

**Några allmänna kommentarer gällande flera av lösningarna:**

- Genomgående används kausala signaler och kausala system, vilket innebär att det är den **enkelsidiga laplacetransformen** som används.  
Bokens författare väljer oftast att *inte* ange konvergensområde i transformberäkningarna, eftersom detta vanligen är uppenbart när det är den enkelsidiga laplacetransformen som används. Vid beräkningarna bör dock **du** ange konvergensområdet, åtminstone i de fall där man skall
  - 1) Dra slutsatser om systemegenskaper baserat på  $H(s)$ .
  - 2) Inverstransformera laplacetransformer för att erhålla motsvarande tidssignal eller impulssvar.
- I lösningarna använder sig författaren av olika transformpar och olika transformegenskaper, men anger inte alltid vilka samband som används. Det bör dock framgå vid koll i tabellerna vilka de aktuella sambanden är.
- **OBS:** Där det i lösningsförslagen står  $H(j\omega)$ , så kan du lika gärna skriva  $H(\omega)$ !  
Författaren skriver  $H(j\omega)$  eftersom boken går igenom laplacetransformen före fouriertransformen, men i kursen behandlar vi fouriertransformen *före* laplacetransformen.

4.3-1. (a)

$$(s^2 + 3s + 2)Y(s) = s\left(\frac{1}{s}\right)$$

$$Y(s) = \frac{1}{s^2 + 3s + 2} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$y(t) = (e^{-t} - e^{-2t})u(t)$$

(b)

$$(s^2Y(s) - 2s - 1) + 4(sY(s) - 2) + 4Y(s) = (s+1)\frac{1}{s+1}$$

or

$$(s^2 + 4s + 4)Y(s) = 2s + 10$$

and

$$Y(s) = \frac{2s + 10}{s^2 + 4s + 4} = \frac{2s + 10}{(s+2)^2} = \frac{2}{s+2} + \frac{6}{(s+2)^2}$$

$$y(t) = (2 + 6t)e^{-2t}u(t)$$

(c)

$$(s^2Y(s) - s - 1) + 6(sY(s) - 1) + 25Y(s) = (s+2)\frac{25}{s} = 25 + \frac{50}{s}$$

or

$$(s^2 + 6s + 25)Y(s) = s + 32 + \frac{50}{s} = \frac{s^2 + 32s + 50}{s}$$

$$\Rightarrow Y(s) = \frac{s^2 + 32s + 50}{s(s^2 + 6s + 25)} = \frac{2}{s} + \frac{-s + 20}{s^2 + 6s + 25} = 2 \cdot \frac{1}{s} - \frac{s + 3}{(s + 3)^2 + 4^2} + \frac{23}{4} \cdot \frac{4}{(s + 3)^2 + 4^2}$$

Konvergensområde:  $\text{Re}\{s\} > 0$

Tab. 4.1, transformpar 2, 9a & 9b ger då

$$y(t) = \left( 2 - e^{-3t} \cos(4t) + \frac{23}{4} e^{-3t} \sin(4t) \right) u(t) = \left( 2 - e^{-3t} \left( \cos(4t) - \frac{23}{4} \sin(4t) \right) \right) u(t)$$

- 4.3-2. (a) All initial conditions are zero. The zero-input response is zero. The entire response found in Prob4.3-2a is zero-state response, that is

$$\begin{aligned} y_{zs}(t) &= (e^{-t} - e^{-2t})u(t) \\ y_{zi}(t) &= 0 \end{aligned}$$

- (b) The Laplace transform of the differential equation is

$$(s^2 Y(s) - 2s - 1) + 4(sY(s) - 2) + 4Y(s) = (s + 1) \frac{1}{s + 1}$$

or  $(s^2 + 4s + 4)Y(s) - (2s + 9) = 1$

or  $(s^2 + 4s + 4)Y(s) = \underbrace{2s + 9}_{\text{i.c. terms}} + \underbrace{1}_{\text{input}}$

$$\begin{aligned} Y(s) &= \underbrace{\frac{2s + 9}{s^2 + 4s + 4}}_{\text{zero-input}} + \underbrace{\frac{1}{s^2 + 4s + 4}}_{\text{zero-state}} \\ &= \underbrace{\frac{2}{s + 2}}_{\text{zero-input}} + \underbrace{\frac{5}{(s + 2)^2}}_{\text{zero-input}} + \underbrace{\frac{1}{(s + 2)^2}}_{\text{zero-state}} \end{aligned}$$

$$y(t) = \left( \underbrace{(2 + 5t)e^{-2t}}_{\text{zero-input}} + \underbrace{te^{-2t}}_{\text{zero-state}} \right) u(t)$$

(c) The Laplace transform of the equation is

$$(s^2 Y(s) - s - 1) + 6(sY(s) - 1) + 25Y(s) = 25 + \frac{50}{s}$$

or

$$(s^2 + 6s + 25)Y(s) = \underbrace{s+7}_{\text{i.c. terms}} + \underbrace{25 + \frac{50}{s}}_{\text{input}}$$

$$Y(s) = \underbrace{\frac{s+7}{s^2+6s+25}}_{\text{zero-input}} + \underbrace{\frac{25s+50}{s(s^2+6s+25)}}_{\text{zero-state}}$$

Dvs.  $Y(s) = Y_{zi}(s) + Y_{zs}(s)$ , där

$$Y_{zi}(s) = \frac{s+7}{s^2+6s+25} = \frac{s+3}{(s+3)^2+4^2} + \frac{4}{(s+3)^2+4^2}, \quad \text{Konv.område: } \operatorname{Re}\{s\} > -3$$

$$\begin{aligned} Y_{zs}(s) &= \frac{25s+50}{s(s^2+6s+25)} = \frac{2}{s} + \frac{-2s+13}{s^2+6s+25} \\ &= 2 \cdot \frac{1}{s} - 2 \cdot \frac{s+3}{(s+3)^2+4^2} + \frac{19}{4} \cdot \frac{4}{(s+3)^2+4^2}, \quad \text{Konv.område } \operatorname{Re}\{s\} > 0 \end{aligned}$$

Tab. 4.1, transformpar 2, 9a & 9b ger då

$$\begin{aligned} y(t) &= y_{zi}(t) + y_{zs}(t) \\ &= \underbrace{e^{-3t}(\cos(4t) + \sin(4t))u(t)}_{y_{zi}(t)} + \underbrace{\left(2 - e^{-3t}\left(2\cos(4t) - \frac{19}{4}\sin(4t)\right)\right)u(t)}_{y_{zs}(t)} \end{aligned}$$

4.3-5. (a)  $\frac{5s+3}{s^2+11s+24}$

(b)  $\frac{3s^2+7s+5}{s^3+6s^2-11s+6}$

(c)  $\frac{3s+2}{s(s^3+4)}$

(d)  $\frac{1}{s+1}$

4.3-6. (a)  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 8y(t) = \frac{dx}{dt} + 5x(t)$   
 (b)  $\frac{d^3y}{dt^3} + 8\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 7y(t) = \frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 5x(t)$   
 (c)  $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y(t) = 5\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 2x(t)$

4.3-6 & 4.3-7:  
 ”Controllable and observable systems”, enligt uppgift (se sid. 123)  $\Rightarrow$  eventuella sammanfallande/överlappande poler och nollställen får inte elimineras!

4.3-7. (a) (i)  $X(s) = \frac{10}{s}$

$$Y(s) = X(s)H(s) = \frac{10(2s+3)}{s(s^2+2s+5)} = \frac{6}{s} + \frac{-6s+8}{s^2+2s+5} = 6 \cdot \frac{1}{s} - 6 \frac{s+1}{(s+1)^2+2^2} + 7 \frac{2}{(s+1)^2+2^2}$$

Konv.område  $\text{Re}\{s\} > 0$

Tab. 4.1, transformpar 2, 9a & 9b ger då

$$y(t) = (6 - e^{-t}(6\cos(2t) - 7\sin(2t)))u(t)$$

(ii)  $x(t) = u(t - 5)$  and  $X(s) = \frac{1}{s}e^{-5s}$

$$Y(s) = \frac{2s+3}{s(s^2+2s+5)}e^{-5s} = \left[ \frac{0.6}{s} + \frac{1}{10} \left( \frac{-6s+8}{s^2+2s+5} \right) \right] e^{-5s}$$

$$= \frac{1}{10} \left( 6 \cdot \frac{1}{s} - 6 \frac{s+1}{(s+1)^2+2^2} + 7 \frac{2}{(s+1)^2+2^2} \right)$$

$$= \frac{1}{10} Y_{(i)}(s)e^{-5s}, \text{ där } Y_{(i)}(s) \text{ är lika med } Y(s) \text{ i fall (i) ovan}$$

$$\Rightarrow y(t) = \frac{1}{10} y_{(i)}(t-5) = \frac{1}{10} (6 - e^{-(t-5)}(6\cos(2(t-5)) - 7\sin(2(t-5))))u(t-5)$$

Egentligen behövs inga transformberäkningar i det här fallet.

Eftersom  $x_{(i)}(t) = 10u(t) \rightarrow y_{(i)}(t)$ , så ger LTI-egenskapen att

$$x_{(ii)}(t) = u(t-5) = \frac{1}{10} x_{(i)}(t-5) \rightarrow y_{(ii)}(t) = \frac{1}{10} y_{(i)}(t-5)$$

(b)  $\ddot{y}(t) + 2\dot{y}(t) + 5y(t) = 2\dot{x}(t) + 3x(t)$

4.3-8 (a)

$$X(s) = \frac{1}{s} - \frac{1}{s+1} = \frac{1}{s(s+1)}, \text{ konvergensområde } \text{Re}\{s\} > 0$$

$$Y(s) = X(s)H(s) = \frac{1}{(s+1)(s^2+9)} = \frac{1}{10} \left( \frac{1}{s+1} - \frac{s}{s^2+3^2} + \frac{1}{3} \frac{3}{s^2+3^2} \right), \text{ konv.område } \text{Re}\{s\} > 0$$

Tab. 4.1, transformpar 5, 8a & 8b  $\Rightarrow y(t) = \frac{1}{10} \left( e^{-t} - \cos(3t) + \frac{1}{3} \sin(3t) \right) u(t)$

(b)  $\ddot{y}(t) + 9y(t) = \dot{x}(t)$

4.3-9. (a) (i)  $X(s) = \frac{1}{s+3}$  and

$$Y(s) = \frac{s+5}{(s+3)(s^2+5s+6)} = \frac{s+5}{(s+2)(s+3)^2} = \frac{3}{s+2} - \frac{3}{s+3} - \frac{2}{(s+3)^2}$$

$$y(t) = (3e^{-2t} - 3e^{-3t} - 2te^{-3t})u(t)$$

(ii)  $X(s) = \frac{1}{s+4}$

$$Y(s) = \frac{s+5}{(s+2)(s+3)(s+4)} = \frac{3/2}{s+2} - \frac{2}{s+3} + \frac{1/2}{s+4}$$

$$y(t) = \left( \frac{3}{2}e^{-2t} - 2e^{-3t} + \frac{1}{2}e^{-4t} \right) u(t)$$

(iii) The input here is the input in (ii) delayed by 5 secs. Therefore  $X(s) = \frac{1}{s+4}e^{-5s}$

$$Y(s) = \frac{s+5}{(s+2)(s+3)(s+4)}e^{-5s} = \left[ \frac{3/2}{s+2} - \frac{2}{s+3} + \frac{1/2}{s+4} \right] e^{-5s}$$

$$y(t) = \left[ \frac{3}{2}e^{-2(t-5)} - 2e^{-3(t-5)} + \frac{1}{2}e^{-4(t-5)} \right] u(t-5)$$

(iv) The input here is equal to the input in (ii) multiplied by  $e^{20}$  because  $e^{-4(t-5)} = e^{20}e^{-4t}$ . Therefore the output is equal to the output in (ii) multiplied by  $e^{20}$ .

$$y(t) = e^{20} \left[ \frac{3}{2}e^{-2t} - 2e^{-3t} + \frac{1}{2}e^{-4t} \right] u(t)$$

(v) The input here is equal to the input in (iii) multiplied by  $e^{-20}$  because  $e^{-4t}u(t-5) = e^{-20}e^{-4(t-5)}u(t-5)$ . Therefore

$$y(t) = e^{-20} \left[ \frac{3}{2}e^{-2(t-5)} - 2e^{-3(t-5)} + \frac{1}{2}e^{-4(t-5)} \right] u(t-5)$$

(b)  $(D^2 + 5D + 6)y(t) = (D + 5)x(t)$

**Kommentar till 4.3-9 (a), fall (iii):** I lösningstexten till (iii) ovan, så står det att insignalen är en fördröjd version av insignalen i fall (ii). Eftersom systemet är tidsinvariant, så erhåller vi direkt utsignalen som motsvarande förskjutning av utsignalen i fall (ii) – dvs. utan att använda transformberäkningar!

4.3-12 Alla system i uppgiften är kausala,  $\Rightarrow$   
 Konvergensområdet för  $H(s)$  är  $\text{Re}\{s\} > \sigma_0$ ,  
 där  $\sigma_0$  är realdelen av den pol i  $H(s)$  som ligger  
 längst till höger i  $s$ -planet.

(a)

$$(i) \quad H(s) = \frac{s+5}{s^2+3s+2} = \frac{s+5}{(s+1)(s+2)}, \quad \text{Re}\{s\} > -1$$

$j\omega$ -axeln ligger i konvergensområdet  $\Rightarrow$  Stabilt system

$$(ii) \quad H(s) = \frac{s+5}{s^2(s+2)}, \quad \text{Re}\{s\} > 0$$

$j\omega$ -axeln är rand till konv.omr. men dubbelpol på  $j\omega$ -axeln  
 $\Rightarrow$  Instabilt system

$$(iii) \quad H(s) = \frac{s(s+2)}{s+5}, \quad \text{Re}\{s\} > -5$$

Med avseende på polen i  $s = -5$  och konvergensområdets utseende,  
 så ligger  $j\omega$ -axeln i konv.området och systemet är stabilt.

Doch är det fler nollställen än poler  $\Rightarrow$

Systemet är ej stabilt

Här är det ett nollställe fler än antal poler (vilket motsvarar ett  
 $H(s)$  har en enkelpol  
 i  $s = \infty$ )  
 Systemet är marginellt stabilt.

$$(iv) \quad H(s) = \frac{s+5}{s(s+2)}; \quad \text{Re}\{s\} > 0$$

$j\omega$ -axeln utgör en rand till konv.området och det finns en  
 enkelpol på  $j\omega$ -axeln  $\Rightarrow$  Systemet är marginellt stabilt

$$(v) \quad H(s) = \frac{s+5}{s^2-2s+3} = \frac{s+5}{(s-1)^2+(\sqrt{2})^2}; \quad \text{Re}\{s\} > 1$$

(Dvs.  $H(s)$  har ett polpar i  $s = 1 \pm j\sqrt{2}$ )

$j\omega$ -axeln ligger inte i konvergensområdet för  $H(s)$   
 $\Rightarrow$  Instabilt system

$$(b) (i) (D^2 + 3D + 2)y(t) = (D + 3)x(t) \Rightarrow$$

$$(s^2 + 3s + 2)Y(s) = (s + 3)X(s) \Rightarrow$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s + 3}{(s + 1)(s + 2)} ; \operatorname{Re}\{s\} > -1$$

$j\omega$ -axeln ligger i konv.området  $\Rightarrow$  Stabilt system

$$(ii) (D^2 + 3D + 2)y(t) = (D + 1)x(t) \Rightarrow$$

$$(s^2 + 3s + 2)Y(s) = (s + 1)X(s) \Rightarrow$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s + 1}{(s + 1)(s + 2)} = \frac{1}{s + 2} \operatorname{Re}\{s\} > -2$$

$j\omega$ -axeln ligger i konv.området  $\Rightarrow$  Stabilt system

$$(iii) (D^2 + D - 2)y(t) = (D - 1)x(t) \Rightarrow$$

$$(s^2 + s - 2)Y(s) = (s - 1)X(s) \Rightarrow$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s - 1}{(s - 1)(s + 2)} = \frac{1}{s + 2} \operatorname{Re}\{s\} > -2$$

$j\omega$ -axeln ligger i konv.området  $\Rightarrow$  Stabilt system

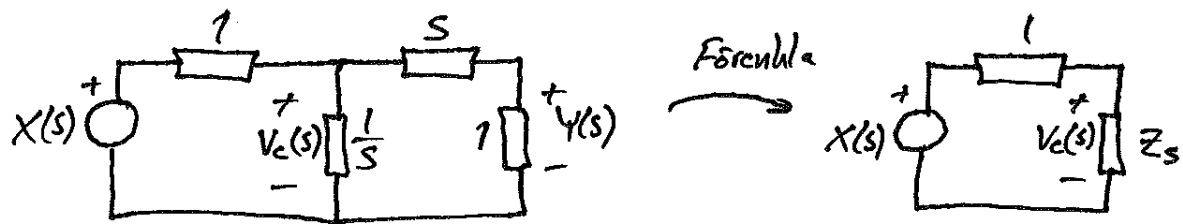
$$(iv) (D^2 - 3D + 2)y(t) = (D - 1)x(t) \Rightarrow$$

$$(s^2 - 3s + 2)Y(s) = (s - 1)X(s) \Rightarrow$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s - 1}{(s - 1)(s - 2)} = \frac{1}{s - 2} \operatorname{Re}\{s\} > 2$$

$j\omega$ -axeln ligger inte i konv.området  $\Rightarrow$  Instabilt system

# 4.4.1 Rita operatorschema:



$$Z_s = \frac{1}{\frac{1}{s} \parallel (s+1)} = \frac{\frac{1}{s}(s+1)}{\frac{1}{s} + (s+1)} = \frac{s+1}{s^2+s+1}$$

Sp. delning:

$$Y(s) = V_c(s) \cdot \frac{1}{s+1}, \text{ där } V_c(s) = X(s) \cdot \frac{Z_s}{1+Z_s} =$$

$$= X(s) \cdot \frac{s+1}{s^2+2s+2}$$

$$\Rightarrow Y(s) = X(s) \cdot \frac{s+1}{s^2+2s+2} \cdot \frac{1}{s+1}$$

$$\Rightarrow \underline{H(s)} = \frac{Y(s)}{X(s)} = \underline{\frac{1}{s^2+2s+2}}$$

Konv. omr.  $\text{Re}\{s\} > -1$   
(ty poler i  $s = -1 \pm j$ , se nedan)

$$\Rightarrow (s^2+2s+2)Y(s) = X(s) \Leftrightarrow \underline{\underline{\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 2y(t) = X(t)}}$$

$$X(t) = t \cdot e^{-t} \cdot u(t) \quad \Rightarrow \quad X(s) = \frac{1}{(s+1)^2}; \text{ Re}\{s\} > -1$$

(Tab. 4.1-6)

$$Y(s) = X(s) \cdot H(s) = \frac{1}{(s+1)^2} \cdot \frac{1}{s^2+2s+2} = \frac{1}{(s+1)^2} - \frac{1}{(s+1)^2+1^2}$$

Konv. omr.  $\text{Re}\{s\} > -1$

Tab. 4.1-6 & 4.1-9b ger då

$$\underline{y(t)} = t \cdot e^{-t} \cdot u(t) - e^{-t} \cdot \sin(t) \cdot u(t) = \underline{\underline{e^{-t}(t - \sin(t))u(t)}}$$

(=  $y_{ZS}(t)$ )



4.4-3. The impedance seen by the source  $x(t)$  is

$$Z(s) = \frac{Ls(1/Cs)}{Ls + (1/Cs)} = \frac{Ls}{LCs^2 + 1} = \frac{Ls\omega_0^2}{s^2 + \omega_0^2}$$

The current  $Y(s)$  is given by

$$Y(s) = \frac{X(s)}{Z(s)} = \frac{s^2 + \omega_0^2}{Ls\omega_0^2} X(s)$$

(a)

$$X(s) = \frac{As}{s^2 + \omega_0^2}, \quad Y(s) = \frac{A}{L\omega_0^2} \quad \text{and} \quad y(t) = \frac{A}{L\omega_0^2} \delta(t)$$

(b)

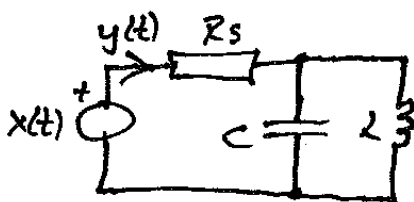
$$X(s) = \frac{A\omega_0}{s^2 + \omega_0^2}, \quad Y(s) = \frac{A}{L\omega_0 s} \quad \text{and} \quad y(t) = \frac{A}{L\omega_0} u(t)$$

$$c) \quad H(s) = \frac{Y(s)}{X(s)} = \frac{1}{Z(s)} = \frac{s^2 + \omega_0^2}{L\omega_0^2 \cdot s}$$

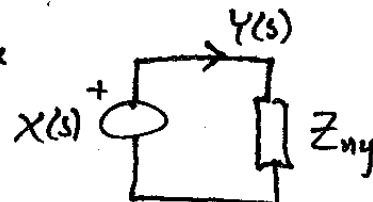
Antal nollställen > antal poler  $\Rightarrow$  instabilt system,  
(både internt & externt).

(I deluppg. b) ser vi iöck för sig en insignal (begränsad) som ger en begränsad utsignal, men det är bara för att  $X(s)$  har poler där  $H(s)$  har sina två nollställen ( $s = \pm j\omega_0$ )

d)  $\omega_0 = 4 \text{ rad/s}$ ,  $L = 1 \text{ H}$ ,  $R_s = 1 \text{ } \Omega$ :



op. schema  $\Rightarrow$



$$\text{där } Z_{my} = R_s + Z = 1 + \frac{16s}{s^2 + 16} = \frac{s^2 + 16s + 16}{s^2 + 16}$$

$$\Rightarrow \underline{H_{ny}(s)} = \frac{\psi(s)}{x(s)} = \frac{1}{Z_{ny}} \approx \frac{s^2 + 16}{(s + 14,9)(s + 1,1)}$$

Antal poler (2)  $\geq$  antal nollställen (2)  $\Rightarrow$  OK.

Både polerna i vänster halvplan  $\Rightarrow$   
 både insignal-utsignal-stabilt och  
asymptotiskt stabilt (inga poler har förkortats bort)

Anm: I praktiken har alla fysikaliska spänningshällor en  
 inre resistans  $> 0$ , vilket gör en krets motsvarande  
 den i uppgiften stabil.  
 Man kopplar aldrig heller en kapacitans (kondensator)  
 direkt till en spänningshälla – det kan vara farligt...

4.4-11. (a) (i)

$$Y(s) = \frac{6s^2 + 3s + 10}{s(2s^2 + 6s + 5)}$$

$$y(0^+) = \lim_{s \rightarrow \infty} sY(s) = 3$$

$$y(\infty) = \lim_{s \rightarrow 0} sY(s) = 2$$

(ii)

$$Y(s) = \frac{6s^2 + 3s + 10}{(s + 1)(2s^2 + 6s + 5)}$$

$$y(0^+) = \lim_{s \rightarrow \infty} sY(s) = 3$$

$$y(\infty) = \lim_{s \rightarrow 0} sY(s) = 0$$

(b) (i)

$$Y(s) = \frac{s^2 + 5s + 6}{s^2 + 3s + 2}$$

This  $Y(s)$  is not strictly proper. We can express it as

$$Y(s) = 1 + \frac{2s + 4}{s^2 + 3s + 2}$$

Hence

$$y(0^+) = \lim_{s \rightarrow \infty} \frac{s(2s + 4)}{s^2 + 3s + 2} = 2$$

and

$$y(\infty) = \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} \frac{s^3 + 5s^2 + 6s}{s^2 + 3s + 2} = 0$$

(ii)

$$Y(s) = \frac{s^3 + 4s^2 + 10s + 7}{s^2 + 2s + 3}$$

Because  $Y(s)$  is improper, we shall find its strictly proper component.

$$Y(s) = (s + 2) + \frac{s + 1}{s^2 + 2s + 3}$$

Hence

$$y(0^+) = \lim_{s \rightarrow \infty} s \left( \frac{s + 1}{s^2 + 2s + 3} \right) = 1$$

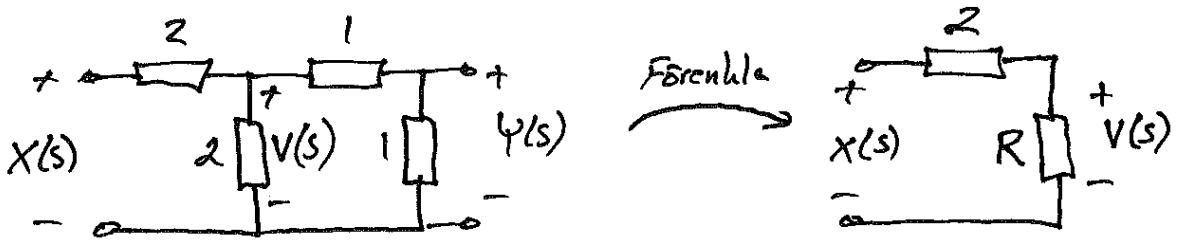
$$y(\infty) = \lim_{s \rightarrow 0} s \left( \frac{s^3 + 4s^2 + 10s + 7}{s^2 + 2s + 3} \right) = 0$$

**Kommentarer till 4.4-11 (b):** "Proper" innebär att nämnarens gradtal > täljarens gradtal

I fall (i) bidrar inte termen 1 till  $y(0^+)$ , för inverstransformen till 1:an är en dirac-impuls och den är noll vid  $t = 0^+$ . Motsvarande gäller i fall (ii), där inverstransformen till  $(s+2)$  är lika med dirac:ens derivata plus  $2 \cdot \text{dirac:en}$  – ingen av dessa är nollskilda vid  $t = 0^+$ .

4.5 -1

a-b)



$$\psi(s) = V(s) \cdot \frac{1}{1+1} = \frac{1}{2} V(s)$$

$$\text{där } V(s) = X(s) \cdot \frac{R}{2+R}$$

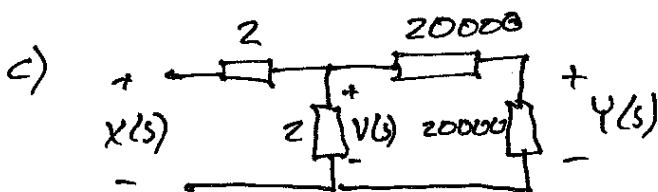
spänningsdelningar

$$\text{där } R = 2 \parallel (1+1) = 1 \Omega \Rightarrow V(s) = \frac{1}{3} X(s)$$

$$\Rightarrow \psi(s) = \frac{1}{2} \cdot \frac{1}{3} X(s) \Rightarrow \underline{\underline{H(s) = \frac{\psi(s)}{X(s)} = \frac{1}{6}}}$$

Svar: Nej, här för vi inte  $H_{\text{tot}}(s) = H_1(s) \cdot H_2(s) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ ,

för det efterföljande systemet i kaskadkopplingen belastar det första systemet (dvs. det drar ström från den). Då ändras det första systemets systemfunktion (det minskar från  $\frac{1}{2}$  till  $\frac{1}{3}$ ).



$$\psi(s) = V(s) \cdot \frac{20000}{20000 + 20000} = \frac{1}{2} V(s)$$

$$V(s) \approx \frac{1}{1.9999} \cdot X(s) \approx \frac{1}{2} X(s)$$

Nytt R ovan blir  $2 \parallel (20000 + 20000) \approx 1.9999 \Omega$

$$\Rightarrow H_{\text{tot}}(s) \approx \frac{1}{1.9999} \cdot \frac{1}{2} \approx \frac{1}{4} \quad \nabla$$

Anledningen är att system 2 knappt drar någon ström alls från system 1.

$$\text{Då } R_3 = R_4 = R \rightarrow \infty \Rightarrow H(s) \rightarrow \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Lärdom: Vid kaskadkoppling erhålls  $H(s) = H_1(s) \cdot H_2(s)$  ( motsvaras av  $h(t) = h_1(t) * h_2(t)$  ) endast om det efterföljande systemet inte belastar systemet/systemen innan!

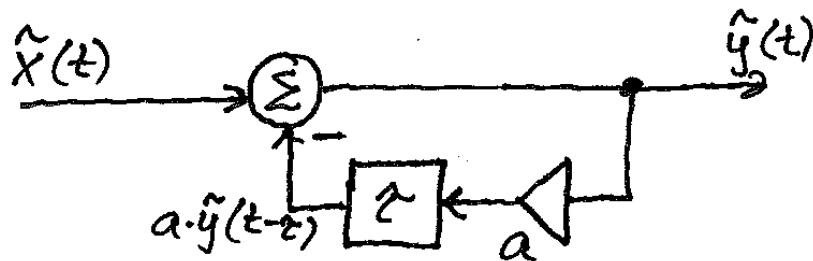
4.5-2. The transfer function of the two paths are  $e^{-sT}$  and  $ae^{-s(T+\tau)}$ . The two paths are in parallel. Hence the transfer function of this communication channel is

$$\begin{aligned} H(s) &= e^{-sT} + ae^{-s(T+\tau)} \\ &= e^{-sT}(1 + ae^{-s\tau}) \end{aligned}$$

For distortionless transmission, it is adequate to undo only the term  $(1 + ae^{-s\tau})$  in  $H(s)$  because  $e^{-sT}$  represents pure delay. Clearly, we need an equalizer with transfer function

$$H_{eq}(s) = \frac{1}{1 + a \cdot e^{-s\tau}} = \frac{\hat{Y}(s)}{\hat{X}(s)} \Rightarrow \hat{Y}(s) = \hat{X}(s) - a \cdot e^{-s\tau} \cdot \hat{Y}(s)$$

Comparing this form with the transfer function of the feedback system in Eq. (4.59) or Figure 4.18d, it is immediately obvious that such an equalizer can be realized by the following system



When this equalizer is placed in cascade with the communication channel, the effective transfer function is given by

$$H_c(s) = \frac{e^{-sT}(1 + ae^{-s\tau})}{1 + ae^{-s\tau}} = e^{-sT}$$

The effective system represents a pure delay of  $T$  seconds, which makes it distortionless. Moreover, the equalizer is realizable.

4.5-3. (a) The system transfer function is

$$H(s) = \frac{\frac{1}{s-1}}{1 + \frac{2}{s-1}} = \frac{1}{s+1}$$

The system is BIBO stable.

$$b) \quad H(s) = \frac{\frac{K}{s^2+2s}}{1 + \frac{K}{s^2+2s}} = \frac{K}{s^2+2s+K} \Rightarrow \text{Polar i } s = -1 \pm \sqrt{1-K}$$

- $K=1 \Rightarrow H(s) = \frac{1}{(s+1)^2} \Rightarrow$  Dubbelpol i  $s = -1$ : insignal-utsignal-stabilt  
(båda i v. halvplan)
- $K=-3 \Rightarrow$  Polar i  $s=1$  &  $s=-3$ , dvs. en pol i höger halvplan  
 $\Rightarrow$  insignal-utsignal-instabilt
- $K=2 \Rightarrow$  Polar i  $s = -1 \pm j$ , dvs. i v. halvplan  $\Rightarrow$  insignal-utsignal-stabilt
- $K=0 \Rightarrow H(s) = \frac{0}{s(s+2)}$ : Ser ut att vara marg. stabilt system, men  
nivåkonst. = 0  $\Rightarrow Y(s)=0$ ;  $y(t)=0$  för alla  $x(t)$ !

## 4.7-1

- a) Allmänt: Systemet med systemfunktion  $G(s) = \frac{\omega_c}{s + \omega_c}$  återkopplas med ett system som har systemfunktion  $H(s)$ , se Fig. 4.18d i boken. Kausalt system antas, vilket innebär att konvergensområdet för  $G(s)$  är  $\text{Re}\{s\} > -\omega_c$ , dvs. det omsluter imaginära axeln.

Frekvensfunktionen existerar därför och är  $G(j\omega) = \frac{\omega_c}{j\omega + \omega_c}$ , vilket innebär att

$$G(j0) = \left| \frac{\omega_c}{j0 + \omega_c} \right| = 1 \quad (\text{= amplitudkaraktäristikens maxvärde}) \quad \text{och} \quad |G(j\omega_c)| = \left| \frac{\omega_c}{j\omega_c + \omega_c} \right| = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} |G(j\omega)|_{\text{max}}, \quad \text{dvs. motsvarande 3 dB under maxvärdet} \quad \left( 20 \cdot 10 \log \frac{1}{\sqrt{2}} \approx -3 \right).$$

Följaktligen är 3 dB-gränsvinkelfrekvensen  $\omega_c$ .

- b) Det återkopplade systemets systemfunktion är  $H_{\text{tot}}(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{\frac{\omega_c}{s + \omega_c}}{1 + \frac{\omega_c}{s + \omega_c} \cdot 0.9} = \frac{\omega_c}{s + 10\omega_c}$ .

På motsvarande sätt som i a) erhåller vi  $H_{\text{tot}}(j0) = \left| \frac{\omega_c}{j0 + 10\omega_c} \right| = 0.1$  och

$$|H_{\text{tot}}(j10\omega_c)| = \left| \frac{\omega_c}{j10\omega_c + 10\omega_c} \right| = \frac{1}{\sqrt{2}} \cdot 0.1 = \frac{1}{\sqrt{2}} |H_{\text{tot}}(j\omega)|_{\text{max}}, \quad \text{dvs. det återkopplade systemets}$$

3 dB-gränsvinkelfrekvens är  $10\omega_c$ .

- c) (Notera att det i återkopplingen i bokens figur P4.7-1c) skall stå "-0.9" och inte "0.9".)

Det nya återkopplade systemet har systemfunktion  $H_{\text{tot}}(s) = \frac{\frac{\omega_c}{s + \omega_c}}{1 + \frac{\omega_c}{s + \omega_c} \cdot (-0.9)} = \frac{\omega_c}{s + 0.1\omega_c}$ ,

$$\text{vilket ger } H_{\text{tot}}(j0) = \left| \frac{\omega_c}{j0 + 0.1\omega_c} \right| = 10 \quad \text{och} \quad |H_{\text{tot}}(j0.1\omega_c)| = \left| \frac{\omega_c}{j0.1\omega_c + 0.1\omega_c} \right| = \frac{1}{\sqrt{2}} \cdot 10, \quad \text{dvs.}$$

3 dB-gränsvinkelfrekvens är  $0.1\omega_c$ .

- d) Produkten mellan förstärkningen  $H(0)$  och bandbredden  $\omega_c$  (the gain-bandwidth product) är konstant ( $= 1 \cdot \omega_c = 0.1 \cdot 10\omega_c = 10 \cdot 0.1\omega_c$ ) i alla tre fallen ovan.

(Anm: den nämnda produkten är ett mått på en förstärkares förmåga att förstärka vid olika frekvenser och ingår inte som ett särskilt moment i kursen!)

## 4.8-1.

$$H(j\omega) = \frac{j\omega + 2}{(j\omega)^2 + 5j\omega + 4} = \frac{j\omega + 2}{(4 - \omega^2) + j5\omega}$$

$$|H(j\omega)| = \sqrt{\frac{\omega^2 + 4}{(4 - \omega^2)^2 + (5\omega)^2}} = \sqrt{\frac{\omega^2 + 4}{\omega^4 + 17\omega^2 + 16}}$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{5\omega}{4 - \omega^2}\right) \quad (\pm\pi \text{ om realdelen } 4 - \omega^2 \text{ är negativ})$$

**Kommentar till 4.8-1:** Antag att systemet är kausalt (vilket allmänt kan antas, om inget annat framgår eller motsäger detta). Då är systemet stabilt (ty alla polerna i vänster halvplan), dvs. imaginära axeln ingår i systemfunktionens konvergensområde, vilket innebär att frekvensfunktionen  $H(j\omega)$  existerar.

Då kan vi beräkna utsignalen enligt nedan!

$$a) \quad x(t) = 5 \cos(2t + 30^\circ) = 5 \cos\left(2t + \frac{\pi}{6}\right) \Rightarrow \omega = 2 \text{ rad/s.}$$

$$|H(j2)| = \sqrt{\frac{2^2 + 4}{2^4 + 17 \cdot 2^2 + 16}} = \sqrt{\frac{2}{25}} = \frac{\sqrt{2}}{5}, \quad \arg H(j2) = \arctan\left(\frac{2}{2}\right) - \arctan(\infty) = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4} \text{ rad.}$$

**Alternativ** (enklare! :) erhålls amplitudskalningen och fäsförskjutningen direkt från

$$H(j2) = \frac{j2 + 2}{(4 - 2^2) + j5 \cdot 2} = \frac{1 + j}{j5} = \frac{\sqrt{2} \cdot e^{j\frac{\pi}{4}}}{5 \cdot e^{j\frac{\pi}{2}}} = \frac{\sqrt{2}}{5} \cdot e^{j\left(\frac{\pi}{4} - \frac{\pi}{2}\right)} = |H(j2)| \cdot e^{j\arg H(j2)}$$

Det stabila LTI-systemet genererar därför den stationära utsignalen

$$y(t) = 5 |H(j2)| \cos\left(2t + \frac{\pi}{6} + \arg H(j2)\right) = 5 \frac{\sqrt{2}}{5} \cos\left(2t + \frac{\pi}{6} - \frac{\pi}{4}\right) = \sqrt{2} \cos\left(2t - \frac{\pi}{12}\right)$$

$$b) \quad x(t) = 10 \sin(2t + 45^\circ) = 10 \sin\left(2t + \frac{\pi}{4}\right) \Rightarrow \omega = 2 \text{ rad/s igen!}$$

$$\Rightarrow y(t) = 10 |H(j2)| \sin\left(2t + \frac{\pi}{4} + \arg H(j2)\right) = 10 \frac{\sqrt{2}}{5} \sin\left(2t + \frac{\pi}{4} - \frac{\pi}{4}\right) = 2\sqrt{2} \sin(2t)$$

$$c) \quad x(t) = 10 \cos(3t + 40^\circ) = 10 \cos\left(2t + \frac{2\pi}{9}\right) \Rightarrow \omega = 3 \text{ rad/s.}$$

$$H(j3) = \frac{j3 + 2}{(4 - 3^2) + j5 \cdot 3} = \frac{2 + j3}{5(-1 + j3)} = \frac{\sqrt{2^2 + 3^2} \cdot e^{j\arctan\frac{3}{2}}}{5\sqrt{(-1)^2 + 3^2} \cdot e^{j\left(\arctan\frac{3}{-1} - \pi\right)}} = \frac{\sqrt{13}}{5\sqrt{10}} e^{j\left(\arctan\frac{3}{2} - \arctan\frac{3}{-1} - \pi\right)}$$

$$\approx 0.23 \cdot e^{-j0.91} = |H(j3)| \cdot e^{j\arg H(j3)}$$

$$\Rightarrow y(t) \approx 10 \cdot 0.23 \cos\left(2t + \frac{2\pi}{9} - 0.91\right) \approx 2.3 \cos(2t - 0.21)$$

4.8-2

$$H(j\omega) = \frac{j\omega + 3}{(j\omega + 2)^2} \Rightarrow$$

$$|H(j\omega)| = \frac{\sqrt{\omega^2 + 9}}{\omega^2 + 4}, \quad \arg H(j\omega) = \arctan \frac{\omega}{3} - 2 \cdot \arctan \frac{\omega}{2}$$

$$(a) \quad x(t) = 10u(t) = 10e^{0t}u(t) \Rightarrow y(t) = y_\phi(t) = 10H(0)e^{0t}u(t) = 10 \frac{3}{4} e^{0t}u(t) = \frac{15}{2} u(t)$$

$$\left( \begin{array}{l} (a) \ \& \ (b): \text{ Den motsvarande homogena utsignalskomponenten} \\ y_\phi(t) = \text{"steady-state response"} = \text{den stationära utsignalskomponenten} \end{array} \right)$$



(b)  $x(t) = \cos(2t + 60^\circ)u(t)$ . Here  $\omega = 2$

$$|H(j2)| = \frac{\sqrt{13}}{8} \quad \text{and} \quad \angle H(j2) = 33.69^\circ - 90^\circ = -56.31^\circ$$

Författaren anger oftast fasen i grader i stället för radianer. **Använd radianer!**

Therefore

$$y(t) = y_\phi(t) = \frac{\sqrt{13}}{8} \cos(2t + 60^\circ - 56.31^\circ)u(t) = \frac{\sqrt{13}}{8} \cos(2t + 3.69^\circ)u(t)$$

(c)  $x(t) = \sin(3t - 45^\circ)u(t)$  Here  $\omega = 3$  and

$$|H(j3)| = \frac{\sqrt{18}}{13} \quad \text{and} \quad \angle H(j3) = 45^\circ - 112.62^\circ = -67.62^\circ$$

Therefore

$$y(t) = y_\phi(t) = \frac{\sqrt{18}}{13} \sin(3t - 45^\circ - 67.62^\circ)u(t) = \frac{\sqrt{18}}{13} \sin(3t - 112.62^\circ)u(t)$$

(d)  $x(t) = e^{j3t}u(t)$

$$y(t) = y_\phi(t) = H(j3)e^{j3t} = |H(j3)|e^{j[3t + \angle H(j3)]}u(t) = \frac{\sqrt{18}}{13}e^{j[3t - 67.62^\circ]}u(t)$$

4.8-4. (a) From the graph, the two system zeros are at  $s = \pm j1.5$ . Thus,  $s^2 + b_1s + b_2 = (s + j1.5)(s - j1.5) = s^2 + 2.25$ . The two system poles are at  $s = -1 \pm j0.5$ . Thus,  $s^2 + a_1s + a_2 = (s + 1 + j0.5)(s + 1 - j0.5) = s^2 + 2s + 1.25$ . At DC, the system function is  $H(j0) = -1 = k \frac{b_2}{a_2} = k \frac{2.25}{1.25} = k \frac{9}{5}$ . Therefore,

$$k = -\frac{5}{9}, b_1 = 0, b_2 = \frac{9}{4}, a_1 = 2, \text{ and } a_2 = \frac{5}{4}.$$

(b) The DC gain is given as  $H(0) = -1$ . Thus, the input of 4 just becomes  $-4$ . To compute the output to  $\cos(t/2 + \pi/3)$ ,  $H(j0.5)$  is required. Graphically,  $|H(j0.5)| = |k| \frac{(1)(2)}{(1)(\sqrt{2})} = \frac{10}{9\sqrt{2}}$  and  $\angle H(j0.5) = \pi - \pi/2 + \pi/2 - (0 + \pi/4) = 3\pi/4$ . Thus, the output to  $\cos(t/2 + \pi/3)$  is just  $\frac{10}{9\sqrt{2}} \cos(t/2 + \pi/3 + 3\pi/4)$ . Thus, the output to  $x(t) = 4 + \cos(t/2 + \pi/3)$  is

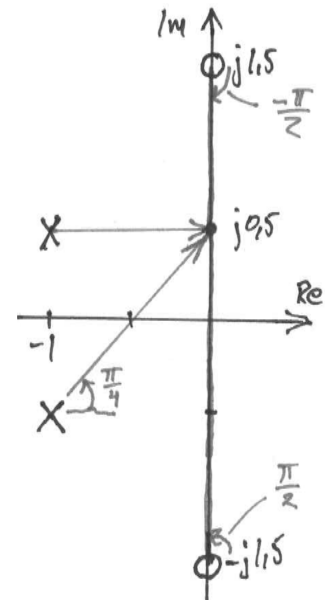
$$y(t) = -4 + \frac{10}{9\sqrt{2}} \cos(t/2 + 13\pi/12) \approx -4 + 0.7857 \cos(t/2 + 3.4034).$$

**OBS:**

- I första raden i (b)-uppgiftslösningen ovan står det  $S(j0)$ . Det skall naturligtvis stå  $H(j0)$ .

*Kommentarer fortsätter på nästa sida!*

- I lösningstexten har författaren använt sig av **polvektorer** och **nollställe-vektorer** för att beräkna  $|H(j0.5)|$  och  $\arg H(j0.5)$  – se figuren till höger: Rita, i pol-nollställediagrammet, vektorer från alla poler och nollställen till  $s = j0.5$ .



4.10-1 Pol-nollställediagrammet med inritade nollställevektorer och polvektorer för  $\omega \approx 3$  rad/s visas i figuren till vänster nedan. Eftersom längderna för varje par av speglade nollställen/poler är lika, så kommer amplitudkaraktäristiken att bli lika med nivåkonstanten för alla vinkelfrekvenser, dvs.

$$|H(j\omega)| = 1. \text{ Detta är således ett allpassfilter.}$$

Faskaraktäristiken: Vid  $\omega = 0$  gäller  $\alpha = -\gamma$  (dvs. summan av polvinklarna blir 0) och

$$\varphi = 2\pi - \beta, \text{ vilket innebär att } \arg H(j0) = \beta + \varphi - (\gamma + \alpha) = \beta + (2\pi - \beta) - (\gamma - \gamma) = 2\pi \text{ rad.}$$

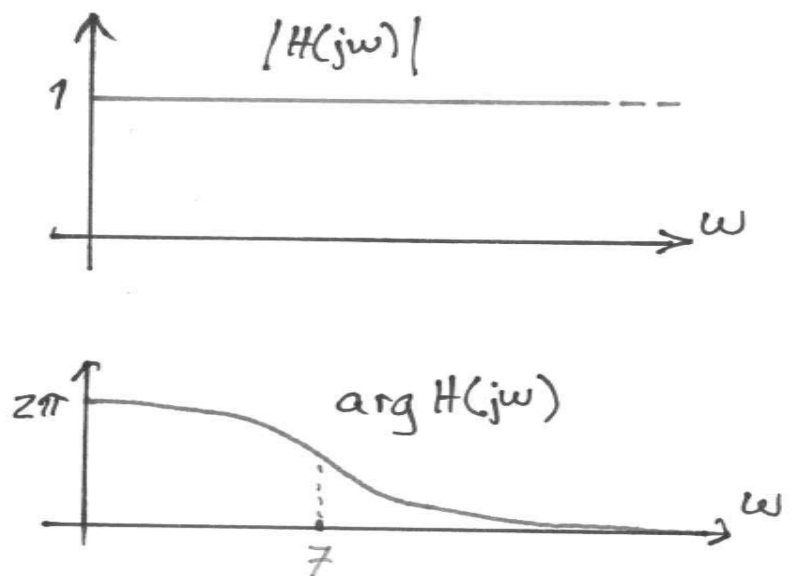
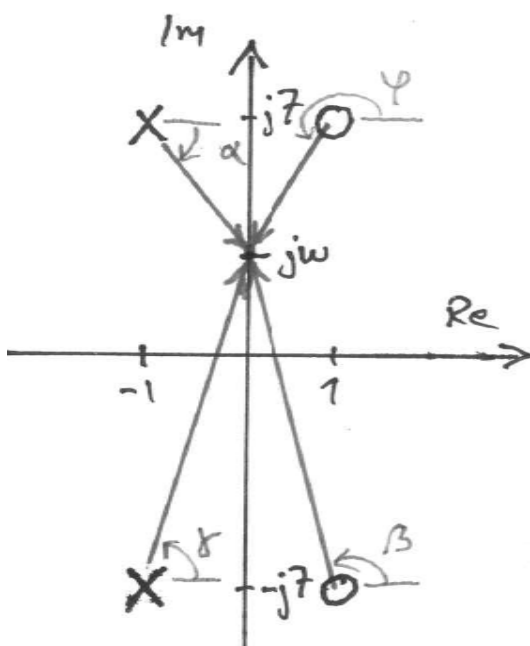
(Notera att  $\alpha$  är negativ i figuren nedan!)

När  $\omega \rightarrow \infty$  så kommer alla nollställe- och polvektorer att vara riktade rakt upp, vilket innebär att

$$\alpha = \gamma = \varphi = \beta = \frac{\pi}{2} \text{ rad. Vi får alltså (med } M = \text{antal nollställen och } N = \text{antal poler) det allmänna}$$

$$\text{sambandet } \lim_{\omega \rightarrow \infty} \arg H(j\omega) = \arg(\text{nivåkonst.}) + \frac{\pi}{2}(M - N) = 0 + \frac{\pi}{2}(2 - 2) = 0.$$

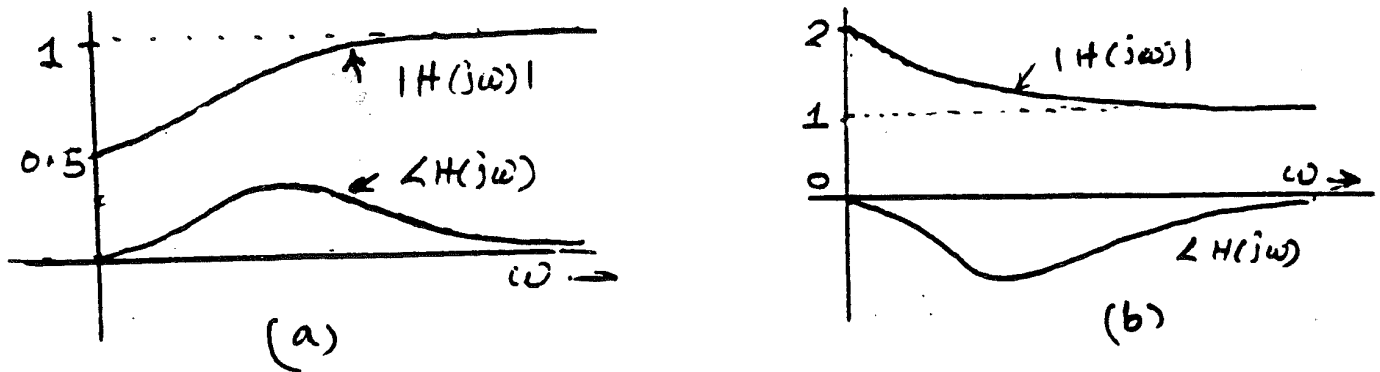
Faskaraktäristiken ändras kraftigast runt  $\omega = 7$  rad/s, på grund av att både polen i  $s = -1 + j7$  och nollstället i  $s = 1 + j7$  har tydligast inverkan på fasen där (vinklarna för motsvarande pol- respektive nollställevektor ändras relativt snabbt när man längs  $\omega$ -axeln ”passerar” polen/nollstället.



4.10-2. (a) If  $r$  and  $d$  are the distances of the zero and pole, respectively from  $j\omega$ , then the amplitude response  $|H(j\omega)|$  is the ratio  $r/d$  corresponding to  $j\omega$ . This ratio is 0.5 for  $\omega = 0$ . Therefore, the dc gain is 0.5. Also the ratio  $r/d = 1$  for  $\omega = \infty$ . Thus, the gain is unity at  $\omega = \infty$ . Also the angles of the line segments connecting the zero and pole to the point  $j\omega$  are both zero for  $\omega = 0$ , and are both  $\pi/2$  for  $\omega = \infty$ . Therefore  $\angle H(j\omega) = 0$  at  $\omega = 0$  and  $\omega = \infty$ . In between the angle is positive as shown in Figure S4.10-2a.

(b) In this case the ratio  $r/d$  is 2 for  $\omega = 0$ . Therefore, the dc gain is 2. Also the ratio  $r/d = 1$  for  $\omega = \infty$ . Thus, the gain is unity at  $\omega = \infty$ . Also the angles

of the line segments connecting the zero and pole to the point  $j\omega$  are both zero for  $\omega = 0$ , and are both  $\pi/2$  for  $\omega = \infty$ . Therefore  $\angle H(j\omega) = 0$  at  $\omega = 0$  and  $\omega = \infty$ . In between the angle is negative as shown in Figure S4.10-2b.



4.10-3. The poles are at  $-a \pm j10$ . Moreover zero gain at  $\omega = 0$  and  $\omega = \infty$  requires that there be a single zero at  $s = 0$ . This clearly causes the gain to be zero at  $\omega = 0$ . Also because there is one excess pole over zero, the gain for large values of  $\omega$  is  $1/\omega$ , which approaches 0 as  $\omega \rightarrow \infty$ . therefore, the suitable transfer function is

$$H(s) = \frac{s}{(s + a + j10)(s + a - j10)} = \frac{s}{s^2 + 2as + (100 + a^2)}$$

The amplitude response is high in the vicinity of  $\omega = 10$  provided  $a$  is small. Smaller the  $a$ , more pronounced the gain in the vicinity of  $\omega = 10$ . For  $a = 0$ , the gain at  $\omega = 10$  is  $\infty$ .

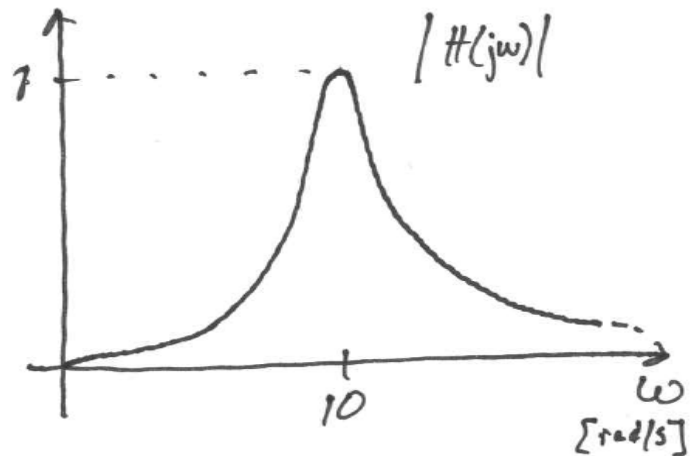
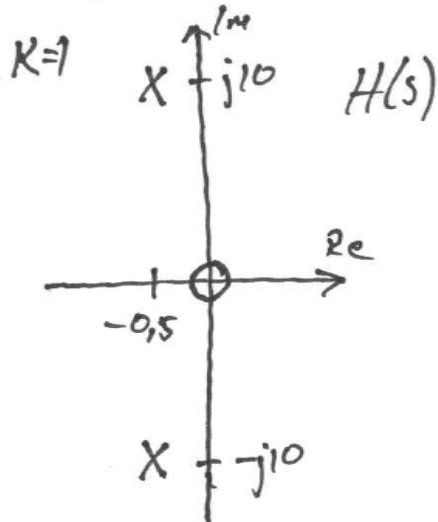
Tilläggsuppgiftens lösning: se nästa sida!

Tilläggsuppgift:

$$|H(j\omega)| = \left| \frac{j\omega}{-10^2 + j20a + 100 + a^2} \right| = \frac{10}{|a^2 + j20a|} = \frac{10}{\sqrt{a^4 + 400a^2}} = 1$$

$$\Rightarrow a^4 + 400a^2 - 100 = 0 \Rightarrow a^2 = \frac{-200 \pm \sqrt{200^2 + 100}}{2} \approx 0,25 \Rightarrow a \approx \pm 0,5$$

(alternativt erhålls samma utgående från polfullställe-vektorens längder vid/för  $\omega=10$ )



4.10-4. Cynthia is correct. Although the system is all-pass and has  $|H(j\omega)| = 1$ , the phase response is not zero. Thus, the output generally has different phase than the input. Furthermore, the output can also include transient components that would not be present in the original input.

4.10-5. Both Amy and Jeff are correct. By definition, a zero is any value  $s$  that forces  $H(s) = 0$  and a pole is any value  $s$  that forces  $H(s) = \infty$ . Thus, the system  $H(s) = s = \frac{1}{s^{-1}}$  has both a zero at  $s = 0$  and a pole at  $s = \infty$ . Remember, a rational system function always has the same number of poles and zeros; if  $H(s) = s$  has an obvious zero at  $s = 0$  there must be a matching pole somewhere, even if it is not finite. By similar argument, the system  $H(s) = \frac{1}{s}$  has a pole at  $s = 0$  and a zero at  $s = \infty$ .

4.10-7. At high frequencies, the highest powers of  $s$  dominate both the numerator and denominator of  $H(s)$ . That is,  $\lim_{s \rightarrow \infty} H(s) = \lim_{s \rightarrow \infty} \frac{b_0 s^M}{s^N}$ . Thus, the log magnitude response at high frequencies is given by  $\lim_{\omega \rightarrow \infty} \log |H(j\omega)| = \log(b_0) + M \log(\omega) - N \log(\omega)$ . The fastest attenuation as a function of frequency requires  $M$  to be as small as possible. Thus, for a given  $N$ , the attenuation rate of an all-pole lowpass filter ( $M = 0$ ) is faster than the attenuation rate of any filter with a finite number of zeros ( $M \neq 0$ ).

4.10-8. No, it is not possible for such a system to function as a lowpass filter. For any choice of  $([k, b_1, b_2, a_1, a_2] \in \mathcal{R})$ , the system function  $H(s) = k \frac{s^2 + b_1 s + b_2}{s^2 + a_1 s + a_2}$  is proper. Thus, the system function always has high-frequency gain of  $k$ . For  $k \neq 0$ , the system cannot be lowpass. Furthermore, for  $k = 0$  the system becomes a useless “nopass filter” (again, not lowpass).

4.10-9. Nick is more correct than his professor. A cascade of two identical filters, each with system response  $H(j\omega)$ , gives a total response of  $H^2(j\omega)$ . Since realizable filters, such as Butterworth filters, are not ideal, the cascade system will tend to have a faster transition band and greater stopband attenuation. In a sense, the resulting fourth-order system really does provide “twice the filtering” of the original second-order system.

Unfortunately, there are also problems with Nick’s approach. Simply cascading a designed lowpass filter twice has negative consequences. For example, the cutoff frequency shifts to a lower frequency than desired. As the cascaded  $RC$  example in

MATLAB Session 4 suggests, a cascade of low-order filters is inferior to a carefully designed, equivalent-order filter. In general, a fourth-order Butterworth filter performs better than a cascade of two second-order Butterworth filters.

4.10-12. (a) Yes, it is possible for the system to output  $y(t) = \sin(100\pi t)u(t)$  in response to  $x(t) = \cos(100\pi t)u(t)$ . Noting  $Y(s) = \frac{100\pi}{s^2 + (100\pi)^2}$  and  $X(s) = \frac{s}{s^2 + (100\pi)^2}$ ,

one way to obtain  $y(t)$  from  $x(t)$  is using the system  $H(s) = Y(s)/X(s) = \frac{100\pi}{s^2 + (100\pi)^2} \frac{s^2 + (100\pi)^2}{s} = \frac{100\pi}{s}$ .

(b) Yes, it is possible for the system to output  $y(t) = \sin(100\pi t)u(t)$  in response to  $x(t) = \sin(50\pi t)u(t)$ . Noting  $Y(s) = \frac{100\pi}{s^2 + (100\pi)^2}$  and  $X(s) = \frac{50\pi}{s^2 + (50\pi)^2}$ , one way to obtain  $y(t)$  from  $x(t)$  is using the system  $H(s) = Y(s)/X(s) = \frac{100\pi}{s^2 + (100\pi)^2} \frac{s^2 + (50\pi)^2}{50\pi} = 2 \cdot \frac{s^2 + (50\pi)^2}{s^2 + (100\pi)^2}$

(c) Yes, it is possible for the system to output  $y(t) = \sin(100\pi t)$  in response to  $x(t) = \cos(100\pi t)$ . To do this, the system must have  $H(j100\pi) = e^{-j\pi/2}$ . That is, the magnitude response at  $\omega = 100\pi$  must be unity, and the phase response at  $\omega = 100\pi$  must be  $-\pi/2$ .

(d) No, it is not possible for the system to output  $y(t) = \sin(100\pi t)$  in response to  $x(t) = \sin(50\pi t)$ . In an LTI system, an everlasting sinusoidal input of frequency  $50\pi$  cannot produce a different frequency output.

4.11-5. (a)

$$x(t) = e^{-\frac{|t|}{2}}, \quad H(s) = \frac{1}{s+1} \quad \sigma > -1$$

$$\text{And} \quad X(s) = \frac{1}{s+0.5} - \frac{1}{s-0.5} \quad -\frac{1}{2} < \sigma < \frac{1}{2}$$

$$\text{hence: } Y(s) = H(s)X(s) = \frac{1}{s+1} \left[ \frac{1}{s+0.5} - \frac{1}{s-0.5} \right] \quad -\frac{1}{2} < \sigma < \frac{1}{2}$$

$$\begin{aligned} Y(s) &= \frac{-2}{s+1} + \frac{2}{s+0.5} + \frac{\frac{2}{3}}{s+1} - \frac{\frac{2}{3}}{s-0.5} \\ &= \frac{-\frac{4}{3}}{s+1} + \frac{2}{s+0.5} - \frac{\frac{2}{3}}{s-0.5} \quad -\frac{1}{2} < \sigma < \frac{1}{2} \end{aligned}$$

The poles  $-1$  and  $-0.5$ , which are to the left of the strip of convergence, yield the causal signal, and the pole  $0.5$ , which is to the right of the strip of convergence, yields the anticausal signal. Hence

$$y(t) = \left( -\frac{4}{3}e^{-t} + 2e^{-t/2} \right) u(t) + \frac{2}{3}e^{t/2}u(-t)$$

(b)

$$x(t) = e^t u(t) + e^{2t} u(-t)$$

$$\begin{aligned} X(s) &= \frac{1}{s-1} - \frac{1}{s-2} \quad 1 < \sigma < 2 \\ &= \frac{-1}{(s-1)(s-2)} \end{aligned}$$

$$\text{And } H(s) = \frac{1}{s+1} \quad \sigma > -1$$

$$\text{Hence: } Y(s) = H(s)X(s) = \frac{-1}{(s+1)(s-1)(s-2)} \quad 1 < \sigma < 2$$

$$Y(s) = \frac{-1/6}{s+1} + \frac{1/2}{s-1} - \frac{1/3}{s-2} \quad 1 < \sigma < 2$$

$$\text{Hence } y(t) = \left( -\frac{1}{6}e^{-t} + \frac{1}{2}e^t \right) u(t) + \frac{1}{3}e^{2t}u(-t)$$

(c)

$$x(t) = e^{-t/2}u(t) + e^{-t/4}u(-t)$$

$$X(s) = \frac{1}{s+0.5} - \frac{1}{s+0.25} = \frac{-\frac{1}{4}}{(s+0.5)(s+0.25)} \quad -\frac{1}{2} < \sigma < -\frac{1}{4}$$

$$\text{Also } H(s) = \frac{1}{s+1} \quad \sigma > -1$$

$$\begin{aligned} \text{Hence: } Y(s) = H(s)X(s) &= \frac{-\frac{1}{4}}{(s+1)(s+0.5)(s+0.25)} \quad -\frac{1}{2} < \sigma < -\frac{1}{4} \\ &= \frac{-\frac{2}{3}}{s+1} + \frac{2}{s+0.5} - \frac{\frac{4}{3}}{s+0.25} \quad -\frac{1}{2} < \sigma < -\frac{1}{4} \end{aligned}$$

$$\text{and } y(t) = \left( -\frac{2}{3}e^{-t} + 2e^{-t/2} \right) u(t) + \frac{4}{3}e^{-t/4}u(-t)$$

(d)

$$x(t) = e^{2t}u(t) + e^t u(-t) = x_1(t) + x_2(t)$$

$$X_1(s) = \frac{1}{s-2} \quad \sigma > 2$$

$$X_2(s) = \frac{-1}{s-1} \quad \sigma < 1$$

$$\text{and} \quad H(s) = \frac{1}{s+1} \quad \sigma > -1$$

In this case, there is no region of convergence that is common to  $X_1(s)$  and  $X_2(s)$ . However, each of  $X_1(s)$  and  $X_2(s)$  have a region of convergence that is common to  $H(s)$ . Hence the output can be computed by finding the system response to  $x_1(t)$  and  $x_2(t)$  separately, and then adding these two components. This means we need not worry about the common region of convergence for  $X_1(s)$  and  $X_2(s)$ . Thus:

$$Y(s) = Y_1(s) + Y_2(s) \quad \text{where}$$

$$\begin{aligned} Y_1(s) = X_1(s)H(s) &= \frac{1}{(s+1)(s-2)} \quad \sigma > 2 \\ &= \frac{-\frac{1}{3}}{s+1} + \frac{\frac{1}{3}}{s-2} \quad \sigma > 2 \end{aligned}$$

Observe that both the poles ( $-1$  and  $2$ ) are to the left of the region of convergence, hence both terms are causal, and:

$$y_1(t) = \left( -\frac{1}{3}e^{-t} + \frac{1}{3}e^{2t} \right) u(t)$$

$$\begin{aligned} Y_2(s) = X_2(s)H(s) &= \frac{-1}{(s+1)(s-1)} \quad -1 < \sigma < 1 \\ &= \frac{\frac{1}{2}}{s+1} - \frac{\frac{1}{2}}{s-1} \quad -1 < \sigma < 1 \end{aligned}$$

The poles  $-1$  and  $1$  are to the left and the right, respectively, of the strip of convergence. Hence the first term yields causal signal and the second yields anticausal signal. Hence

$$y_2(t) = \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^t u(-t)$$

$$\text{Therefore} \quad y(t) = y_1(t) + y_2(t) = \left( \frac{1}{6}e^{-t} + \frac{1}{3}e^{2t} \right) u(t) + \frac{1}{2}e^t u(-t)$$

(e)

$$x(t) = e^{-\frac{t}{4}}u(t) + e^{-\frac{t}{2}}u(-t) = x_1(t) + x_2(t)$$

$$X(s) = X_1(s) + X_2(s)$$

$$\begin{aligned} \text{where } X_1(s) &= \frac{1}{s+0.25} & \sigma > -\frac{1}{4} \\ X_2(s) &= \frac{-1}{s+0.5} & \sigma < -\frac{1}{2} \\ H(s) &= \frac{1}{s+1} & \sigma > -1 \end{aligned}$$

Here also, we have no common region of convergence, for  $X_1(s)$  and  $X_2(s)$  as in part d. Let  $Y(s) = Y_1(s) + Y_2(s)$  where:

$$\begin{aligned} Y_1(s) &= \frac{1}{(s+1)(s+0.25)} & \sigma > -\frac{1}{4} \\ &= \frac{-\frac{4}{3}}{s+1} + \frac{\frac{4}{3}}{s+0.25} & \sigma > -\frac{1}{4} \end{aligned}$$

$$y_1(t) = \left( -\frac{4}{3}e^{-t} + \frac{4}{3}e^{-\frac{t}{4}} \right) u(t)$$

$$\begin{aligned} Y_2(s) &= \frac{-1}{(s+1)(s+0.5)} & -1 < \sigma < -\frac{1}{2} \\ &= \frac{2}{s+1} - \frac{2}{s+0.5} & -1 < \sigma < -\frac{1}{2} \end{aligned}$$

$$\text{and } y_2(t) = 2e^{-t}u(t) + 2e^{-\frac{t}{2}}u(-t)$$

$$\text{Hence } y(t) = y_1(t) + y_2(t) = \left( \frac{2}{3}e^{-t} + \frac{4}{3}e^{-\frac{t}{4}} \right) u(t) + 2e^{-\frac{t}{2}}u(-t)$$

(f)

$$x(t) = e^{-3t}u(t) + e^{-2t}u(-t) = x_1(t) + x_2(t)$$

$$X(s) = X_1(s) + X_2(s)$$

$$\begin{aligned} \text{where } X_1(s) &= \frac{1}{s+3} & \sigma > -3 \\ X_2(s) &= \frac{-1}{s+2} & \sigma < -2 \\ H(s) &= \frac{1}{s+1} & \sigma > -1 \end{aligned}$$

In this case, there is a common region of convergence for  $X_1(s)$  and  $H(s)$ , but there is no region of convergence common to  $X_2(s)$  and  $H(s)$ . Hence the output  $y_1(t)$  will be finite but  $y_2(t)$  will be  $\infty$ .