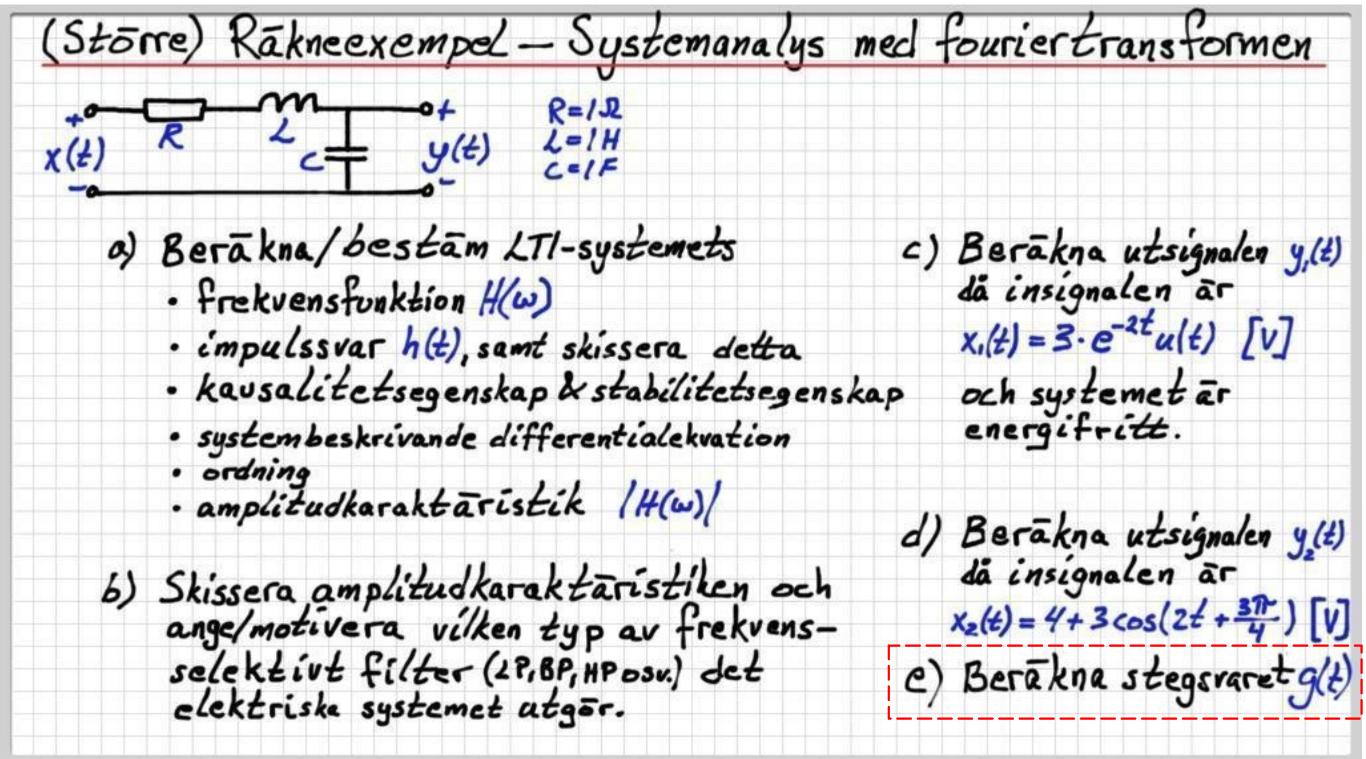
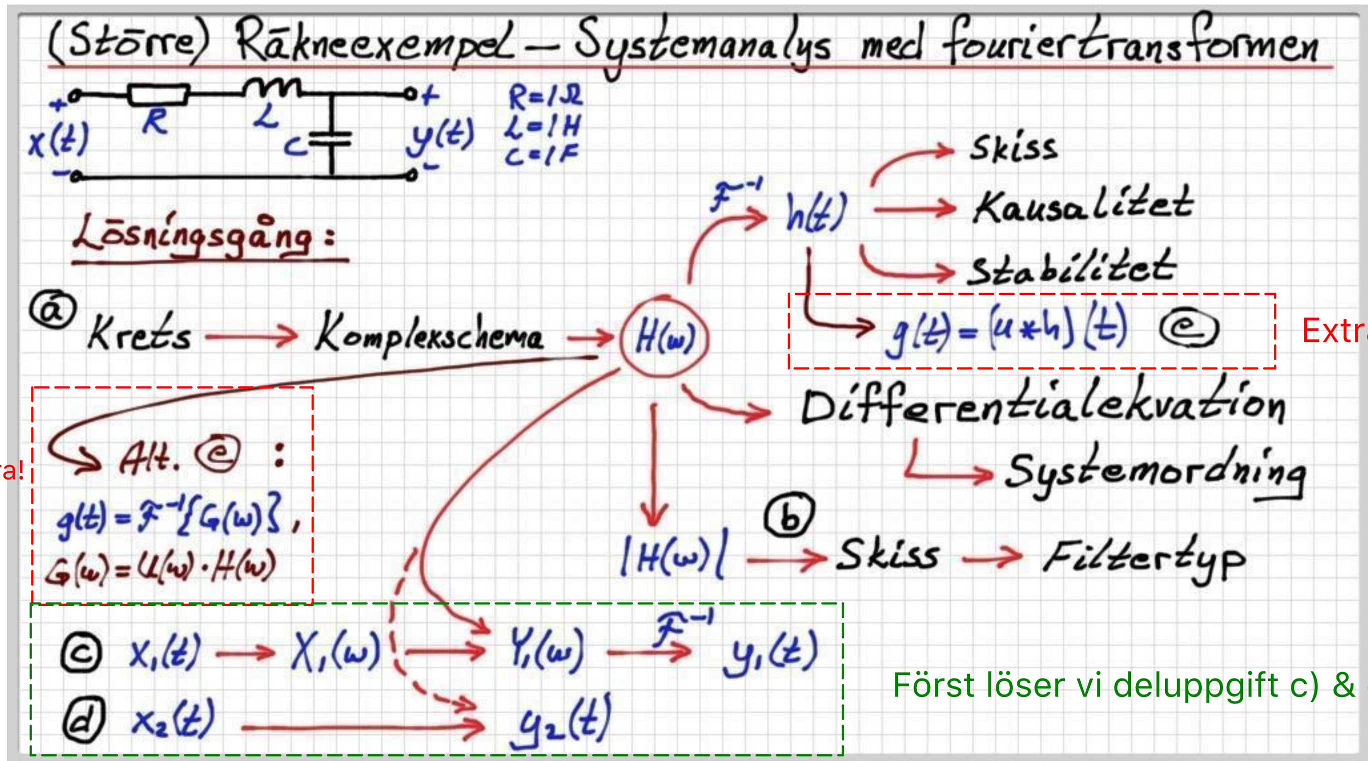
- Forts. på förra föreläsningens stora räkneexempel – fouriertransformanalys av ett LTI-system, deluppgift c) & d) samt extrauppgift e)
    - Video inför deluppgift d): LTI-system och frekvenssignaler
- Tillämpningsexempel, fouriertransformanalys: Amplitudmodulering (AM)

Från förra föreläsningen – då gick vi igenom deluppgift a) & b):



Extra!

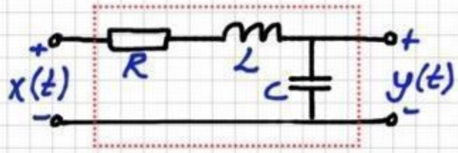


Först löser vi deluppgift c) & d)!

c) Beräkna utsignalen $y_1(t)$ då insignalen är $x_1(t)=3\cdot e^{-2t}u(t)$ [V]

och systemet är energifritt.

(Större) Räkneexempel – Systemanalys med fouriertransformen



$$x(t) \longrightarrow \boxed{h(t),\ H(\omega)} \longrightarrow y(t) \qquad H(\omega)=\frac{1}{1-\omega^2+j\omega}$$

c) $x_1(t)=3\cdot e^{-2t}u(t)$ [V]. Beräkna $y_1(t)$. // $\quad y_1(t)=(x*h)(t) \Rightarrow$

$$Y_1(\omega)=X_1(\omega)H(\omega)=\Big/\ \text{Formels. Tab. 3:5,}\ e^{-at}u(t) \Leftrightarrow \frac{1}{a+j\omega}\Big/ = \frac{3}{2+j\omega}\cdot\frac{1}{1-\omega^2+j\omega} = /\text{P.B.U.}/$$

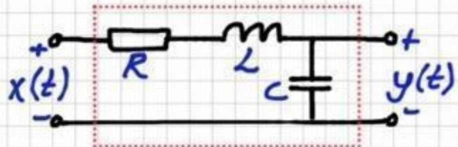
$$=\frac{A}{2+j\omega}+\frac{B(j\omega)+C}{(j\omega)^2+j\omega+1} = \Big/ \left.\begin{matrix} A+B & =0 \\ A+2B+C & =0 \\ A \quad +2C & =3 \end{matrix}\right\} \Rightarrow A=1,\ B=-1,\ C=1 \Big/$$

$$=\frac{1}{2+j\omega}+\frac{-j\omega+1}{(j\omega+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

$\mathcal{F}^{-1}\left\{\frac{1}{2+j\omega}\right\}=e^{-2t}u(t)$ enligt Tab. 3:5

men vad är $\mathcal{F}^{-1}$ av den andra termen?

(Större) Räkneexempel – Systemanalys med fouriertransformen



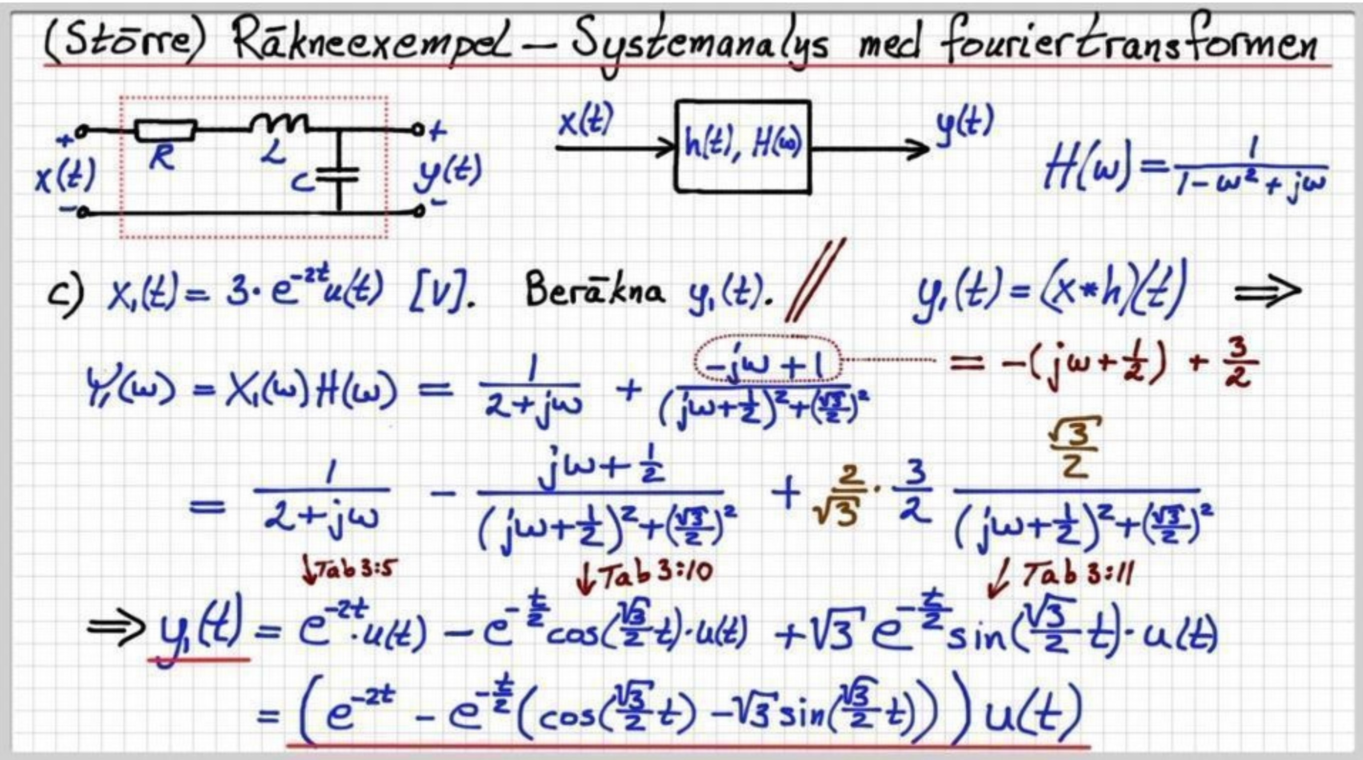
$$x(t) \longrightarrow \boxed{h(t),\ H(\omega)} \longrightarrow y(t) \qquad H(\omega)=\frac{1}{1-\omega^2+j\omega}$$

c) $x_1(t)=3\cdot e^{-2t}u(t)$ [V]. Beräkna $y_1(t)$. // $\quad y_1(t)=(x*h)(t) \Rightarrow$

$$Y_1(\omega)=X_1(\omega)H(\omega)=\frac{1}{2+j\omega}+\frac{-j\omega+1}{(j\omega+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

Formelsaml. Tab. 3:10 & 3:11 $\Rightarrow$

$$\mathcal{F}\{e^{-at}\cos(\omega_0 t)u(t)\}=\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2} \qquad \mathcal{F}\{e^{-at}\sin(\omega_0 t)u(t)\}=\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$$



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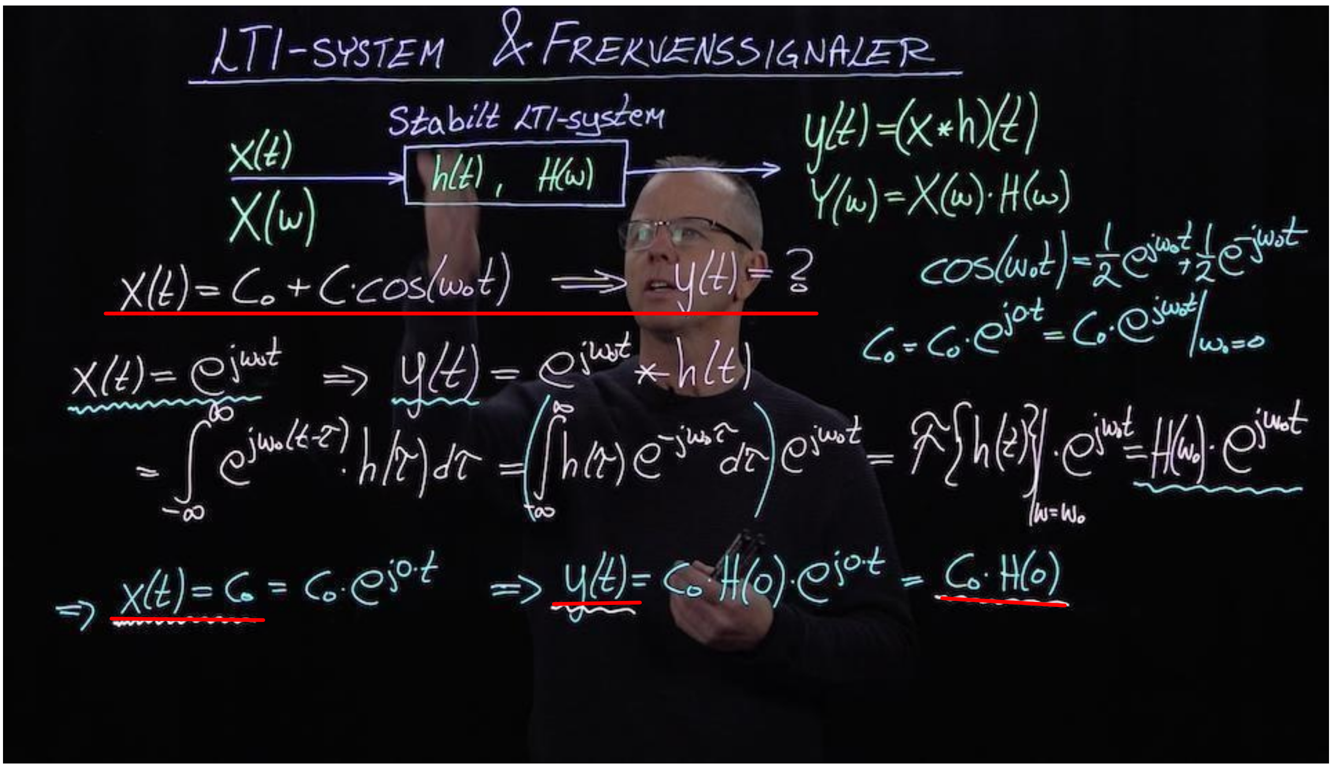
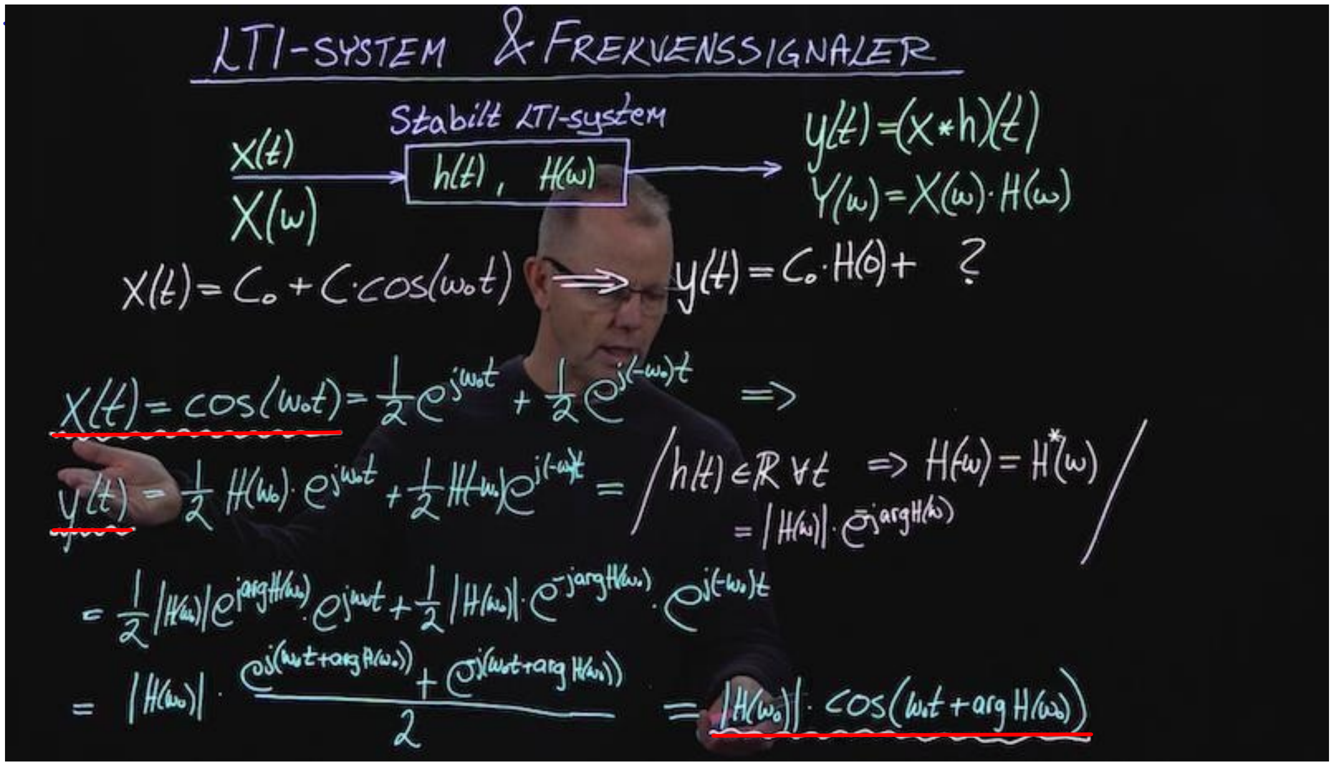
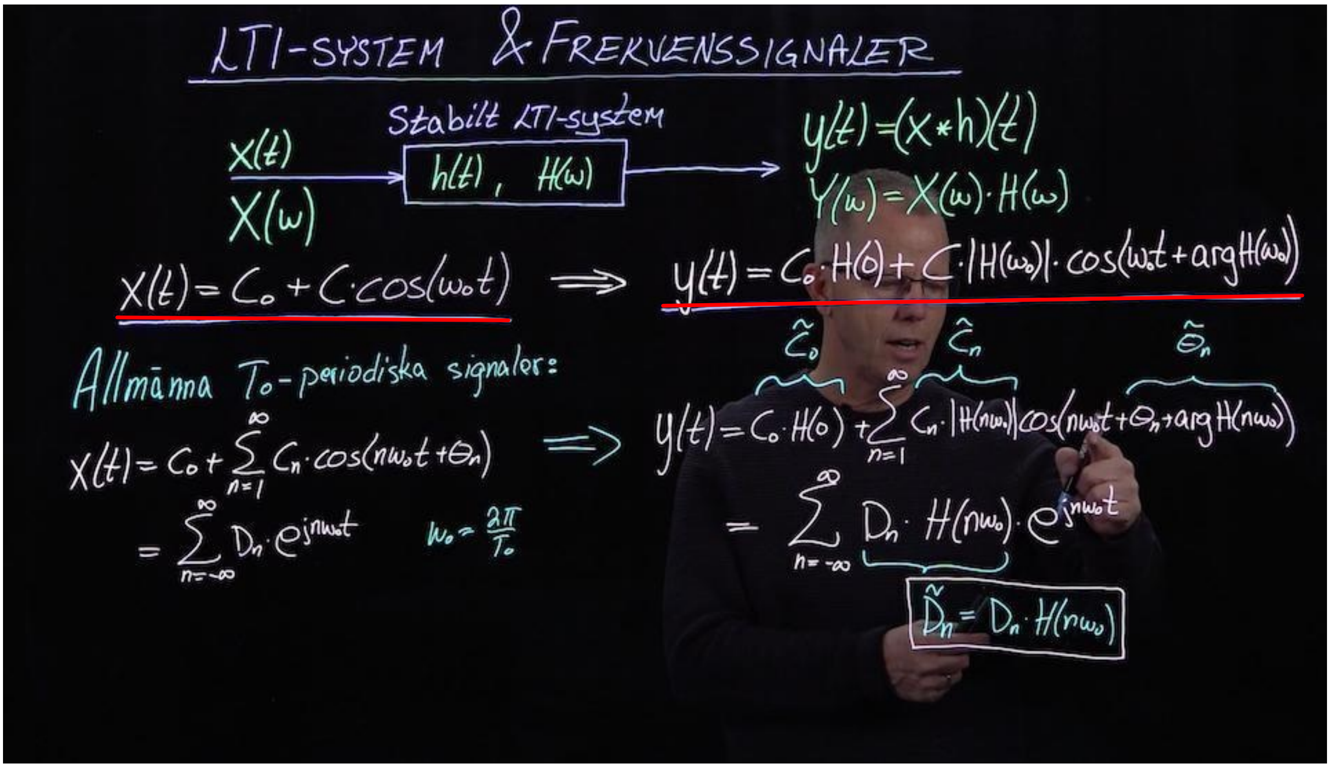
d) Beräkna utsignalen $y_2(t)$ då insignalen är $x_2(t) = 4 + 3\cos(2t + \frac{3\pi}{4})$ [V]

Först den förberedande videon om hur LTI-system amplitudskalar och fasförskjuter frekvenssignaler:

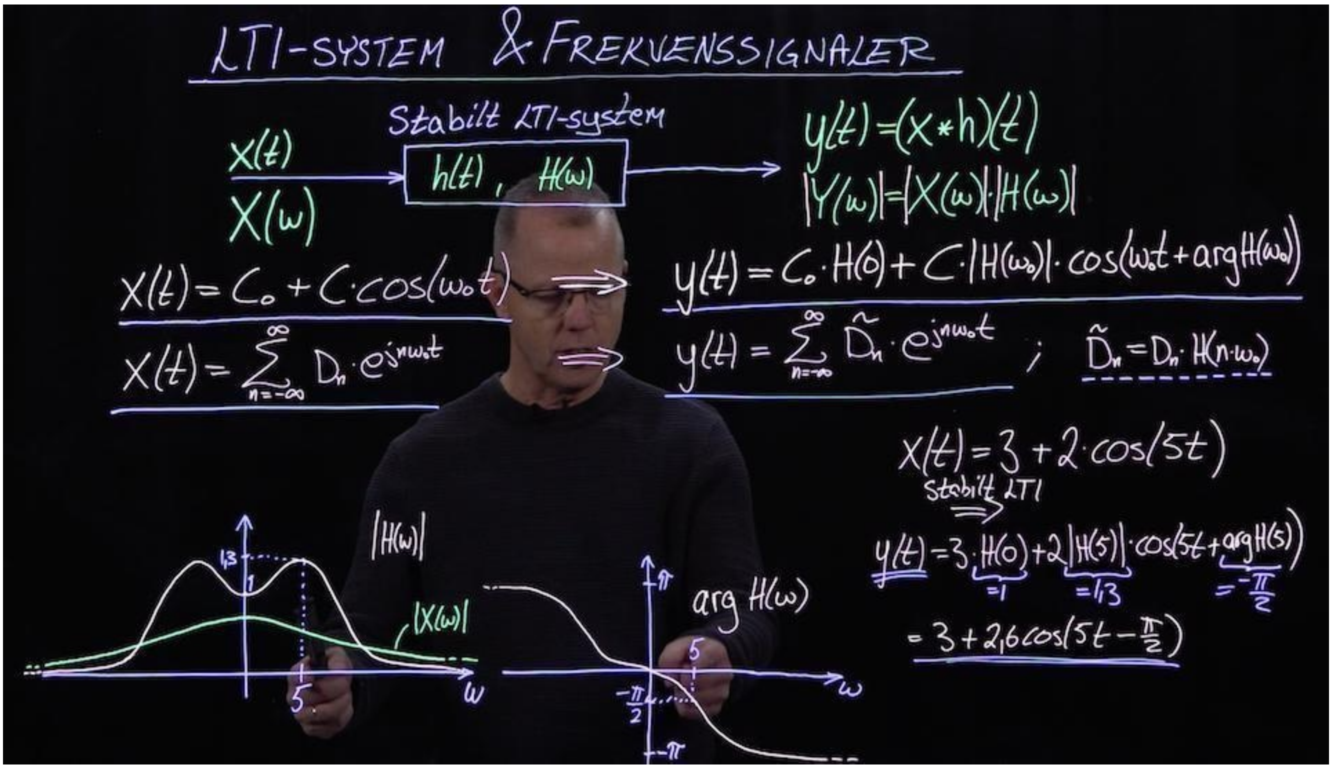
Från föreläsningen om fourierserier känner du redan till hur LTI-system amplitudskalar och fasvrider en insignal x(t) = Co + C*cos(wot). Här är dock en härledning – tillsammans med förtydligande exempel – som baserar sig mer på fouriertransformen.

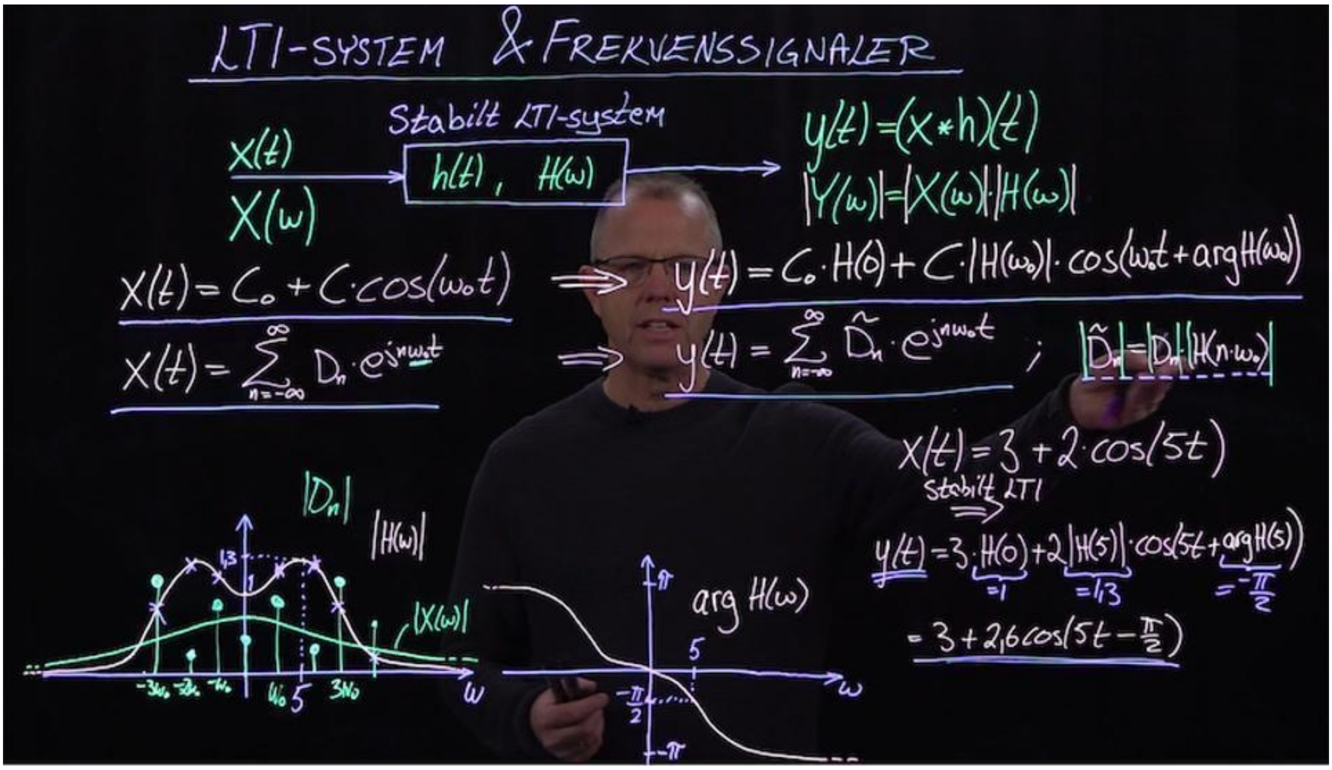




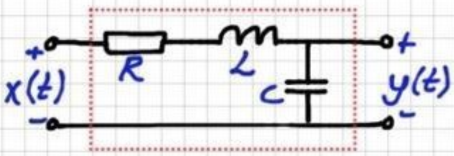


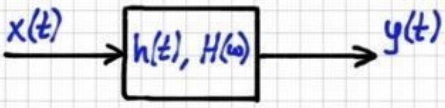
I videon skriver jag $\tilde{D}_n$ i stället för $\hat{D}_n$ — det spelar ingen större roll vilken vi använder.


LTI-SYSTEM & FREKVENSSIGNALER
Stabilt LTI-system
$x(t)$
$X(\omega)$
$h(t)$, $H(\omega)$
$y(t) = (x*h)(t)$
$|Y(\omega)| = |X(\omega)|\cdot|H(\omega)|$
$x(t) = C_0 + C\cdot\cos(\omega_0 t) \Longrightarrow y(t) = C_0\cdot H(0) + C\cdot|H(\omega_0)|\cdot\cos(\omega_0 t + \arg H(\omega_0))$
$x(t) = \sum_{n=-\infty}^{\infty} D_n\cdot e^{jn\omega_0 t} \Longrightarrow y(t) = \sum_{n=-\infty}^{\infty} \tilde{D}_n\cdot e^{jn\omega_0 t}$; $\tilde{D}_n = D_n\cdot H(n\cdot\omega_0)$
$x(t) = 3 + 2\cdot\cos(5t)$
Stabilt LTI
$y(t) = 3\cdot\underbrace{H(0)}_{=1} + 2\underbrace{|H(5)|}_{=1,3}\cdot\cos(5t + \underbrace{\arg H(5)}_{=-\frac{\pi}{2}})$
$= 3 + 2{,}6\cos(5t - \frac{\pi}{2})$
$|H(\omega)|$
$|X(\omega)|$
1,3
1
5
ω
$\arg H(\omega)$
π
$-\frac{\pi}{2}$
$-\pi$


LTI-SYSTEM & FREKVENSSIGNALER
Stabilt LTI-system
$x(t)$
$X(\omega)$
$h(t)$, $H(\omega)$
$y(t) = (x*h)(t)$
$|Y(\omega)| = |X(\omega)|\cdot|H(\omega)|$
$x(t) = C_0 + C\cdot\cos(\omega_0 t) \Longrightarrow y(t) = C_0\cdot H(0) + C\cdot|H(\omega_0)|\cdot\cos(\omega_0 t + \arg H(\omega_0))$
$x(t) = \sum_{n=-\infty}^{\infty} D_n\cdot e^{jn\omega_0 t} \Longrightarrow y(t) = \sum_{n=-\infty}^{\infty} \tilde{D}_n\cdot e^{jn\omega_0 t}$; $|\tilde{D}_n| = |D_n|\cdot|H(n\cdot\omega_0)|$
$x(t) = 3 + 2\cdot\cos(5t)$
Stabilt LTI
$y(t) = 3\cdot\underbrace{H(0)}_{=1} + 2\underbrace{|H(5)|}_{=1,3}\cdot\cos(5t + \underbrace{\arg H(5)}_{=-\frac{\pi}{2}})$
$= 3 + 2{,}6\cos(5t - \frac{\pi}{2})$
$|D_n|$
$|H(\omega)|$
$|X(\omega)|$
1,3
1
$-3\omega_0$ $-2\omega_0$ $-\omega_0$ ω_0 5 $3\omega_0$
ω
$\arg H(\omega)$
π
$-\frac{\pi}{2}$
$-\pi$

(Större) Räkneexempel – Systemanalys med fouriertransformen





$$H(\omega)=\frac{1}{1-\omega^2+j\omega}$$

d) $x_2(t)=4+3\cos(2t+\frac{3\pi}{4})$ [V]. Beräkna $y_2(t)$.

Vi vet: För stabilt LTI-system (som här!) med stationär insignal

$x(t)=C_0+C\cdot\cos/\sin(\omega_0 t) \Rightarrow y(t)=C_0\cdot H(0)+C\cdot|H(\omega_0)|\cdot\cos/\sin(\omega_0 t+\arg H(\omega_0))$

Här: $\boxed{y_2(t)=4\cdot H(0)+3\cdot|H(2)|\cdot\cos\left(2t+\frac{3\pi}{4}+\arg H(2)\right)}$

där $H(0)=\frac{1}{1-0^2+j\cdot 0}=1$ & $H(2)=\frac{1}{1-2^2+j2}=\frac{1}{-3+j2}=\frac{1}{\sqrt{(-3)^2+2^2}\cdot e^{j(\arctan\frac{2}{-3}+\pi)}}$

$=\underbrace{\frac{1}{\sqrt{13}}}_{|H(2)|}\cdot e^{-j\overbrace{(\arctan\frac{2}{3}+\pi)}^{\arg H(2)}} \Rightarrow y(t)=4+\frac{3}{\sqrt{13}}\cos(2t+\frac{3\pi}{4}+\arctan\frac{2}{3}+\pi)$ [V]

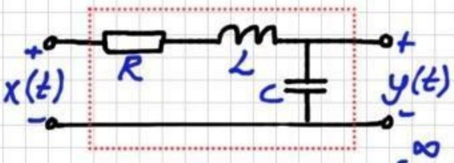
4·1

Ty neg. realdel

---

e) Beräkna systemets stegsvar $g(t)$

(Större) Räkneexempel – Systemanalys med fouriertransformen



$h(t)=\frac{2}{\sqrt{3}}\cdot e^{-\frac{t}{2}}\sin(\frac{\sqrt{3}}{2}t)u(t) \quad H(\omega)=\frac{1}{1-\omega^2+j\omega}$

e) $g(t)=(u*h)(t)=\int_{-\infty}^{\infty}u(t-\tau)h(\tau)\,d\tau=\int_{-\infty}^{t}h(\tau)\,d\tau=A\int_0^t e^{a\tau}\sin(\omega_0\tau)\,d\tau \quad \begin{cases}A=\frac{2}{\sqrt{3}}\\ a=-\frac{1}{2}\\ \omega_0=\frac{1}{A}\end{cases}$

$=\left/\begin{array}{l}\int u\cdot v=u\cdot V-\int u'\cdot V\\ \int\sin=-\cos,\ \int\cos=\sin\end{array}\right/=A\left(\frac{1}{-\omega_0}\left[e^{a\tau}\cos(\omega_0\tau)\right]_0^t-\frac{a}{-\omega_0}\int_0^t e^{a\tau}\cos(\omega_0\tau)\,d\tau\right)$

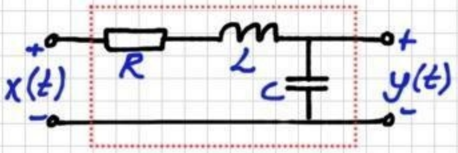
$=A\left(\frac{-1}{\omega_0}\left[e^{a\tau}\cos(\omega_0\tau)\right]_0^t+\frac{a}{\omega_0}\left(\frac{1}{\omega_0}\left[e^{a\tau}\sin(\omega_0\tau)\right]_0^t-\frac{a}{\omega_0}\int_0^t e^{a\tau}\sin(\omega_0\tau)\,d\tau\right)\right)$

$\Rightarrow A\left(1+\frac{a^2}{\omega_0^2}\right)\int_0^t e^{a\tau}\sin(\omega_0\tau)\,d\tau=\frac{A}{\omega_0}\left[e^{a\tau}\left(\frac{a}{\omega_0}\sin(\omega_0 t)-\cos(\omega_0 t)\right)\right]_0^t=\frac{A}{\omega_0}\left(e^{at}(\sin(\omega_0 t)-\cos(\omega_0 t))\underbrace{-e^0(\sin(0)-\cos(0))}_{=-1(0-1)=1}\right)$

$\Rightarrow g(t)==\underbrace{\frac{A\cdot\omega_0^2}{\omega_0(\omega_0^2+a^2)}}_{=1}\left(e^{-\frac{t}{2}}\left(\frac{-1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}t)-\cos(\frac{\sqrt{3}}{2}t)\right)+1\right)u(t)$

Phu, det var jobbigt!

(Större) Räkneexempel – Systemanalys med fouriertransformen



$$h(t) = \frac{2}{\sqrt{3}} \cdot e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2}t\right) u(t) \qquad H(\omega) = \frac{1}{1-\omega^2+j\omega}$$

e) Enklare: Använd fouriertransformen! (Laplacetransformen ännu bättre...!)

$$g(t) = (u * h)(t) \Rightarrow G(\omega) = U(\omega) \cdot H(\omega) = \left(vp\left\{\frac{1}{j\omega}\right\} + \pi\delta(\omega)\right)\frac{1}{1-\omega^2+j\omega}$$

$$= vp\left\{\frac{1}{j\omega((j\omega)^2+j\omega+1)}\right\} + \underbrace{\frac{\pi}{1-\omega^2+j\omega}\delta(\omega)}_{=\frac{\pi}{1-0+j0}\delta(\omega)} = \Big/ \begin{matrix}\text{P.B.U.}\\ \text{som i c)}\end{matrix} \Big/ = vp\left\{\frac{1}{j\omega} - \underbrace{\frac{j\omega+1}{(j\omega)^2+j\omega+1}}_{=\frac{j\omega+\frac{1}{2}+\frac{1}{2}}{(j\omega+\frac{1}{2})^2+\frac{3}{4}}}\right\} + \pi\delta(\omega)$$

Omforma, som i c)

$$= vp\left\{\frac{1}{j\omega}\right\} + \pi\delta(\omega) - \frac{\frac{1}{2}+j\omega}{(\frac{1}{2}+j\omega)^2+(\frac{\sqrt{3}}{2})^2} - \frac{1}{2}\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}/2}{(\frac{1}{2}+j\omega)^2+(\frac{\sqrt{3}}{2})^2}$$

Formels. Tab. 3:2, 3:10, 3:11

$$\Rightarrow \quad g(t) = \left(1 - e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)u(t)$$

Digital kommunikation

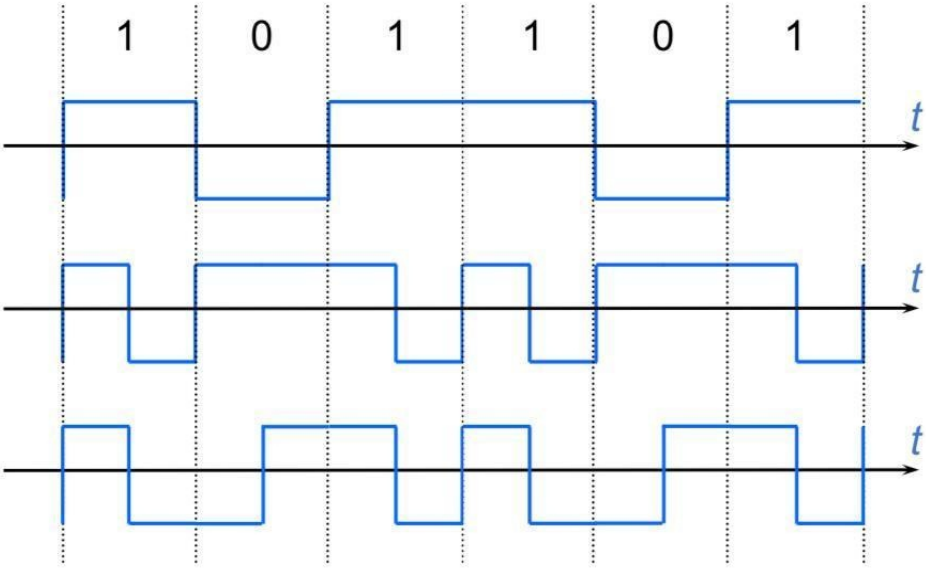
Digital signalering med analoga signalvågformer

Basbandsmodulation,

Exempel 1:

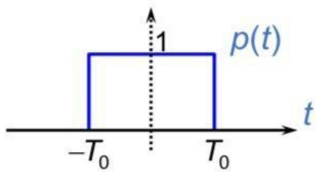
Exempel 2:

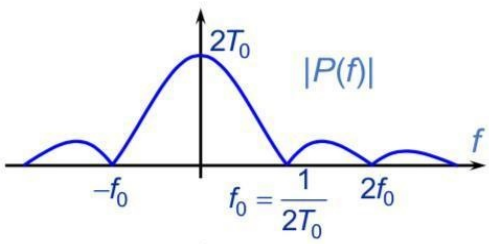
Exempel 3:



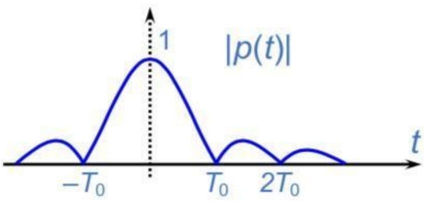
Ex. på signalpulsformer för basbandskanaler:

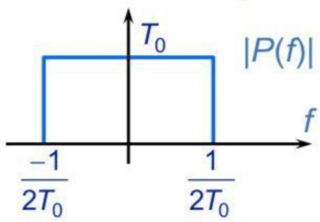
$p(t) = u(t+T_0) - u(t-T_0)$





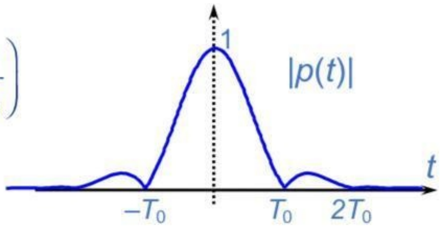
$p(t) = \text{sinc}\left(\frac{t}{T_0}\right)$

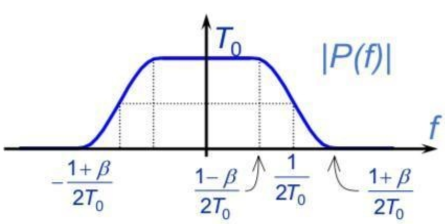




$p(t) = \frac{\cos(2\beta\pi t/T_0)}{1-(4\beta t/T_0)^2}\text{sinc}\left(\frac{t}{T_0}\right)$

”Raised cosine”

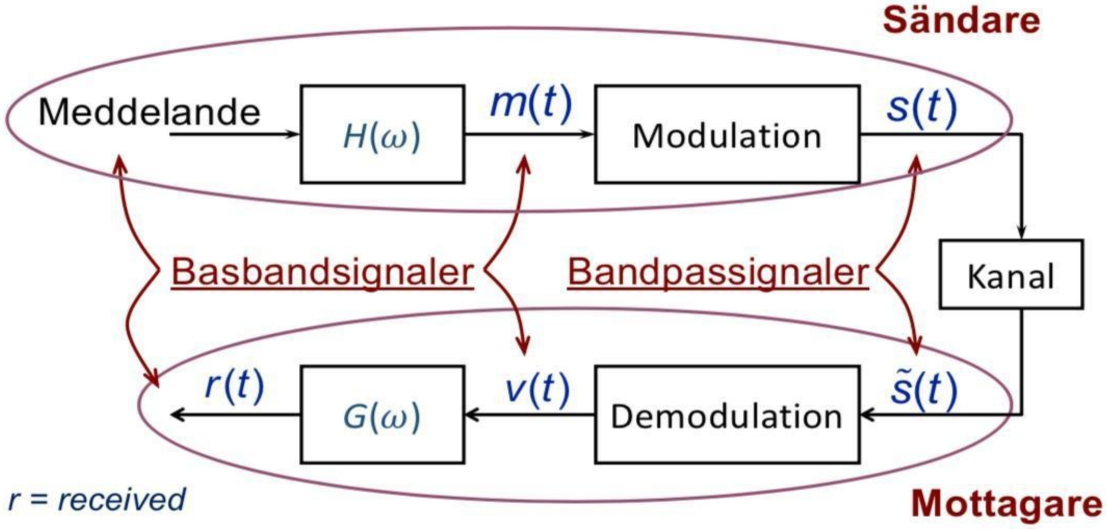




Vanligt: högfrekvent signalerering

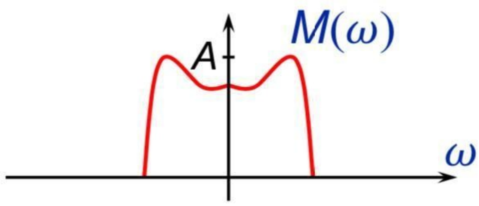
(T.ex. ADSL, mobiltfn, radio, satellit, bluetooth, WLAN m.m.)

Typiskt analogt kommunikationssystem:

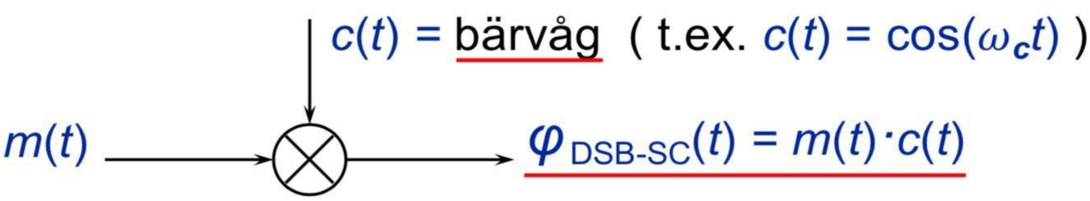


Generell Amplitudmodulering

Basbandsignal (här: meddelandesignalen $m(t)$):

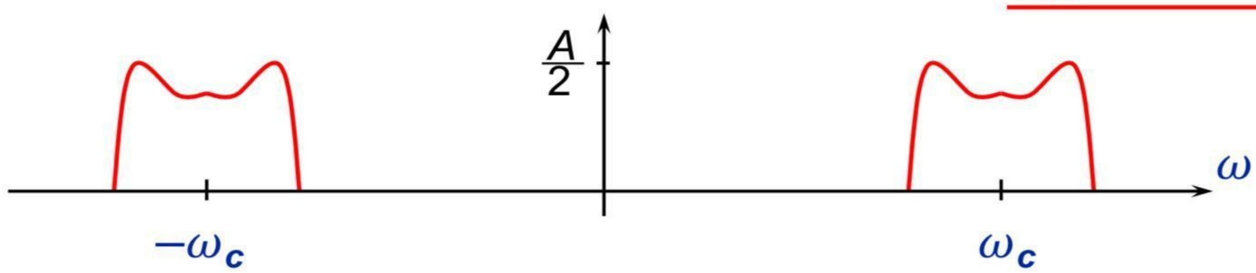


Amplitudmodulering (AM-DSB-SC):



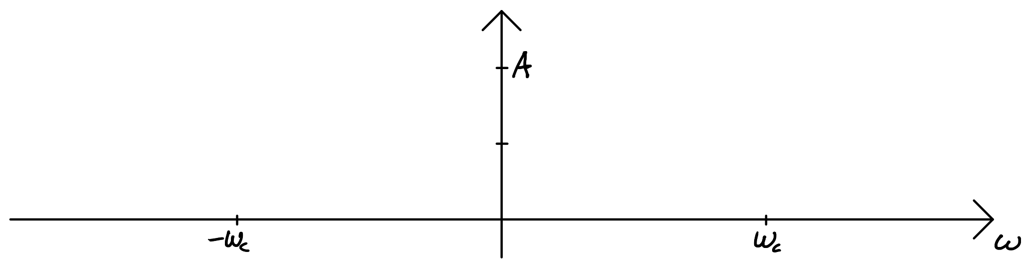
Bandpassignal (AM-DSB-SC):

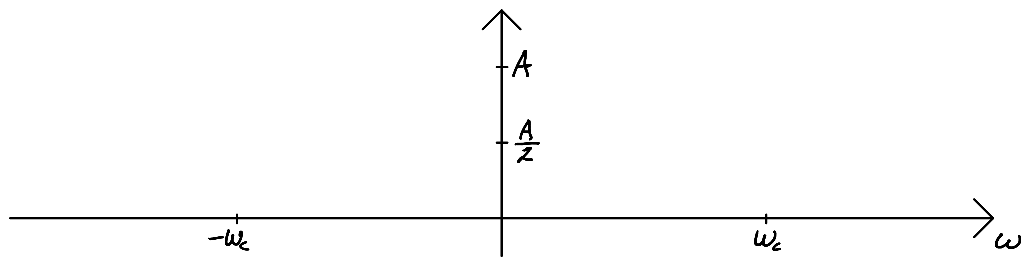
$$\Phi_{\text{DSB-SC}}(\omega) = \mathcal{F}\{m(t)\cdot c(t)\} = \frac{1}{2\pi}(M * C)(\omega)$$



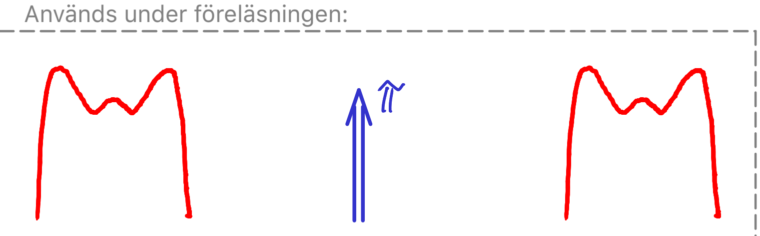
där $\quad C(\omega) = \mathcal{F}\{\cos(\omega_c t)\} = \pi\left(\delta(\omega+\omega_c) + \delta(\omega-\omega_c)\right)$

$$\Rightarrow \quad \boxed{\Phi_{\text{DSB-SC}}(\omega) = \frac{1}{2}\left(M(\omega+\omega_c) + M(\omega-\omega_c)\right)}$$

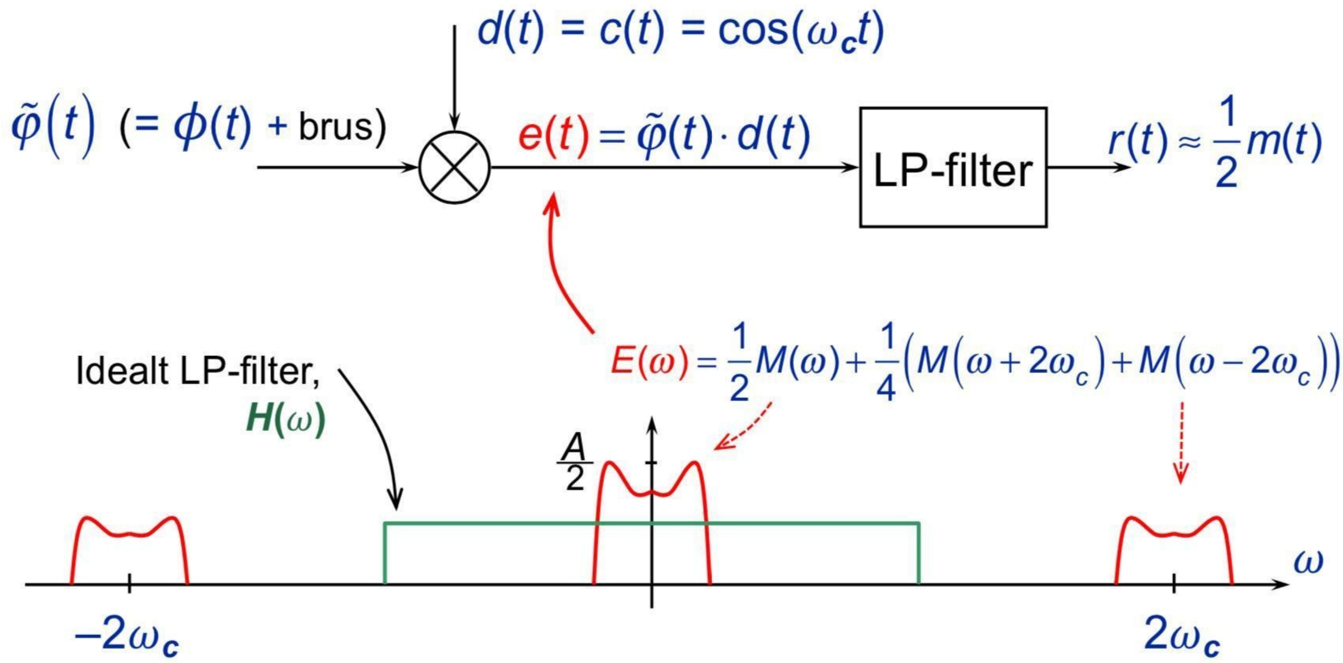


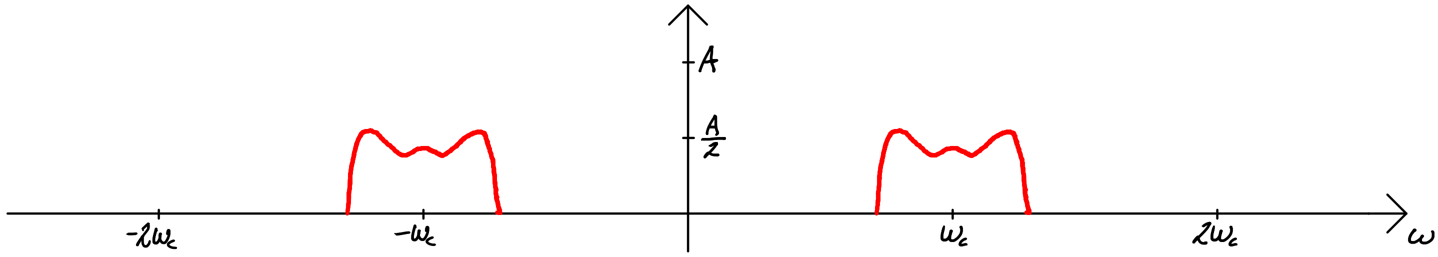


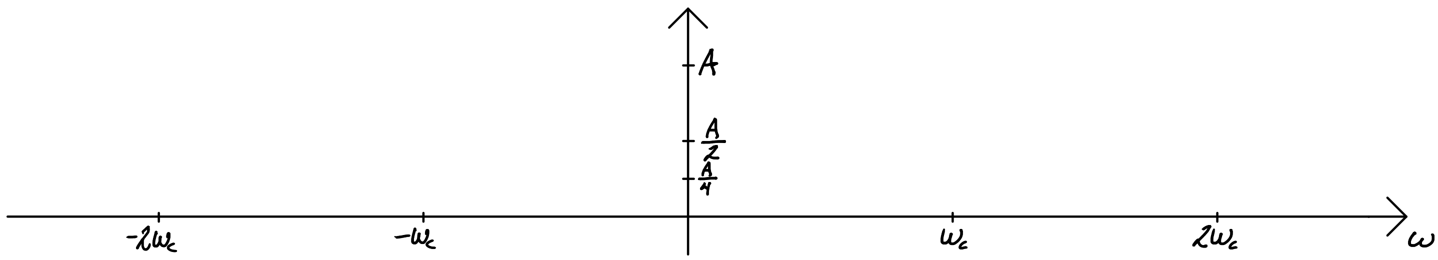
Används under föreläsningen:

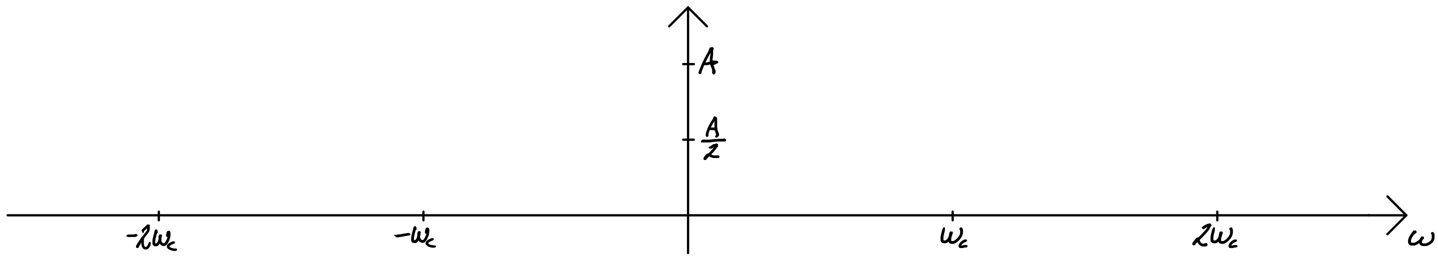


Demodulering + LP-filter:









Används under föreläsningen:

Sidoinformation (ingår inte i kursen):
Vid praktisk demodulering av AM-DSB (där bärvågen ingår i den modulerade signalen) används ofta en enveloppdetektor:

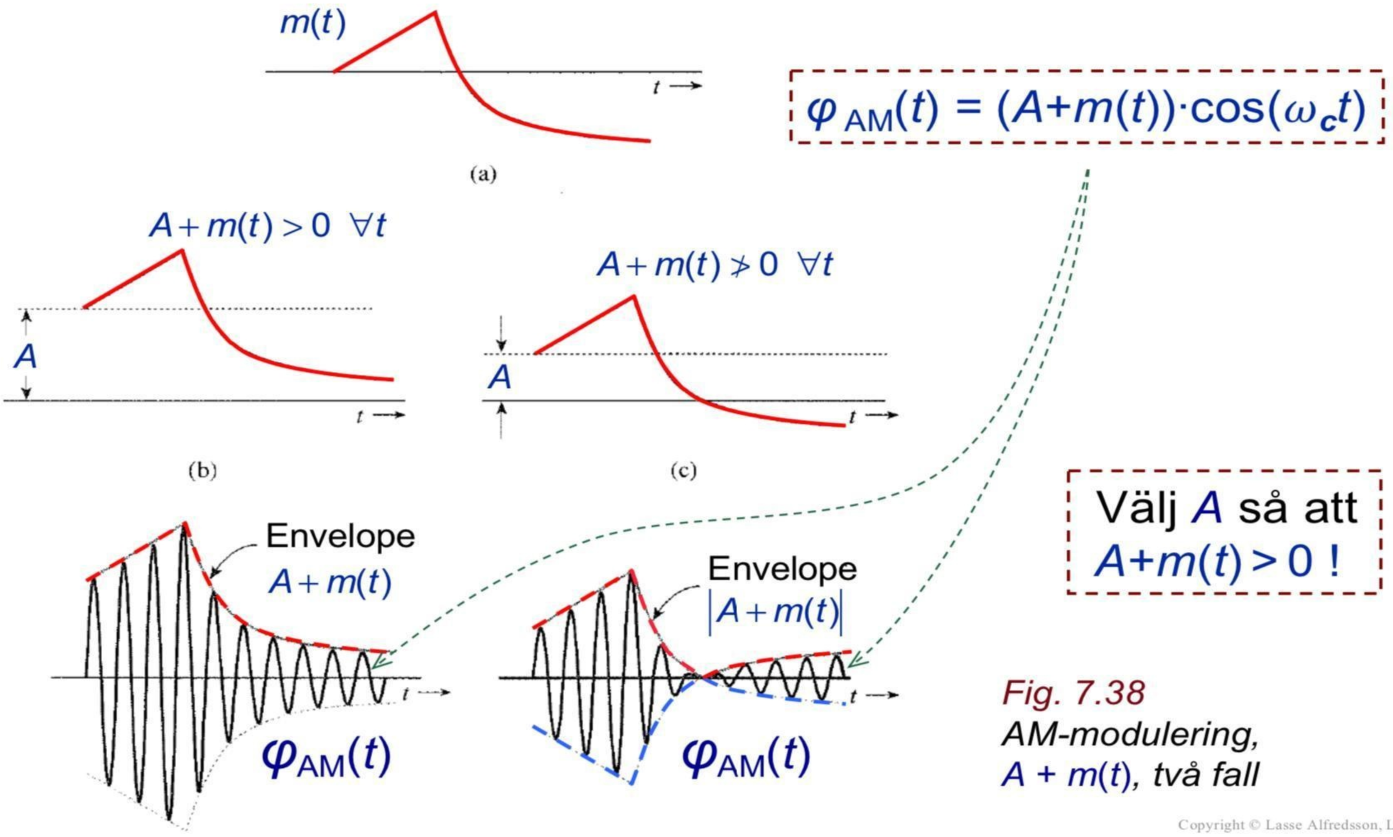


Fig. 7.38
AM-modulering,
$A + m(t)$, två fall