FOURIERTRANSFORMEN

- Fouriertransformen till *x(t)*:

$$\mathcal{F}\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

(Jfr. fourierserie: $C_k = \frac{1}{T}\int_{\alpha}^{\alpha+T} x(t) e^{-jk\omega_1 t} dt$)

- Inversa fouriertransformen till *X(ω)*:

$$\mathcal{F}^{-1}\{X(\omega)\} = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

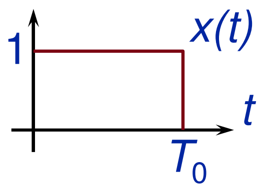
(Jfr. fourierserie: $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_1 t}$)

- Existensvillkor: $\mathcal{F}\{x(t)\}\ \exists$ om $\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$

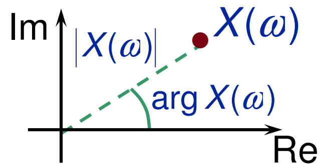
Frekvensegenskap hos signal

- Frekvensspektrum, $X(\omega)$:

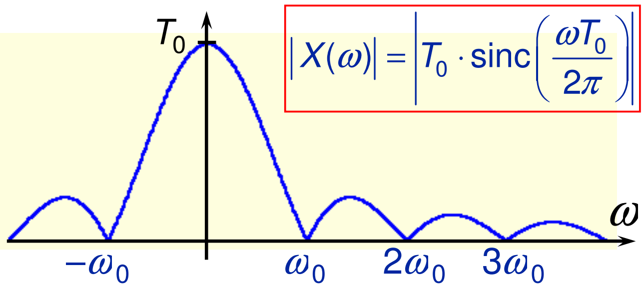
Ex:



$$X(\omega) = e^{-j\frac{\omega T_0}{2}} \cdot T_0 \,\mathrm{sinc}\left(\frac{\omega T_0}{2\pi}\right) = |X(\omega)|\, e^{j\cdot \arg X(\omega)}$$

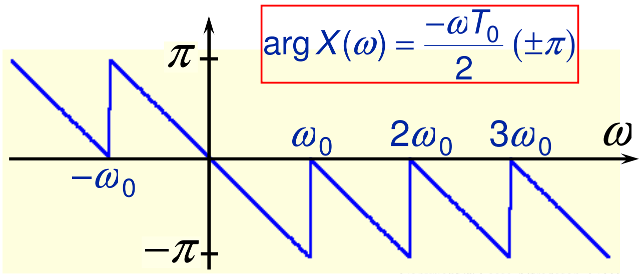


Amplitudspektrum, $|X(\omega)|$:



$$|X(\omega)| = \left|T_0 \cdot \mathrm{sinc}\left(\frac{\omega T_0}{2\pi}\right)\right|$$

Fasspektrum, $\arg X(\omega)$:



$$\arg X(\omega) = \frac{-\omega T_0}{2}\ (\pm\pi)$$

Fouriertransform till distribution

- Utöka klassen av fouriertransformerbara funktioner (fouriertransform till begränsad, ej absolutintegrerbar signal):

Låt $g(t)$ utgöra en snabbt avtagande (och mycket snäll) fouriertransformerbar testfunktion.

Den *distribution* X som då uppfyller sambandet

$$\int_{-\infty}^{\infty} X(\lambda)g(\lambda)\,d\lambda = \int_{-\infty}^{\infty} x(\lambda)G(\lambda)\,d\lambda$$

definieras som fouriertransformen till *distributionen* x.

(även $G(\lambda) = \mathcal{F}\{g(t)\}$ är en snabbt avtagande och mycket snäll testfunktion)

Energispektrum

- Låt $x(t)$ vara en reellvärd spänning (eller ström) som läggs över (går genom) en resistans på 1Ω.

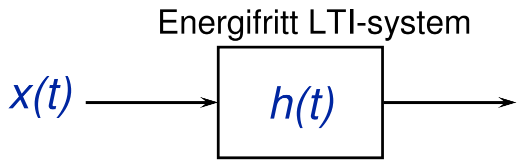
Energiinnehållet i $x(t)$ är då $W = \int_{-\infty}^{\infty} x^2(t)\,dt$

- Parsevals formel gäller generellt för komplexvärd fouriertransformerbar signal $x(t)$:

Signalenergin W = $$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega$$

$|X(\omega)|^2$: Energispektrum

SYSTEMANALYS



$$y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$Y(\omega) = \mathcal{F}\{y(t)\} = \cdots \quad \cdots \quad \cdots = X(\omega)H(\omega)$$

$\Rightarrow$ **Faltningsteoremet:** $\mathcal{F}\{f_1 * f_2\} = F_1(\omega)F_2(\omega)$

- Frekvensfunktion: $H(\omega) = \mathcal{F}\{h(t)\} = |H(\omega)|\, e^{j \arg H(\omega)}$
  - Amplitudkaraktäristik: $|H(\omega)|$
  - Faskaraktäristik: $\arg H(\omega)$

Systemanalys, forts.

$$Y(\omega) = X(\omega)H(\omega) \quad \Rightarrow \quad \begin{cases} |Y(\omega)| = |X(\omega)| \cdot |H(\omega)| \\ \arg Y(\omega) = \arg X(\omega) + \arg H(\omega) \end{cases}$$

$$\Rightarrow \quad |Y(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2$$

$|H(\omega)|^2$: Energiöverförings-funktionen

- Stationär sinus:

Insignal: $x(t) = \hat{X}\sin(\omega_0 t + \varphi) = \mathcal{Im}\left\{\hat{X}\, e^{j(\omega_0 t + \varphi)}\right\}$

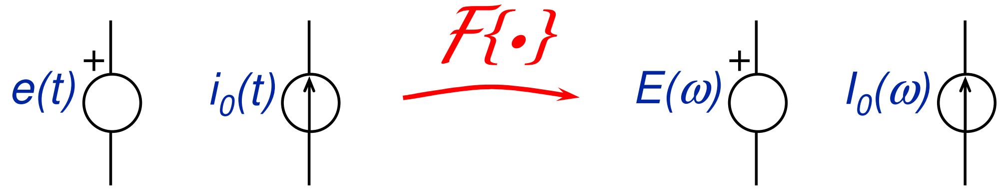
$$\Rightarrow \quad y(t) = (x * h)(t) = \quad \cdots \quad = \mathcal{Im}\left\{\hat{X}\, e^{j(\omega_0 t + \varphi)} \cdot H(\omega_0)\right\}$$

$$= \hat{X} \cdot |H(\omega_0)| \sin(\omega_0 t + \varphi + \arg H(\omega_0))$$

Kretsberäkningar, linjära *RLMC*-nät
(passiva kretselement, fouriertransformerbara källor)

<u>METODIK,</u> beräkna godtycklig nätspänning / -ström:

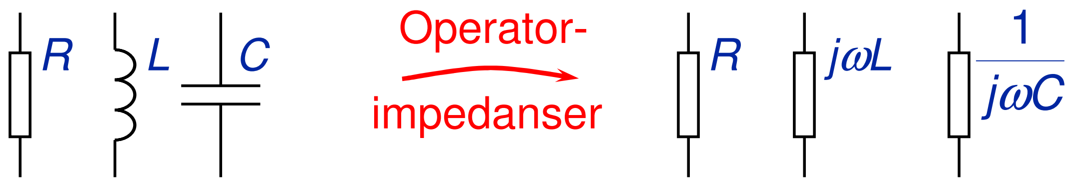
1)



2)

Kretsberäkningar, metodik (forts)

3)



4) Likströmsteori $\Rightarrow$ Sökt storhets fouriertransform ($Y(\omega)$)

5) Inverstransformera $\Rightarrow$ Sökt storhets tidsuttryck ($y(t) = \mathcal{F}^{-1}\{\, Y(\omega) \,\}$)

Digital kommunikation

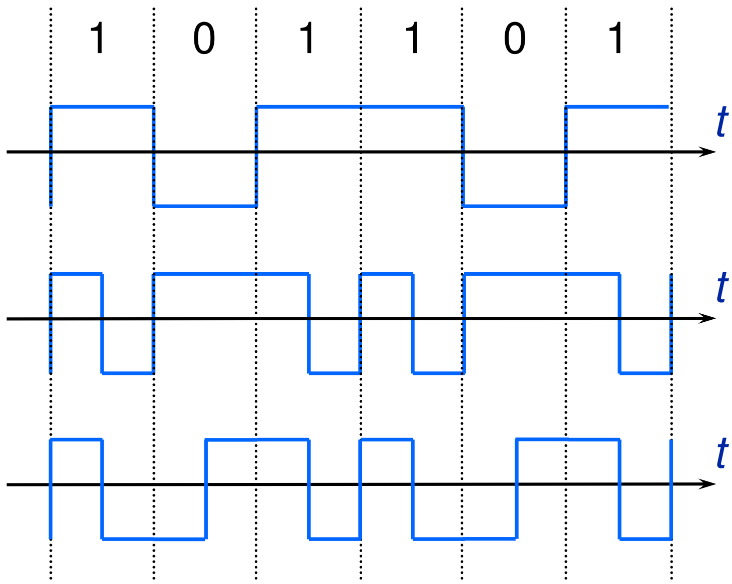
Digital signalering med analoga signalvågformer

Basbandsmodulation,

Exempel 1:

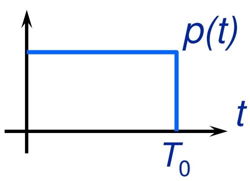
Exempel 2:

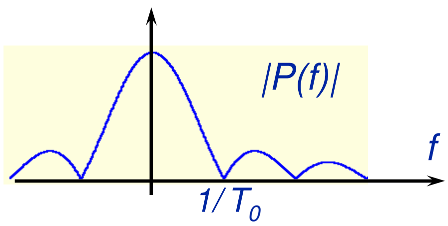
Exempel 3:



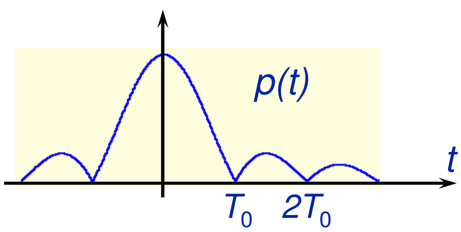
Ex. på signalpulsformer för basbandskanaler:

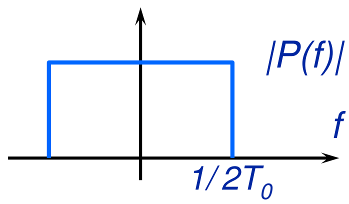
$$p(t) = u(t) - u(t - T_0)$$





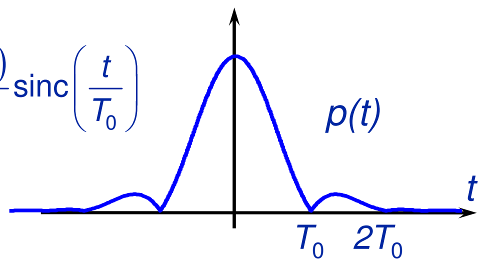
$$p(t) = \text{sinc}\left(\frac{t}{T_0}\right)$$

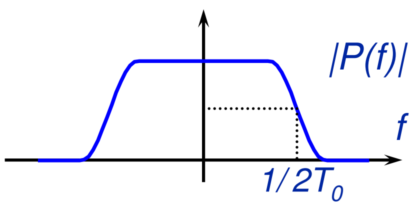




$$p(t) = \frac{\cos(2\beta\pi t/T_0)}{1-(4\beta t/T_0)^2}\text{sinc}\left(\frac{t}{T_0}\right)$$

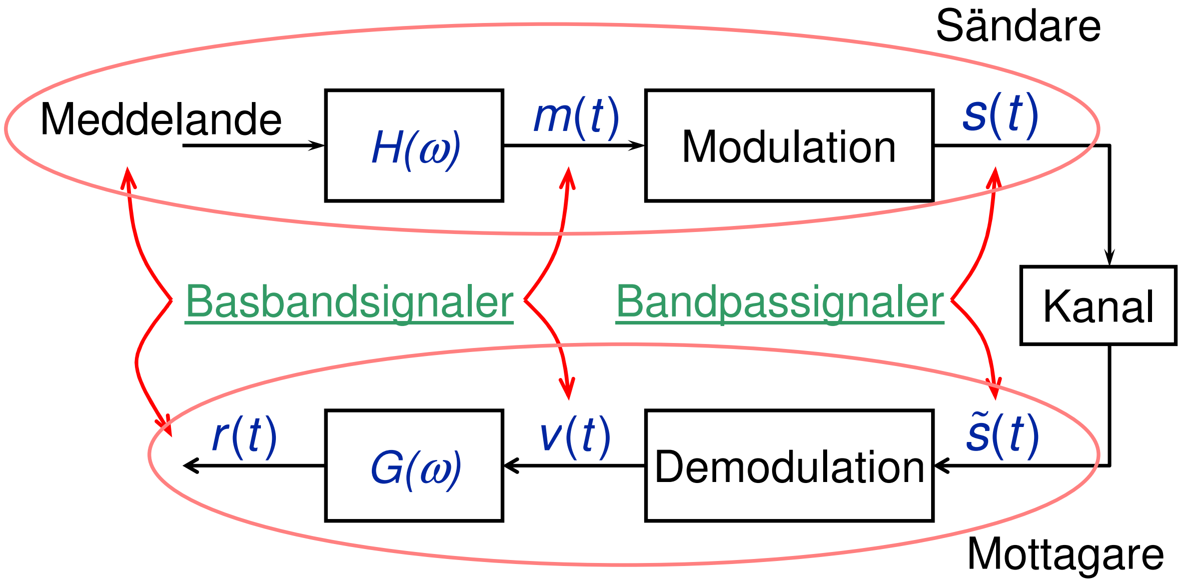
”Raised cosine”





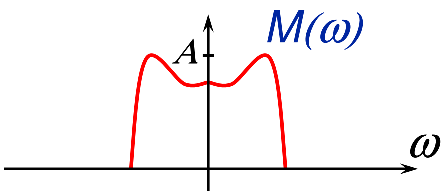
Vanligt: högfrekvent signalerering
(Ex: ADSL, radio- och satellitkommunikation, m.m.)

- Typiskt analogt kommunikationssystem:

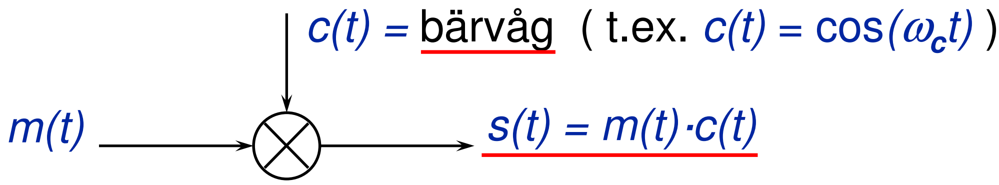


Generell Amplitudmodulering

- Basbandsignal (här: meddelandesignalen $m(t)$):



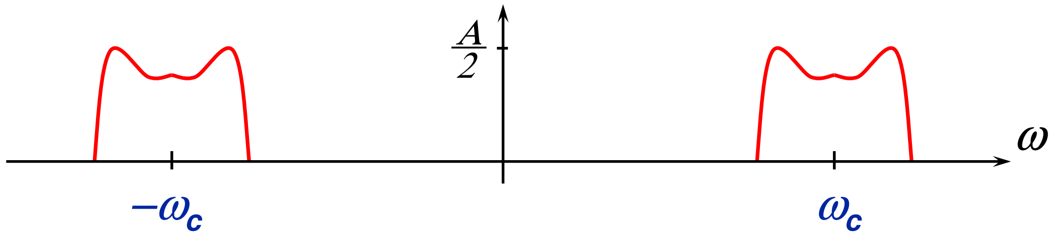
- (Amplitud-)Modulering:



Amplitudmodulering, forts

- Bandpassignal:

$$S(\omega) = \mathcal{F}\{m(t)\cdot c(t)\} = \frac{1}{2\pi}(M * C)(\omega)$$

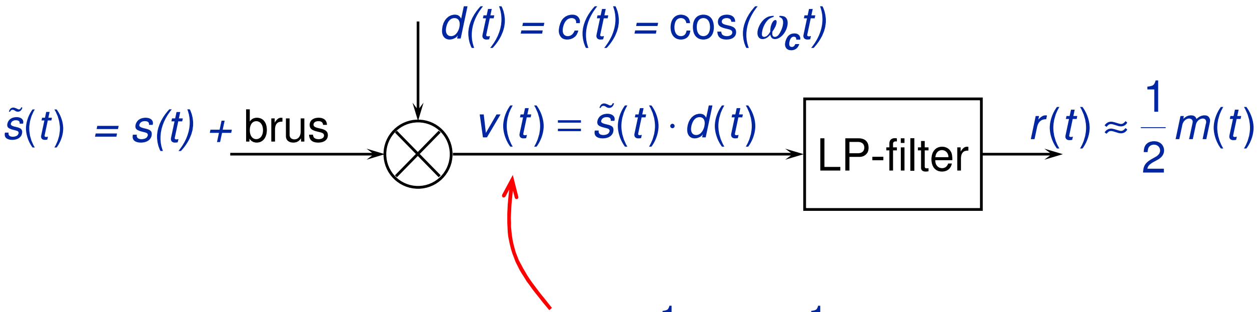


där $C(\omega) = \mathcal{F}\{\cos(\omega_c t)\} = \pi\left(\delta(\omega+\omega_c) + \delta(\omega-\omega_c)\right)$

$$\Rightarrow \quad S(\omega) = \frac{1}{2}\left(M(\omega+\omega_c) + M(\omega-\omega_c)\right)$$

Amplitudmodulering, forts

- Demodulering + LP-filter:



$$V(\omega) = \frac{1}{2}M(\omega) + \frac{1}{4}\left(M(\omega+2\omega_c) + M(\omega-2\omega_c)\right)$$