [V]

e) Beräkna stegsvaret $g(t)$ Extra!

(Större) Räkneexempel – Systemanalys med fouriertransformen



$R=1\Omega$
 $L=1H$
 $C=1F$

Lösningssång:

② Krets → Komplexschema → $H(\omega)$

From $H(\omega)$:

- $h(t)$ (via $\mathcal{F}^{-1}$) → Skiss
- Kausalitet
- Stabilitet
- $g(t) = (u * h)(t)$ ③ Extra!
- Differentialekvation
- Systemordning
- $|H(\omega)|$ ⑥ → Skiss → Filtertyp

Extra! $\Rightarrow$ Alt. ③: $g(t) = \mathcal{F}^{-1}\{G(\omega)\}$, $G(\omega) = U(\omega) \cdot H(\omega)$

③ $x_1(t) \rightarrow X_1(\omega) \rightarrow Y_1(\omega) \xrightarrow{\mathcal{F}^{-1}} y_1(t)$

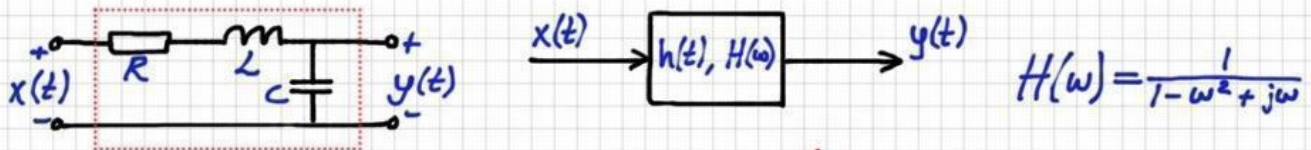
④ $x_2(t) \rightarrow y_2(t)$

Först löser vi deluppgift c) & d)!

c) Beräkna utsignalen $y(t)$ då insignalen är $x_1(t) = 3 \cdot e^{-2t} u(t)$ [V]

och systemet är energifritt.

(Större) Räkneexempel – Systemanalys med Fouriertransformen



c) $x_1(t) = 3 \cdot e^{-2t} u(t)$ [V]. Beräkna $y_1(t)$. // $y_1(t) = (x * h)(t) \Rightarrow$

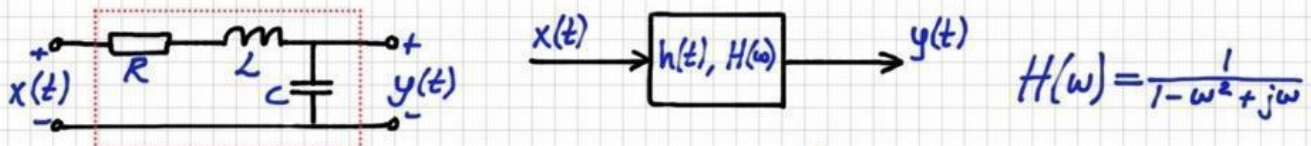
$$Y_1(\omega) = X_1(\omega) H(\omega) = \left/ \begin{array}{l} \text{Formels. Tab. 3:5} \\ e^{at} u(t) \Leftrightarrow \frac{1}{a+j\omega} \end{array} \right/ = \frac{3}{2+j\omega} \cdot \frac{1}{1-\omega^2+j\omega} = \left/ \text{P.B.U.} \right/$$

$$= \frac{A}{2+j\omega} + \frac{B(j\omega) + C}{(j\omega)^2 + j\omega + 1} = \left/ \begin{array}{l} A + B = 0 \\ A + 2B + C = 0 \\ A + 2C = 3 \end{array} \right\} \Rightarrow A=1, B=-1, C=1 \left/$$

$$= \frac{1}{2+j\omega} + \frac{-j\omega + 1}{(j\omega + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$\mathcal{F}^{-1}\left\{\frac{1}{2+j\omega}\right\} = e^{-2t} u(t)$ enligt Tab. 3:5
men vad är $\mathcal{F}^{-1}$ av den andra termen?

(Större) Räkneexempel – Systemanalys med Fouriertransformen



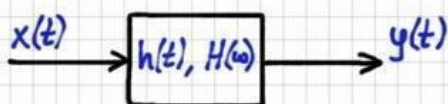
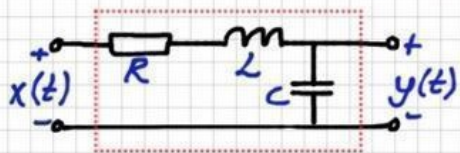
c) $x_1(t) = 3 \cdot e^{-2t} u(t)$ [V]. Beräkna $y_1(t)$. // $y_1(t) = (x * h)(t) \Rightarrow$

$$Y_1(\omega) = X_1(\omega) H(\omega) = \frac{1}{2+j\omega} + \frac{-j\omega + 1}{(j\omega + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

Formelsaml.
Tab. 3:10 & 3:11 $\Rightarrow$

$$\mathcal{F}\{e^{-at} \cos(\omega t) u(t)\} = \frac{a+j\omega}{(a+j\omega)^2 + \omega^2} \quad \mathcal{F}\{e^{-at} \sin(\omega t) u(t)\} = \frac{\omega}{(a+j\omega)^2 + \omega^2}$$

(Större) Räkneexempel – Systemanalys med fouriertransformen



$$H(w) = \frac{1}{1 - w^2 + jw}$$

c) $x_1(t) = 3 \cdot e^{-2t} u(t)$ [V]. Beräkna $y_1(t)$. // $y_1(t) = (x * h)(t) \Rightarrow$

$$Y_1(w) = X_1(w) H(w) = \frac{1}{2 + jw} + \frac{-jw + 1}{(jw + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = -(jw + \frac{1}{2}) + \frac{3}{2}$$

$$= \frac{1}{2 + jw} - \frac{jw + \frac{1}{2}}{(jw + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} + \frac{\frac{2}{\sqrt{3}} \cdot \frac{3}{2}}{(jw + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$\Rightarrow y_1(t) = e^{-2t} u(t) - e^{-\frac{t}{2}} \cos(\frac{\sqrt{3}}{2} t) u(t) + \sqrt{3} e^{-\frac{t}{2}} \sin(\frac{\sqrt{3}}{2} t) u(t)$$

$$= (e^{-2t} - e^{-\frac{t}{2}} (\cos(\frac{\sqrt{3}}{2} t) - \sqrt{3} \sin(\frac{\sqrt{3}}{2} t))) u(t)$$

d) Beräkna utsignalen $y_2(t)$ då insignalen är $x_2(t) = 4 + 3 \cos(2t + \frac{3\pi}{4})$ [V]

Först den förberedande videon om hur LTI-system amplitudskalar och fasförskjuter frekvenssignaler:

Från föreläsningen om fourierserier känner du redan till hur LTI-system amplitudskalar och fasvrider en insignal $x(t) = C_0 + C \cdot \cos(\omega_0 t)$. Här är dock en härledning – tillsammans med förtydligande exempel – som baserar sig mer på fouriertransformen.

LTI-SYSTEM & FREKVENSSIGNALER



$$y(t) = (x * h)(t)$$

$$Y(w) = X(w) \cdot H(w)$$

$$x(t) = C_0 + C \cdot \cos(\omega_0 t) \Rightarrow y(t) = ?$$

$$\cos(\omega_0 t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$$

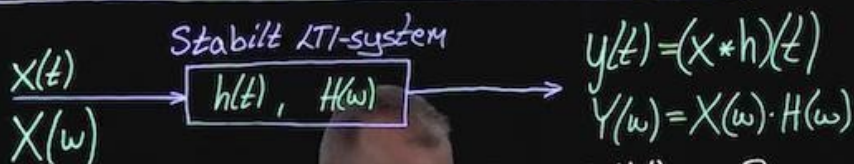
$$C_0 = C_0 \cdot e^{j0t} = C_0 \cdot e^{j\omega_0 t} \Big|_{\omega_0=0}$$

$$x(t) = e^{j\omega_0 t} \Rightarrow y(t) = e^{j\omega_0 t} * h(t)$$

$$= \int_{-\infty}^{\infty} e^{j\omega_0(t-\tau)} h(\tau) d\tau = \left(\int_{-\infty}^{\infty} h(\tau) e^{-j\omega_0 \tau} d\tau \right) e^{j\omega_0 t} = \mathcal{F}\{h(t)\} \Big|_{w=\omega_0} \cdot e^{j\omega_0 t} = H(\omega_0) \cdot e^{j\omega_0 t}$$

$$\Rightarrow x(t) = C_0 = C_0 \cdot e^{j0t} \Rightarrow y(t) = C_0 H(0) \cdot e^{j0t} = C_0 H(0)$$

LTI-SYSTEM & FREKVENSIGNALER



$$x(t) = C_0 + C \cdot \cos(\omega_0 t) \Rightarrow y(t) = C_0 \cdot H(0) + ?$$

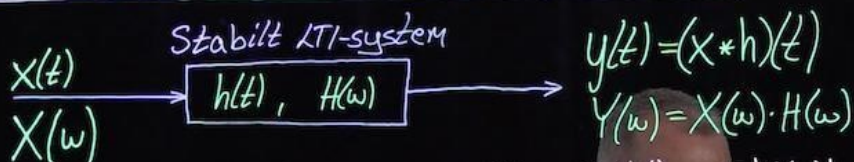
$$\underline{x(t) = \cos(\omega_0 t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{j(-\omega_0)t} \Rightarrow$$

$$\underline{y(t) = \frac{1}{2} H(\omega_0) e^{j\omega_0 t} + \frac{1}{2} H(-\omega_0) e^{j(-\omega_0)t} = \left[\begin{array}{l} h(t) \in \mathbb{R} \forall t \Rightarrow H(\omega) = H^*(\omega) \\ = |H(\omega)| \cdot e^{j \arg H(\omega)} \end{array} \right]$$

$$= \frac{1}{2} |H(\omega_0)| e^{j \arg H(\omega_0)} e^{j\omega_0 t} + \frac{1}{2} |H(\omega_0)| e^{-j \arg H(\omega_0)} e^{j(-\omega_0)t}$$

$$= |H(\omega_0)| \cdot \frac{e^{j(\omega_0 t + \arg H(\omega_0))} + e^{j(-\omega_0 t + \arg H(\omega_0))}}{2} = \underline{|H(\omega_0)| \cdot \cos(\omega_0 t + \arg H(\omega_0))}$$

LTI-SYSTEM & FREKVENSIGNALER



$$\underline{x(t) = C_0 + C \cdot \cos(\omega_0 t)} \Rightarrow \underline{y(t) = C_0 \cdot H(0) + C \cdot |H(\omega_0)| \cdot \cos(\omega_0 t + \arg H(\omega_0))}$$

Allmänna T_0 -periodiska signaler:

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot \cos(n\omega_0 t + \theta_n)$$

$$= \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t} \quad \omega_0 = \frac{2\pi}{T_0}$$

$$y(t) = \underbrace{C_0 \cdot H(0)}_{\tilde{C}_0} + \sum_{n=1}^{\infty} \underbrace{C_n \cdot |H(n\omega_0)|}_{\tilde{C}_n} \cos(n\omega_0 t + \underbrace{\theta_n + \arg H(n\omega_0)}_{\tilde{\theta}_n})$$

$$= \sum_{n=-\infty}^{\infty} \underbrace{D_n \cdot H(n\omega_0)}_{\tilde{D}_n} \cdot e^{jn\omega_0 t}$$

$$\boxed{\tilde{D}_n = D_n \cdot H(n\omega_0)}$$

I videon skriver jag $\tilde{D}_n$ i stället för $\hat{D}_n$ – det spelar ingen större roll vilken vi använder.

LTI-SYSTEM & FREQUENZSIGNALER

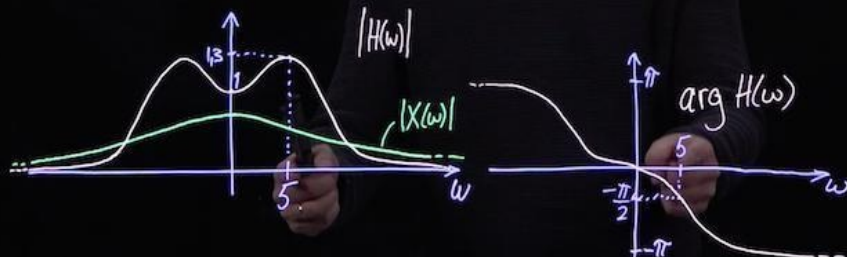
$x(t)$
 $X(\omega)$

Stabilt LTI-system
 $h(t), H(\omega)$

$y(t) = (x * h)(t)$
 $|Y(\omega)| = |X(\omega)| |H(\omega)|$

$x(t) = C_0 + C \cdot \cos(\omega_0 t) \Rightarrow y(t) = C_0 \cdot H(0) + C \cdot |H(\omega_0)| \cdot \cos(\omega_0 t + \arg H(\omega_0))$

$x(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t} \Rightarrow y(t) = \sum_{n=-\infty}^{\infty} \tilde{D}_n \cdot e^{jn\omega_0 t} ; \tilde{D}_n = D_n \cdot H(n\omega_0)$



$x(t) = 3 + 2 \cdot \cos(5t)$
 Stabilt LTI

$y(t) = 3 \cdot \underbrace{H(0)}_{=1} + 2 \cdot \underbrace{|H(5)|}_{=1/3} \cdot \cos(5t + \underbrace{\arg H(5)}_{=-\pi/2})$
 $= 3 + 2/3 \cos(5t - \pi/2)$

LTI-SYSTEM & FREQUENZSIGNALER

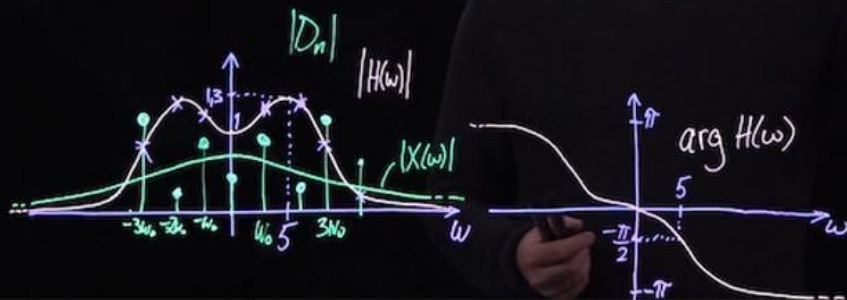
$x(t)$
 $X(\omega)$

Stabilt LTI-system
 $h(t), H(\omega)$

$y(t) = (x * h)(t)$
 $|Y(\omega)| = |X(\omega)| |H(\omega)|$

$x(t) = C_0 + C \cdot \cos(\omega_0 t) \Rightarrow y(t) = C_0 \cdot H(0) + C \cdot |H(\omega_0)| \cdot \cos(\omega_0 t + \arg H(\omega_0))$

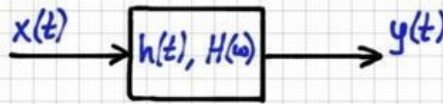
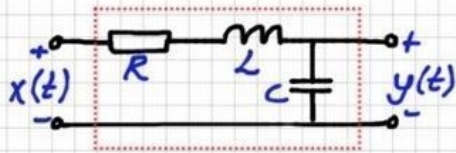
$x(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t} \Rightarrow y(t) = \sum_{n=-\infty}^{\infty} \tilde{D}_n \cdot e^{jn\omega_0 t} ; \tilde{D}_n = D_n \cdot |H(n\omega_0)|$



$x(t) = 3 + 2 \cdot \cos(5t)$
 Stabilt LTI

$y(t) = 3 \cdot \underbrace{H(0)}_{=1} + 2 \cdot \underbrace{|H(5)|}_{=1/3} \cdot \cos(5t + \underbrace{\arg H(5)}_{=-\pi/2})$
 $= 3 + 2/3 \cos(5t - \pi/2)$

(Större) Räkneexempel – Systemanalys med fouriertransformen



$$H(w) = \frac{1}{1 - w^2 + jw}$$

d) $x_2(t) = 4 + 3\cos(2t + \frac{3\pi}{4})$ [V]. Beräkna $y_2(t)$.

Vi vet: För stabilt LTI-system (som här!) med stationär insignal

$$x(t) = C_0 + C \cdot \cos/\sin(\omega t) \Rightarrow y(t) = C_0 \cdot H(0) + C \cdot |H(\omega)| \cdot \cos/\sin(\omega t + \arg H(\omega))$$

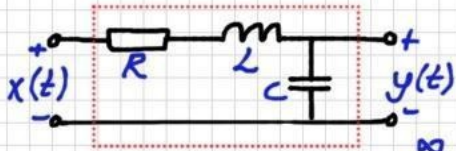
$$\text{Här: } y_2(t) = 4 \cdot H(0) + 3 \cdot |H(2)| \cdot \cos(2t + \frac{3\pi}{4} + \arg H(2))$$

$$\begin{aligned} \text{där } H(0) &= \frac{1}{1 - 0^2 + j \cdot 0} = 1 \quad \& \quad H(2) = \frac{1}{1 - 2^2 + j2} = \frac{1}{-3 + j2} = \frac{1}{\sqrt{(-3)^2 + 2^2}} \cdot e^{j \arctan \frac{2}{-3} + \pi} \\ &= \frac{1}{\sqrt{13}} \cdot e^{j(\arctan \frac{2}{-3} + \pi)} \quad \Rightarrow \quad y(t) = 4 + \frac{3}{\sqrt{13}} \cos(2t + \frac{3\pi}{4} + \arctan \frac{2}{-3} + \pi) \text{ [V]} \end{aligned}$$

Ty neg. realdel

e) Beräkna systemets stegsvar $g(t)$

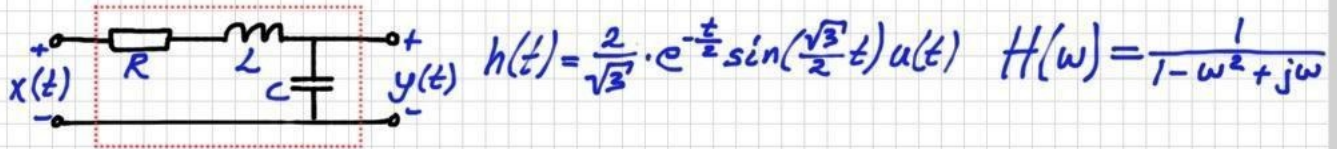
(Större) Räkneexempel – Systemanalys med fouriertransformen



$$h(t) = \frac{2}{\sqrt{3}} \cdot e^{-\frac{t}{2}} \sin(\frac{\sqrt{3}}{2} t) u(t) \quad H(w) = \frac{1}{1 - w^2 + jw}$$

$$\begin{aligned} \text{e) } g(t) &= (u * h)(t) = \int_{-\infty}^{\infty} u(t-\tau) h(\tau) d\tau = \int_{-\infty}^t h(\tau) d\tau = A \int_0^t e^{a\tau} \sin(\omega_0 \tau) d\tau \quad \left\{ \begin{array}{l} A = \frac{2}{\sqrt{3}} \\ a = -\frac{1}{2} \\ \omega_0 = \frac{\sqrt{3}}{2} \end{array} \right. \\ &= \left[\int u \cdot v = u \cdot V - \int u' \cdot V \right] / \left[\int \sin = -\cos, \int \cos = \sin \right] = A \left(\frac{1}{\omega_0} [e^{a\tau} \cos(\omega_0 \tau)]_0^t - \frac{a}{\omega_0} \int_0^t e^{a\tau} \cos(\omega_0 \tau) d\tau \right) \\ &= A \left(\frac{-1}{\omega_0} [e^{a\tau} \cos(\omega_0 \tau)]_0^t + \frac{a}{\omega_0} \left(\frac{1}{\omega_0} [e^{a\tau} \sin(\omega_0 \tau)]_0^t - \frac{a}{\omega_0} \int_0^t e^{a\tau} \sin(\omega_0 \tau) d\tau \right) \right) \\ &\Rightarrow A \left(1 + \frac{a^2}{\omega_0^2} \right) \int_0^t e^{a\tau} \sin(\omega_0 \tau) d\tau = \frac{A}{\omega_0} [e^{a\tau} (\frac{a}{\omega_0} \sin(\omega_0 \tau) - \cos(\omega_0 \tau))]_0^t = \frac{A}{\omega_0} (e^{at} (\sin(\omega_0 t) - \cos(\omega_0 t)) - e^0 (\sin(0) - \cos(0))) \\ &\Rightarrow g(t) = \frac{A \cdot \omega_0^2}{\omega_0 (\omega_0^2 + a^2)} \left(e^{-\frac{t}{2}} \left(\frac{1}{\sqrt{3}} \sin(\frac{\sqrt{3}}{2} t) - \cos(\frac{\sqrt{3}}{2} t) \right) + 1 \right) u(t) \quad \underbrace{-1(0-1)=1}_{\text{Phu, det var jobbigt!}} \end{aligned}$$

(Större) Räkneexempel – Systemanalys med fouriertransformen



c) Enklare: Använd fouriertransformen! (laplacetransformen ännu bättre...!)

$$\begin{aligned}
 g(t) &= (u * h)(t) \Rightarrow G(w) = U(w) \cdot H(w) = \left(\text{vp}\left\{ \frac{1}{jw} \right\} + \pi \delta(w) \right) \frac{1}{1 - w^2 + jw} \\
 &= \text{vp}\left\{ \frac{1}{jw(jw^2 + jw + 1)} \right\} + \frac{\pi}{1 - w^2 + jw} \delta(w) \stackrel{\text{P.B.U. som i c)}}{=} \text{vp}\left\{ \frac{1}{jw} - \frac{jw + 1}{(jw)^2 + jw + 1} \right\} + \pi \delta(w) \\
 &\stackrel{\text{Omforma, som i c)}}{=} \text{vp}\left\{ \frac{1}{jw} \right\} + \pi \delta(w) - \frac{\frac{1}{2} + jw}{\left(\frac{1}{2} + jw\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{2\sqrt{3}} \cdot \frac{\sqrt{3}/2}{\left(\frac{1}{2} + jw\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\
 &\stackrel{\text{Formels. Tab. 3:2, 3:10, 3:11}}{\Rightarrow} \underline{g(t) = \left(1 - e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}} e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2}t\right) \right) u(t)}
 \end{aligned}$$

Digital kommunikation

Digital signalering med analoga signalvågformer

Basbandsmodulation,

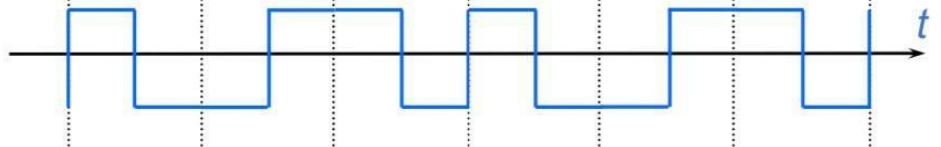
Exempel 1:



Exempel 2:

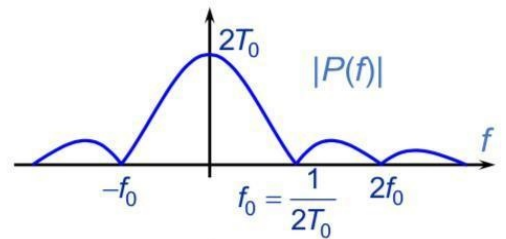
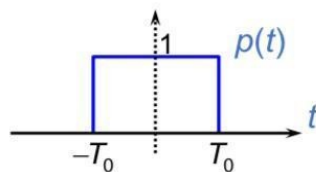


Exempel 3:

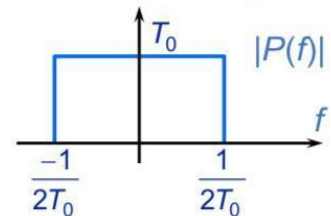
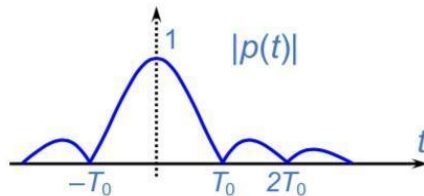


Ex. på signalpulsformer för basbandskanaler:

$$p(t) = u(t + T_0) - u(t - T_0)$$

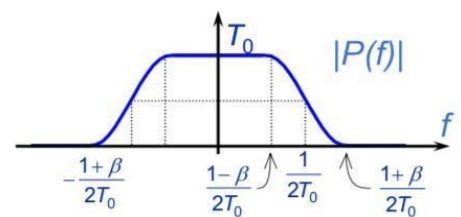
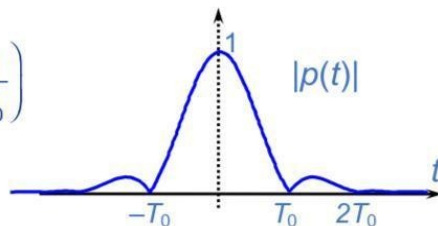


$$p(t) = \text{sinc}\left(\frac{t}{T_0}\right)$$



$$p(t) = \frac{\cos(2\beta\pi t/T_0)}{1 - (4\beta t/T_0)^2} \text{sinc}\left(\frac{t}{T_0}\right)$$

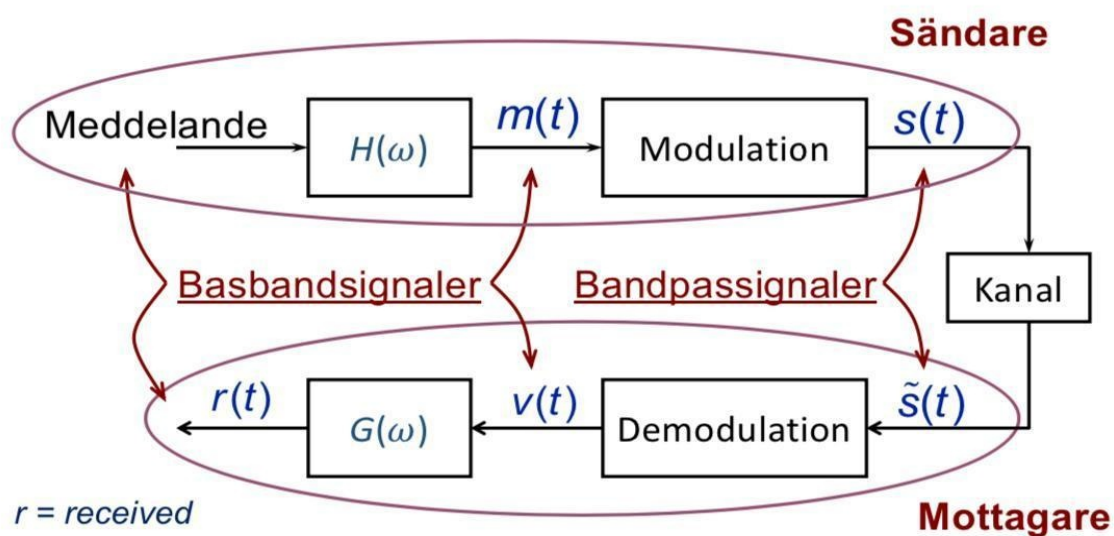
"Raised cosine"



Vanligt: högfrekvent signalering

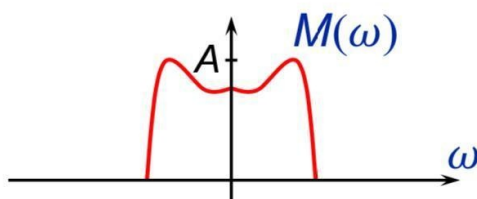
(T.ex. ADSL, mobiltn, radio, satellit, bluetooth, WLAN m.m.)

Typiskt analogt kommunikationssystem:

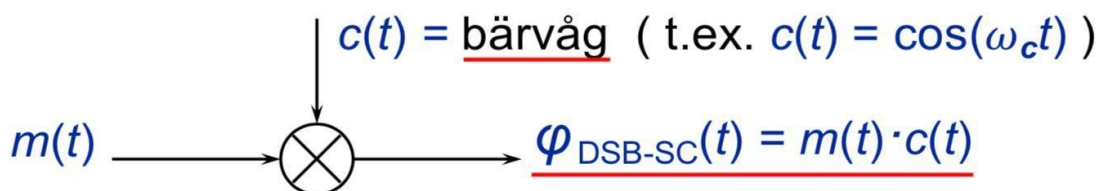


Generell Amplitudmodulering

Basbandsignal (här: meddelandesignalen $m(t)$):

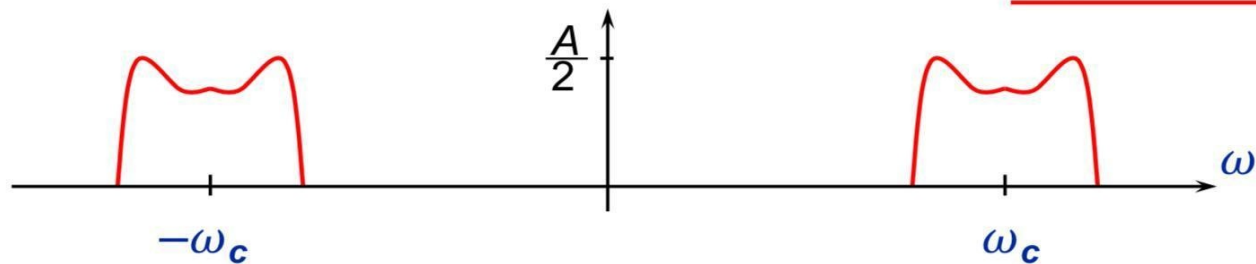


Amplitudmodulering (AM-DSB-SC):



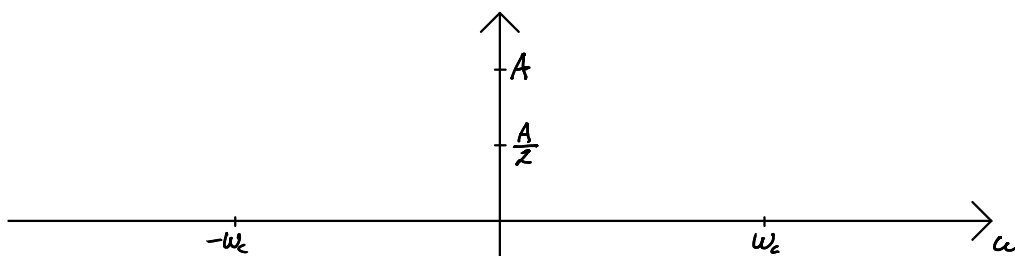
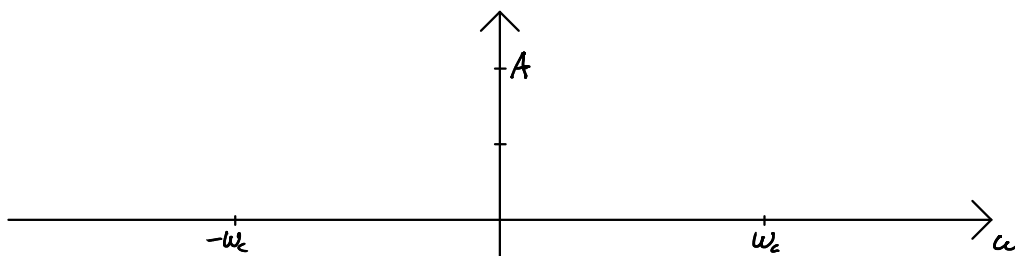
Bandpassignal (AM-DSB-SC):

$$\underline{\Phi_{\text{DSB-SC}}(\omega) = \mathcal{F}\{m(t) \cdot c(t)\} = \frac{1}{2\pi}(M * C)(\omega)}$$

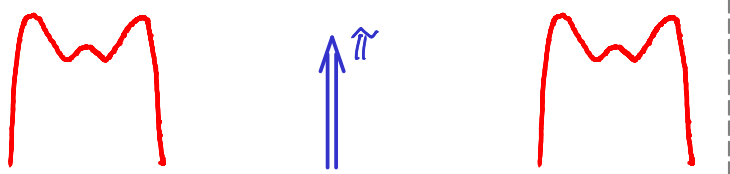


där $C(\omega) = \mathcal{F}\{\cos(\omega_c t)\} = \pi(\delta(\omega + \omega_c) + \delta(\omega - \omega_c))$

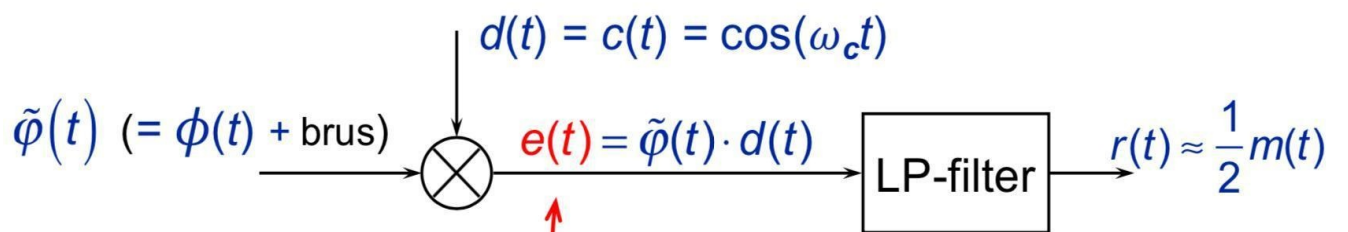
$$\Rightarrow \boxed{\Phi_{\text{DSB-SC}}(\omega) = \frac{1}{2}(M(\omega + \omega_c) + M(\omega - \omega_c))}$$



Används under föreläsningen:

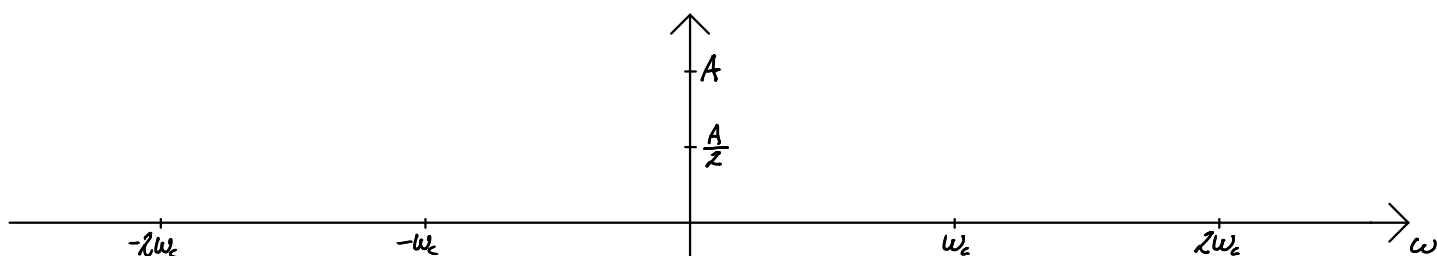
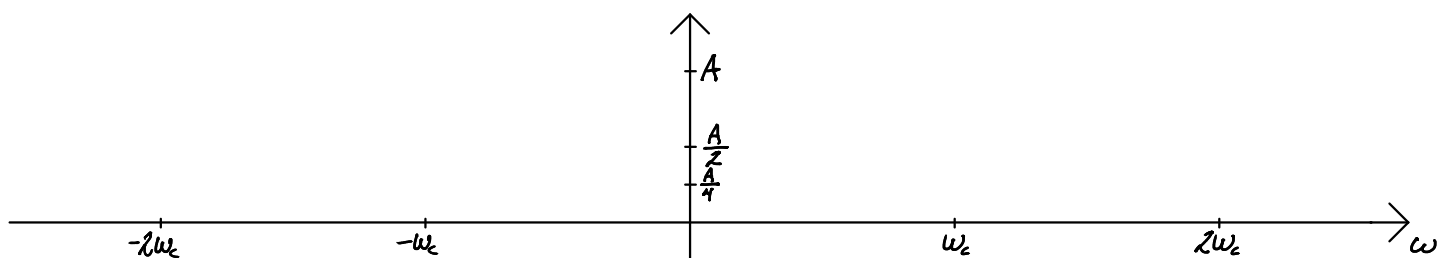
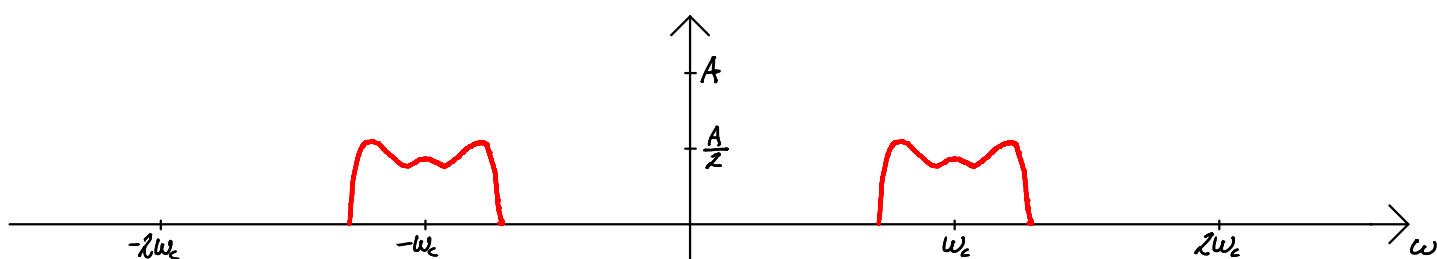
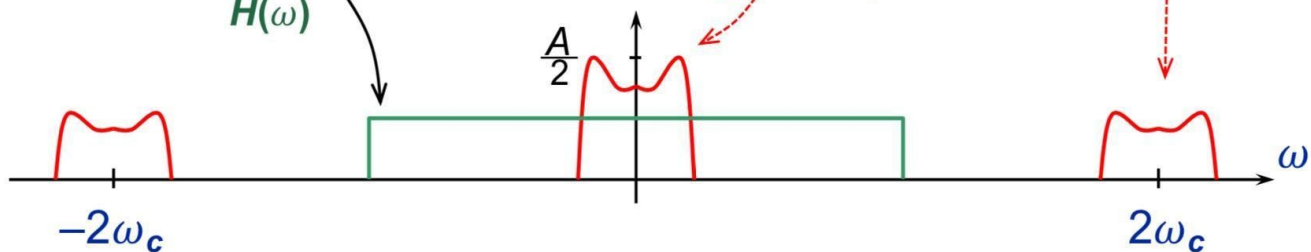


Demodulering + LP-filter:



Idealt LP-filter,
 $H(\omega)$

$$E(\omega) = \frac{1}{2}M(\omega) + \frac{1}{4}(M(\omega + 2\omega_c) + M(\omega - 2\omega_c))$$



Används under föreläsningen:



Sidoinformation (ingår inte i kursen):

Vid praktisk demodulering av AM-DSB (där bärvågen ingår i den modulerade signalen) används ofta en enveloppdetektor:

