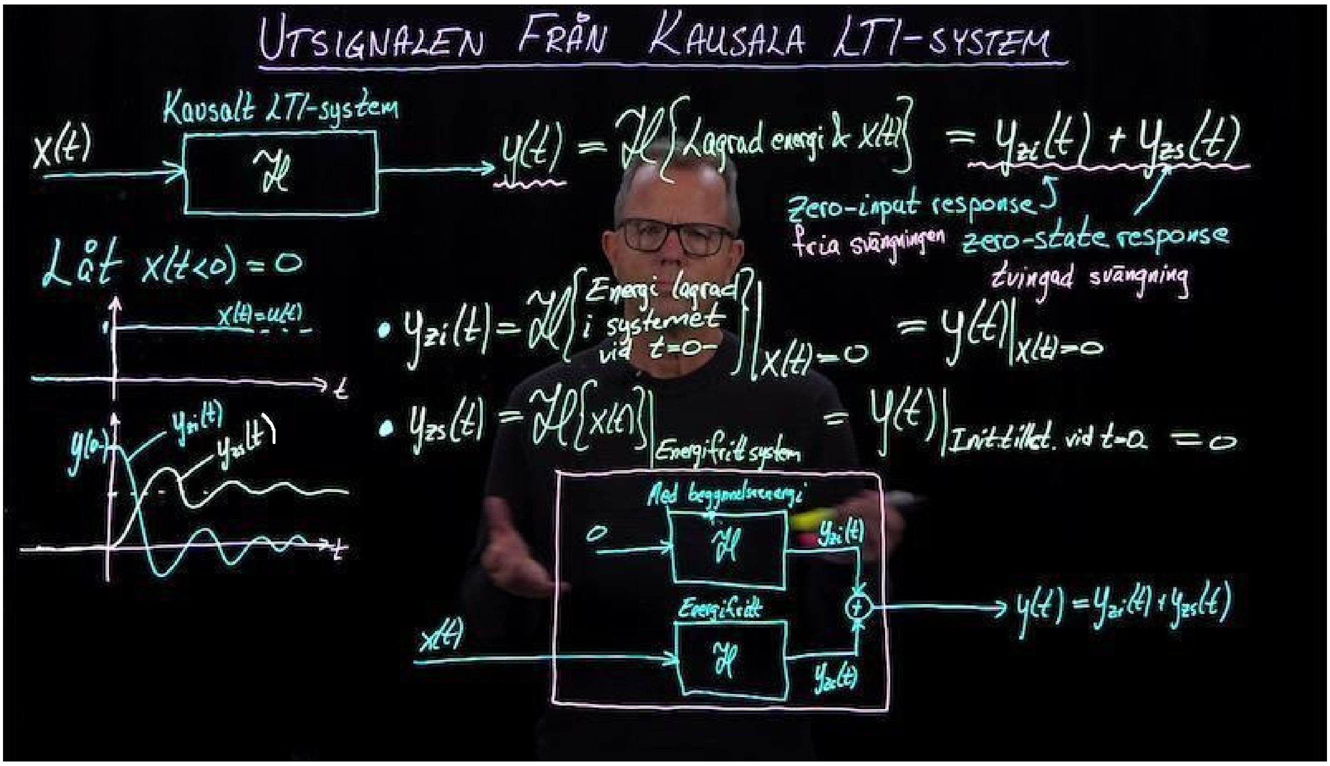
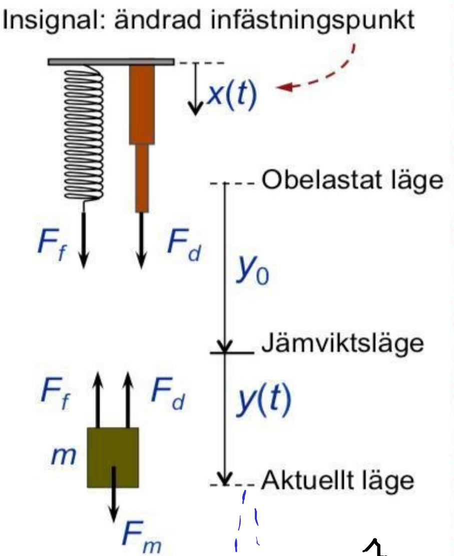
Signaler & System – Föreläsning 2:
Tidsdomänanalys av tidskontinuerliga LTI-system

VIDEO 1



Systemexempel 1 – Mekaniskt svängningssystem, massa i dämpad fjäder

Svängande dämpad fjäder – frilägg och sätt ut krafter:



Fjäderkraften $F_f = k \cdot y_{tot}(t) = k \cdot (y_0 + y(t) - x(t))$

Dämpkraften $F_d = c \cdot (y_{tot}(t))' = c \cdot (y'(t) - x'(t))$

Tyngdkraften $F_m = m \cdot g$ (g = tyngdaccelerationen)

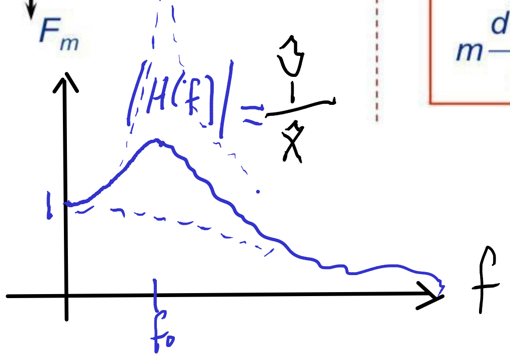
Newtons 2:a lag: $F_m - F_f - F_d = m \cdot y''(t)$

$\Rightarrow m \cdot y''(t) + c \cdot y'(t) + k \cdot y(t) = m \cdot g - k \cdot y_0 + c \cdot x'(t) + k \cdot x(t)$

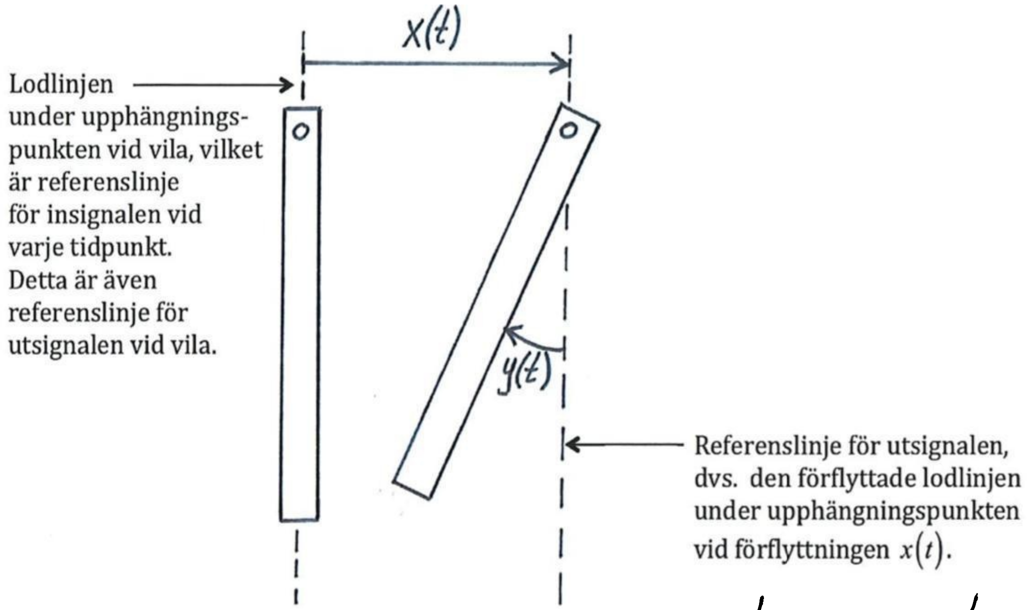
Vid vila är $x=0$, $x'=0$, $y=0$, $y'=0$, $y''=0$ $\Rightarrow$ $m \cdot g = k \cdot y_0$

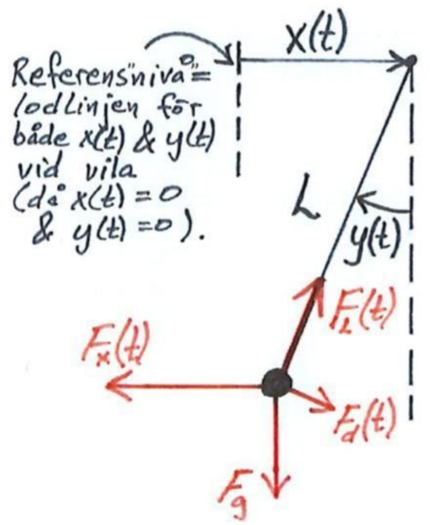
$\Rightarrow$

$$\boxed{m\frac{d^2y(t)}{dt^2} + c\frac{dy(t)}{dt} + k \cdot y(t) = c\frac{dx(t)}{dt} + k \cdot x(t)}$$



Systemexempel 2 – Mekaniskt svängningssystem, pendlande linjal





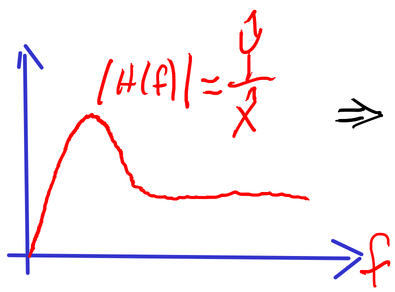
Ideal modell

Newtons 2:a lag $\Longrightarrow$ $y'' + \frac{c}{m} y' + \frac{g}{m}\sin(y) = \frac{\cos(y)}{L} \cdot x''$

Icke-linjärt system, p.g.a. sin(y) och cos(y)

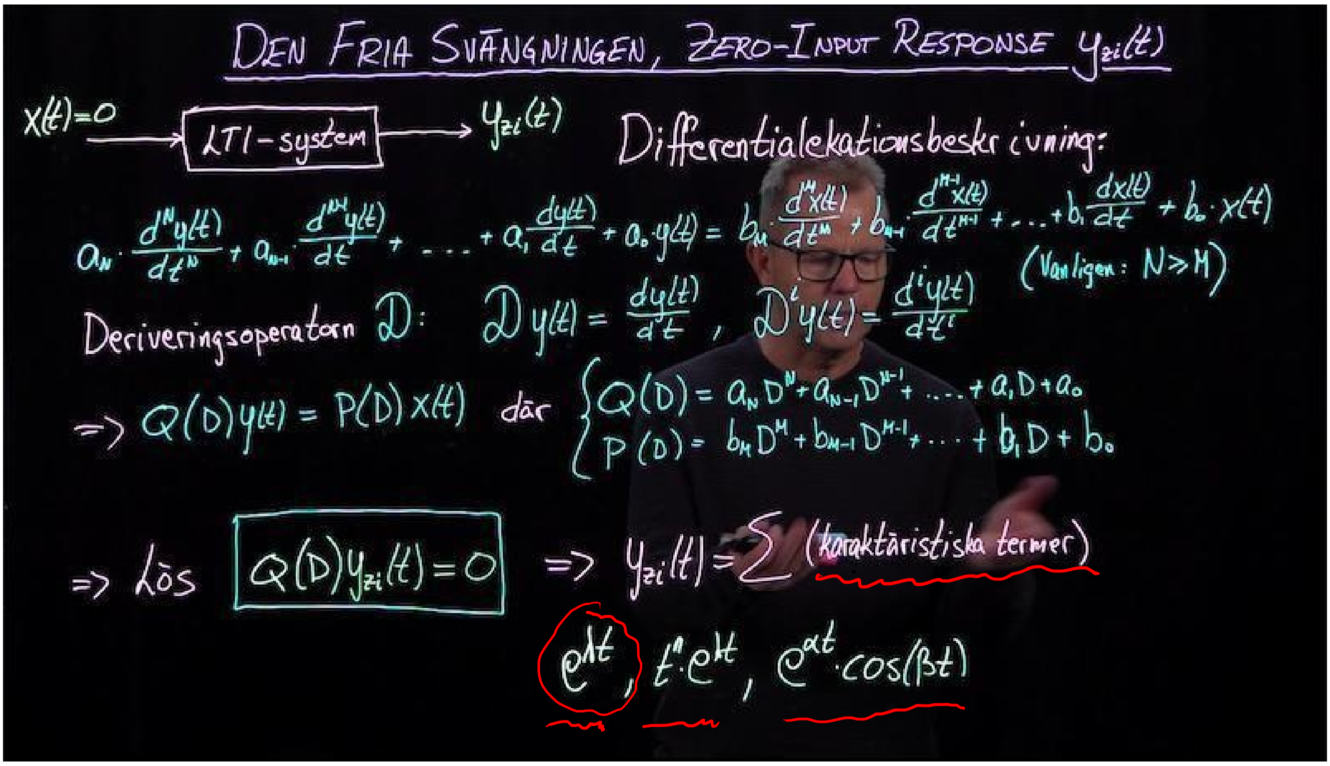
$\Longrightarrow$ Linjärisera, dvs. approximera med linjär modell:

Om vinkeln y är liten $\Longrightarrow$ $\sin(y) \approx y$, $\cos(y) \approx 1$



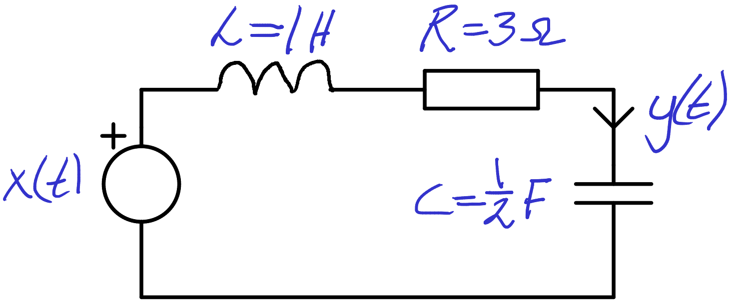
$\Rightarrow$ $\boxed{\frac{d^2y(t)}{dt^2} + \frac{c}{m}\frac{dy(t)}{dt} + \frac{g}{L}y(t) = \frac{1}{L}\frac{d^2x(t)}{dt^2}}$

VIDEO 2



OBS: $y_{zi}(t)$ betecknas $y_0(t)$ i kursboken!

Exempel:

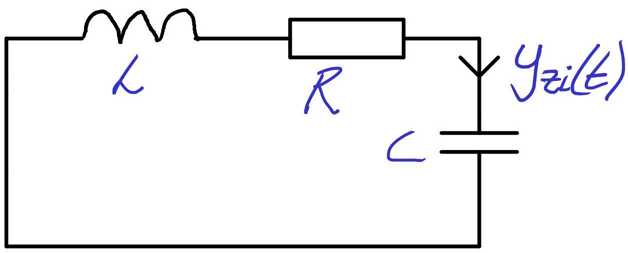


$$\frac{d^2 y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}$$

$y(0^-) = 0$, $y'(0^-) = -5$

Beräkna $y_{zi}(t)$

Ekvivalent krets:

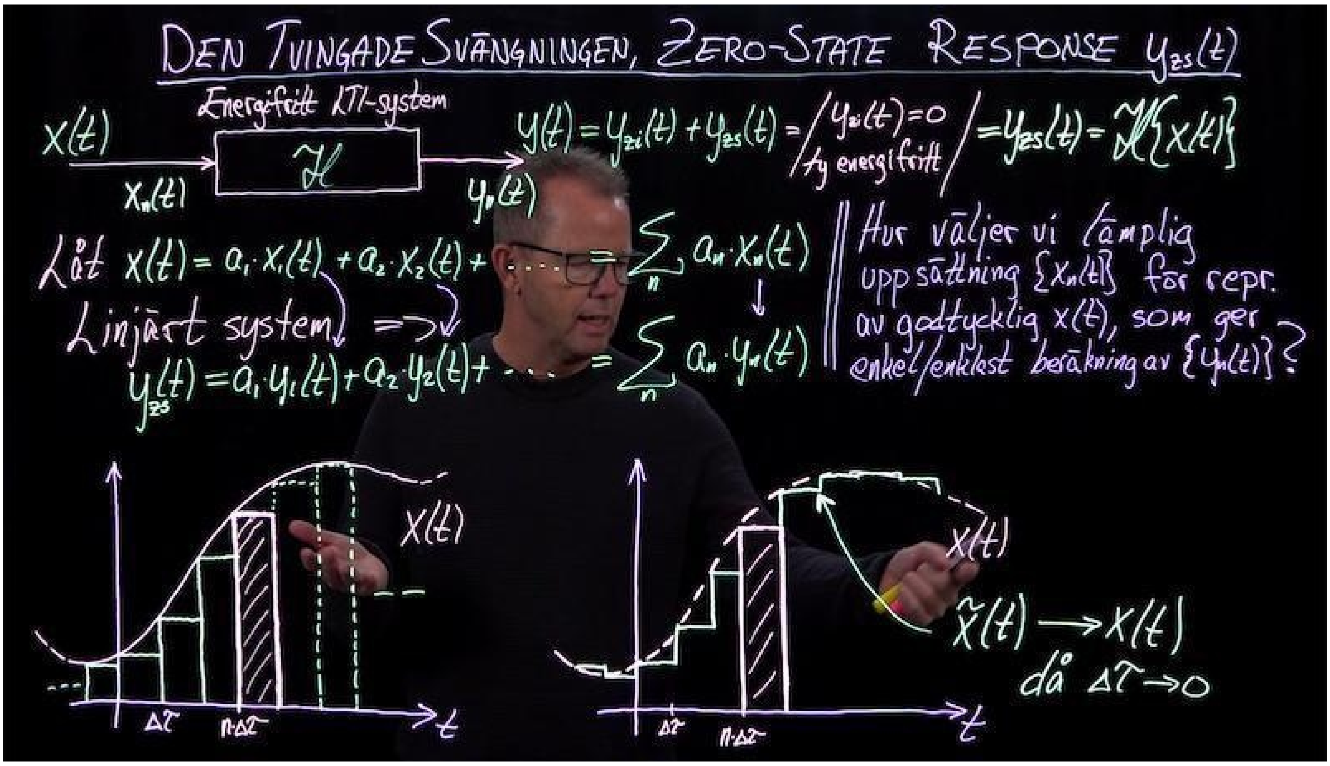


$(D^2 + 3D + 2)y_{zi}(t) = 0$

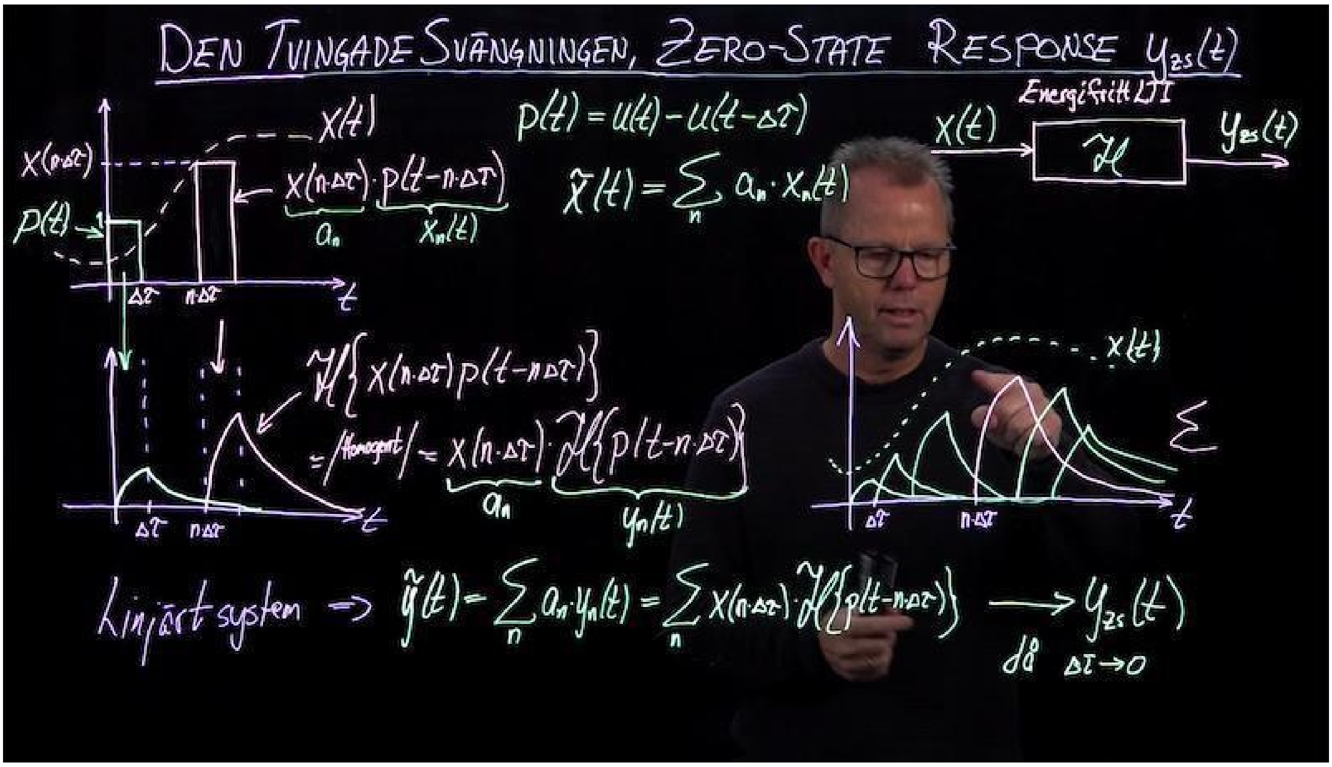
$\Rightarrow$ Kar. ekv. $\lambda^2 + 3\lambda + 2 = 0$

$\Rightarrow \begin{cases} \lambda_1 = -1 \\ \lambda_2 = -2 \end{cases} \Rightarrow y_{zi}(t) = K_1 \cdot e^{\lambda_1 t} + K_2 e^{\lambda_2 t} = \ldots$

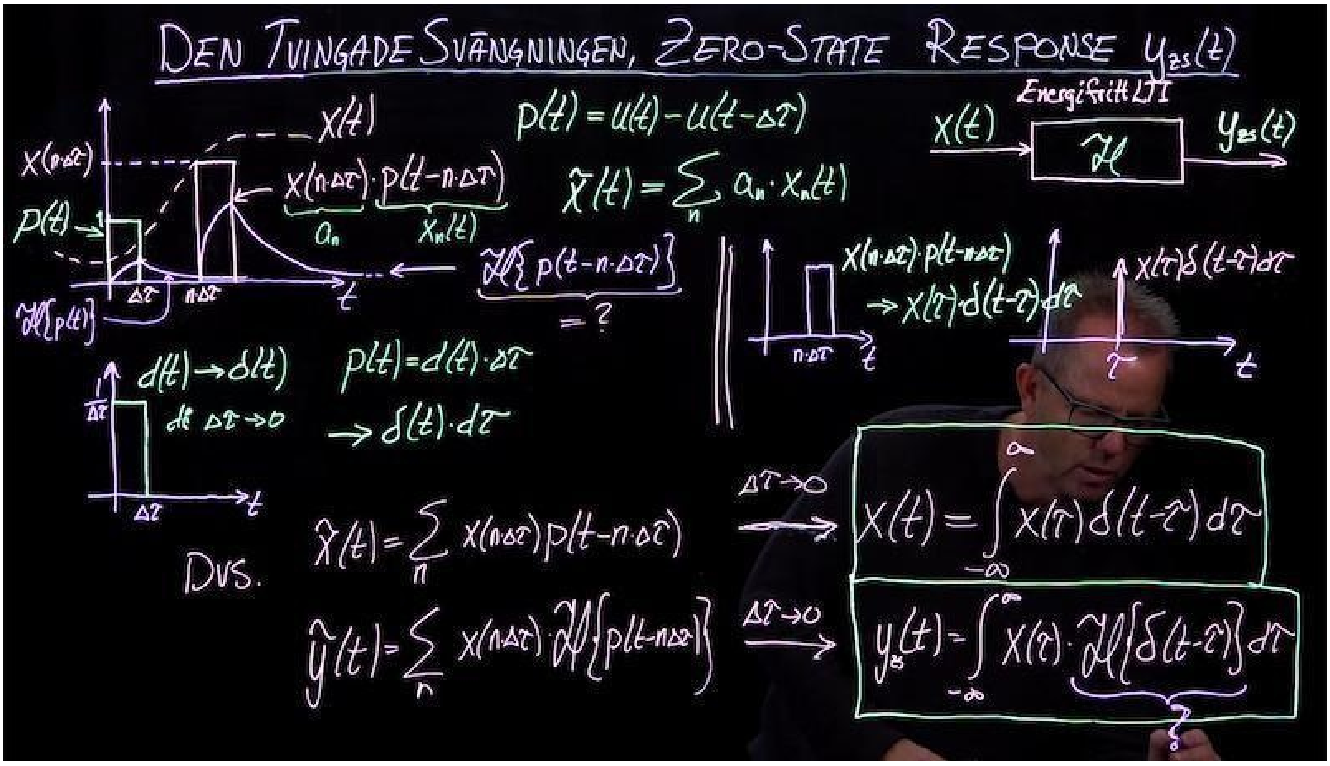
$= -5e^{-t} + 5e^{-2t}$; $t \geq 0$



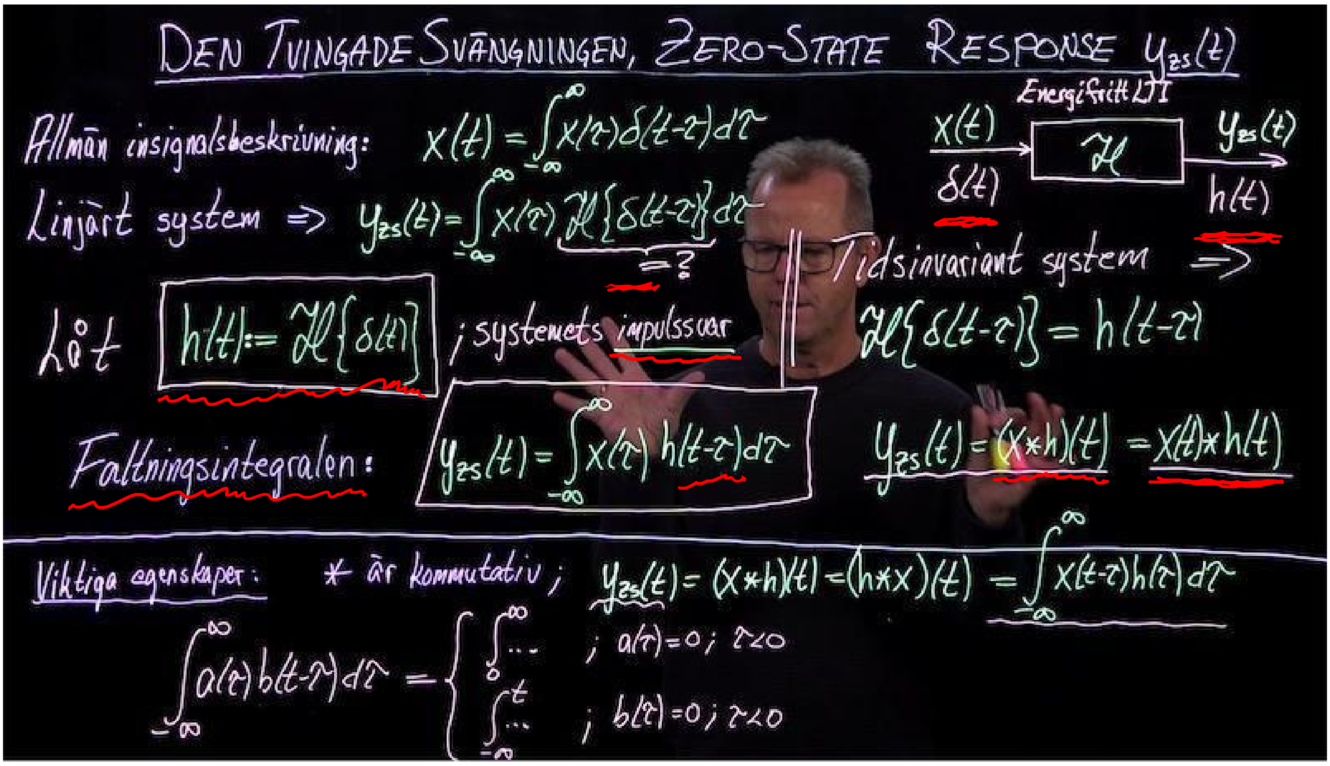
VIDEO 3.1



VIDEO 3.2



VIDEO 3.3



VIDEO 3.4

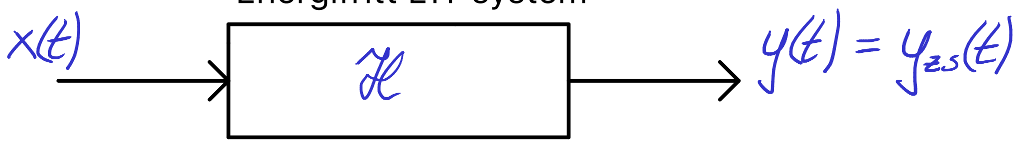
Notera: Om vi utgår från $x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \Rightarrow$

$$y_{zs}(t) = \mathcal{H}\{x(t)\} = \mathcal{H}\left\{\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau\right\} = \Big/\ \text{Linjärt system}\ \Big/$$

$$= \int_{-\infty}^{\infty} x(\tau)\mathcal{H}\{\delta(t-\tau)\}d\tau = \Big/\ \begin{matrix} \mathcal{H}\{\delta(t)\} = h(t)\ \& \\ \text{Tidsinvariant system} \\ \Rightarrow \mathcal{H}\{\delta(t-\tau)\} = h(t-\tau) \end{matrix}\ \Big/ = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Slutsats efter videon ovan:

Utsignalens zero-statekomponent (den tvingade svängningen):



där

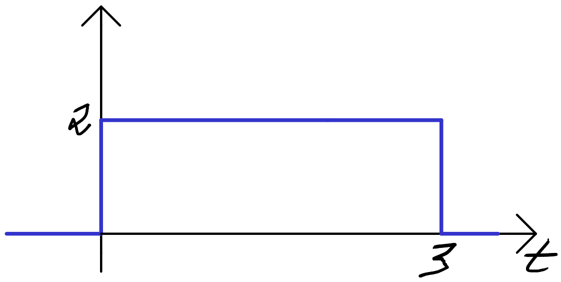
$$y_{zs}(t) = (x*h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau$$

Eller $x(t)*h(t)$

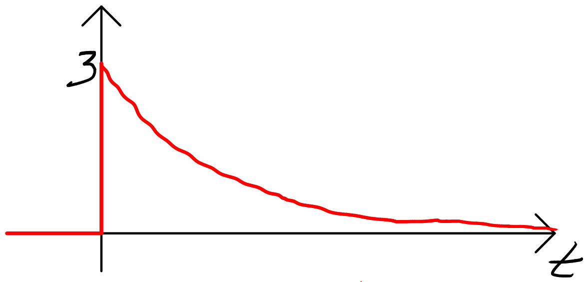
Faltningsintegralen/-erna

där $h(t) = \mathcal{H}\{\delta(t)\}$ är LTI-systemets impulssvar

Exempel:



$x(t) = 2(u(t) - u(t-3))$

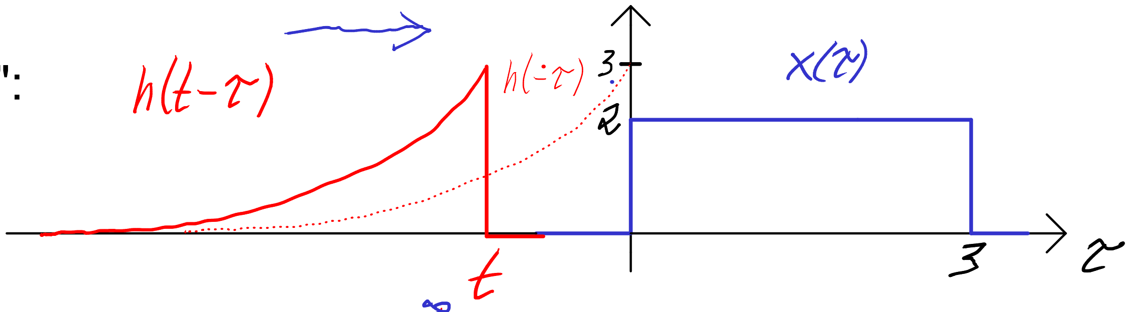


$h(t) = 3e^{-\frac{1}{2}t}\cdot u(t)$

ordn. 1

$$y_{zs}(t) = (x*h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

"Grafisk faltning":

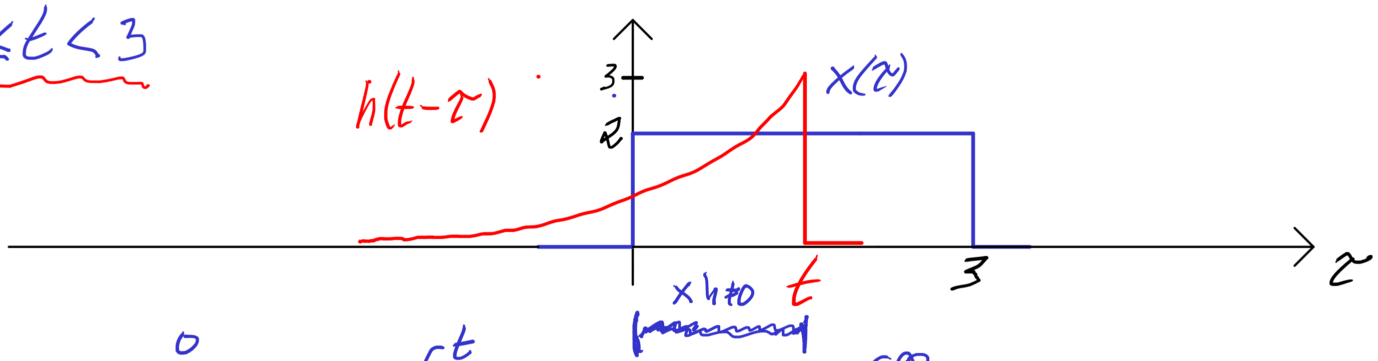


$t<0$

$x(\tau)h(t-\tau) = 0 \quad \forall \tau \Rightarrow y_{zs}(t) = \int_{-\infty}^{\infty} 0\, d\tau = 0$

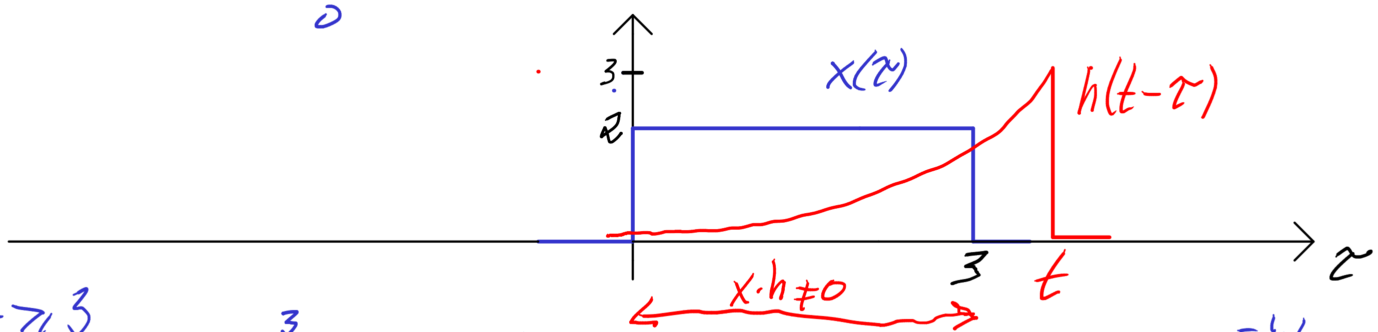
Lasses demo-material under föreläsningen – ingår ej i studentmaterialet:

$0 \leq t < 3$



$$y_{zs}(t) = \int_{-\infty}^{0} 0\,d\tau + \int_{0}^{t} 2\cdot 3e^{-\frac{1}{2}(t-\tau)}\,d\tau + \int_{t}^{\infty} 0\,d\tau$$

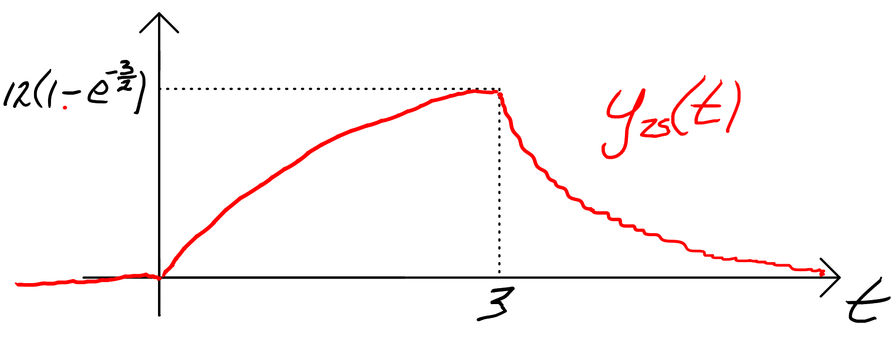
$$= 6e^{-\frac{1}{2}t}\int_{0}^{t} e^{\frac{1}{2}\tau}\,d\tau = \dots = 12\left(1-e^{-\frac{1}{2}t}\right)$$



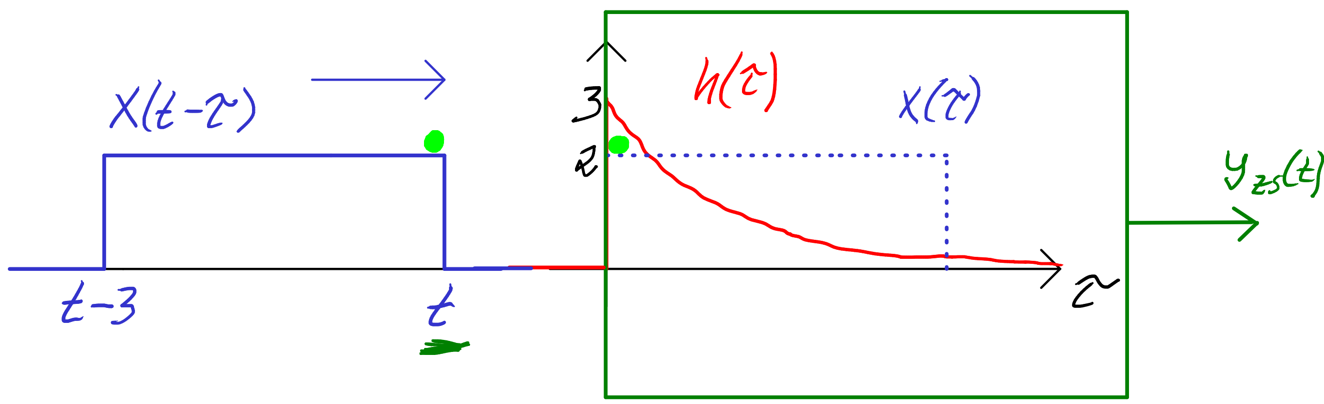
$t \geq 3$

$$y_{zs}(t) = \int_{0}^{3} 2\cdot 3e^{-\frac{1}{2}(t-\tau)}\,d\tau = \dots = 12\left(e^{\frac{3}{2}}-1\right)e^{-\frac{1}{2}t}$$

$$\therefore\ y_{zs}(t) = \begin{cases} 0; & t<0 \\ 12\left(1-e^{-\frac{1}{2}t}\right); & 0\leq t<3 \\ 12\left(e^{\frac{3}{2}}-1\right)e^{-\frac{1}{2}t}; & t\geq 3 \end{cases}$$



Testa gärna själv att beräkna $y_{zs}(t)=\int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau$?



Impulssvaret h(t) och kausalitet

Om $x(t<t_0)=0 \quad \Longrightarrow \quad y_{zs}(t<t_0)=0$

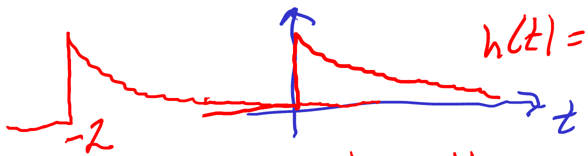
$\boxed{\text{Kausalt system}}$

Ex:

$\delta(t<0)=0 \quad \Longrightarrow \quad \boxed{h(t<0)=0}$

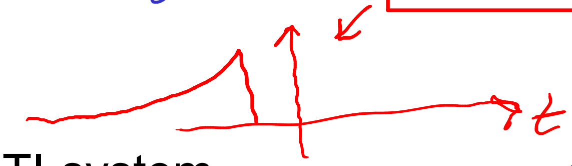
$h(t)=3e^{-\frac{1}{2}t}u(t)$

$h_2(t) = 3e^{-\frac{1}{2}(t+2)}u(t+2)$



Icke-kausalt

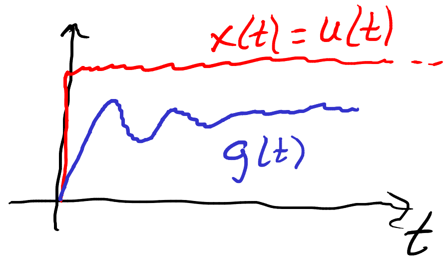
Anti-kausalt system: $\boxed{h(t)=0\ ;\ t\geq 0}$



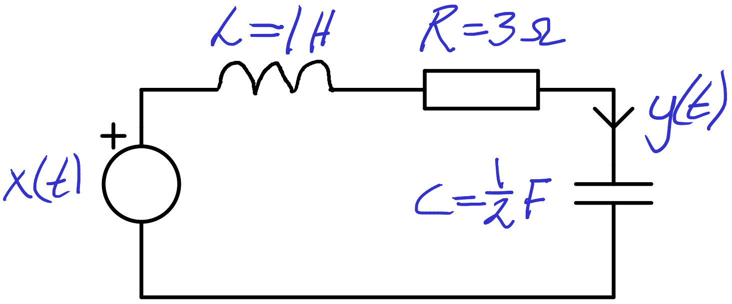
Stegsvar för LTI-system

$$g(t)=\mathcal{H}\{u(t)\}=(u*h)(t)=\int_{-\infty}^{\infty}\underbrace{u(t-\tau)}_{=0;\ t-\tau<0\ \Rightarrow\ \tau>t}h(\tau)\,d\tau=$$

$$=\int_{-\infty}^{t}h(\tau)\,d\tau \quad \Rightarrow \quad h(t)=\frac{dg(t)}{dt}$$



Tidigare exempel:

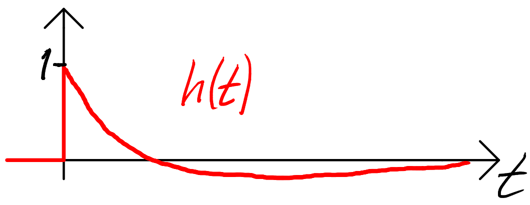


$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}$$

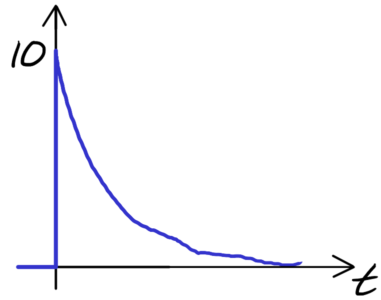
$$y(0^-) = 0, \quad y'(0^-) = -5$$

$$\Rightarrow \quad y_{zi}(t) = -5e^{-t} + 5e^{-2t}; \quad t \geq 0$$

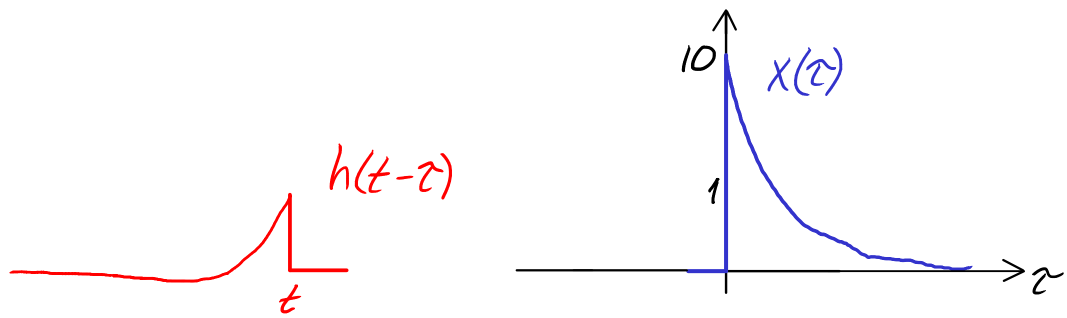
Man kan visa att detta LTI-system har impulssvar $h(t) = (2e^{-2t} - e^{-t})u(t)$



Låt insignalen vara $x(t) = 10e^{-3t} \cdot u(t)$



Beräkna den totala utsignalen $y(t) = y_{zi}(t) + y_{zs}(t)$



$$y_{zs}(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$