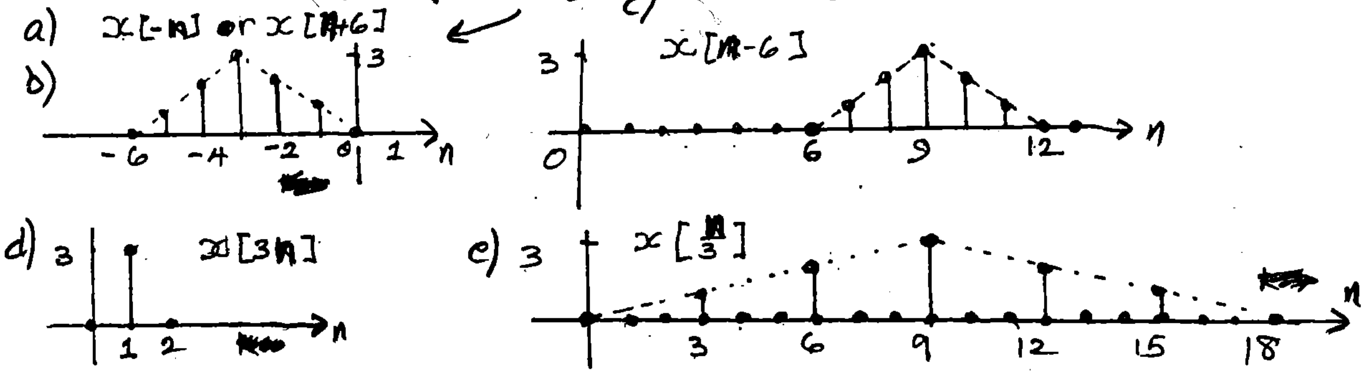
3.2-3:

f): $x[3-n]$ = den $x[n]$-fkn. som ges i uppg. 3.1-1(a), dvs. förskjut $x[-n]$ 3 steg åt höger.



3.3-4. (a) $x[n] = (n+3)\,(u[n+3] - u[n]) + (-n+3)\,(u[n] - u[n-4])$

(b) $x[n] = n\,(u[n] - u[n-4]) + (-n+6)\,(u[n-4] - u[n-7])$

(c) $x[n] = 3n(u[n+3] - u[n-4])$

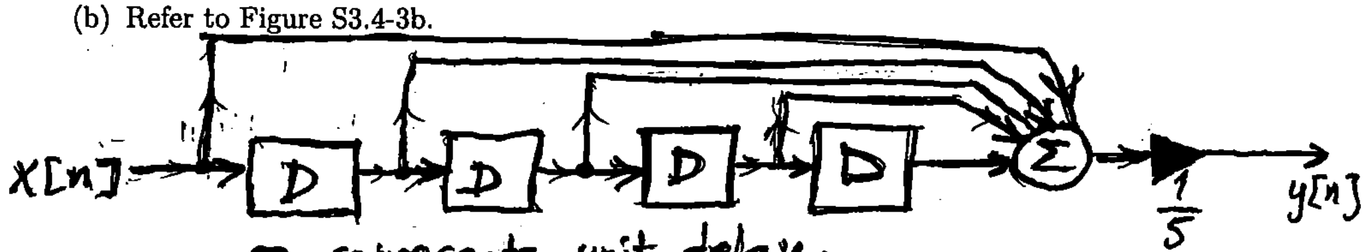
(d) $x[n] = -2n\,(u[n+2] - u[n]) + 2n\,(u[n] - u[n-3])$

In all four cases, $x[n]$ may be represented by several other (slightly different) expressions. For instance, in case (a), we may also use $x[n] = (n+3)\,(u[n+3] - u[n-1]) + (-n+3)\,(u[n-1] - u[n-4])$. Moreover because $x[n] = 0$ at $n = \pm 3$, $u[n+3]$ and $u[n-4]$ may be replaced with $u[n+2]$ and $u[n-3]$, respectively. Similar observations apply to other cases also.

3.4-3. (a)

$$y[n] = \left(x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]\right) * \frac{1}{5}$$

(b) Refer to Figure S3.4-3b.



D represents unit delay.

Figure S3.4-3b

3.4-7. (a) True; all finite power signals have infinite energy, and therefore cannot be energy signals. Energy signals and power signals are mutually exclusive.

(b) False; a signal with infinite energy need not be a power signal. For example, the signal $x[n] = 2^n u[n]$ has infinite energy and infinite power. Thus, it is neither an energy signal nor a power signal.

(c) True; the system is causal. Even though the input is scaled by $(n+1)$, the current output only depends on the current input. Another way to see this is to rewrite the expression as $y[n] = nx[n] + x[n]$.

(d) False; the system is not causal. The current output depends on a future input value. To help see this, substitute $n' = n-1$ to yield $y[n'] = x[n'+1]$; the output at time n' requires the future input value at time $n' + 1$.

(e) False; a signal $x[n]$ with energy E does not guarantee that signal $x[an]$ has energy $\frac{E}{|a|}$. Although this statement is true for continuous-time signals, it is not true for discrete-time signals. Remember, the discrete operation $x[an]$ results in a loss of information and thus a likely loss in energy. For example, consider $x[n] = \delta[n-1]$, which has energy $E = 1$. The signal $y[n] = x[2n] = 0$ has zero energy, not $E/2 = 1/2$.

3.4-8. Notice, $y_1[n] = -\delta[n]+\delta[n-1]+2\delta[n-2]$. Furthermore, $x_2[n] = x_1[n-1]-2x_1[n-2]$. Since the system is LTI,

$$y_2[n] = y_1[n-1] - 2y_1[n-2].$$

MATLAB is used to plot the result.

(Men grafen är även lätt att rita för hand, efter att först ha förenklat $y_2[n]$!)

```
>> y1 = inline('-(n==0)+(n==1)+2*(n==2)'); n = [-2:8];
>> stem(n,y1(n-1)-2*y1(n-2),'k'); axis([-2 8 -4.5 4.5]);
>> xlabel('n'); ylabel('y_2[n]');
```

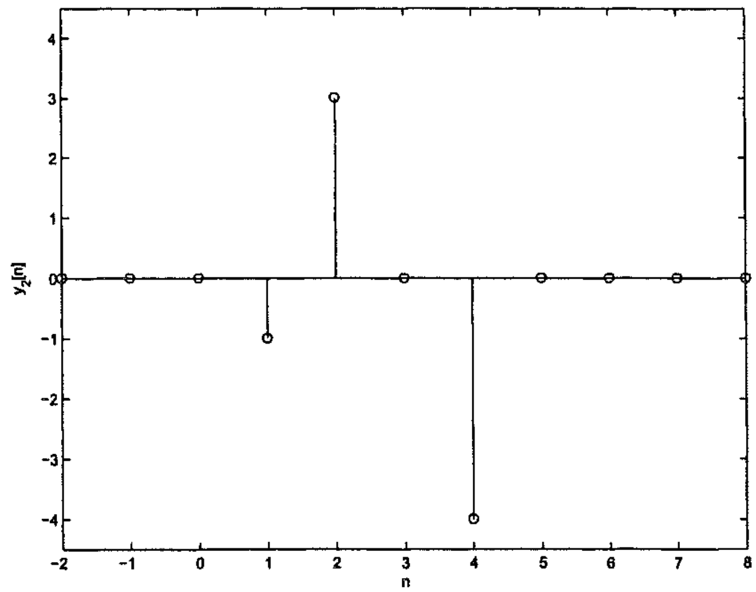


Figure S3.4-8: Plot of $y_2[n] = y_1[n-1] - 2y_1[n-2]$.

3.4-10. It is convenient to substitute $n' = n + 1$ and rewrite the system expression as $y[n'] = x[n' - 1]/x[n']$.

(a) No, the system is not BIBO stable. Input values of zero can result in unbounded outputs. For example, at $n' = 0$ the bounded input $x[n'] = \delta[n']$ yields an unbounded output $y[1] = 1/0 = \infty$.

(b) No, the system is not memoryless. The current output relies on a past input. For example, at $n' = 0$, the output $y[n']$ requires both the current input $x[n']$ and a stored past input $x[n' - 1]$.

(c) Yes, the system is causal. The current output $y[n']$ does not depend on any future value of the input.

3.4-11. The operation $y(t) = x(2t)$ is a one-to-one mapping, where no information is lost. Any one-to-one mapping is invertible. In this case, $x(t)$ is recovered by taking $y(t/2)$.

Since every other sample of $x[n]$ is removed in the operation $y[n] = x[2n]$, one half of $x[n]$ is lost and the process is not invertible. Thought of another way, the operation $y[n] = x[2n]$ is not a one-to-one mapping; many different signals $x[n]$ map to the same signal $y[n]$, which makes inversion impossible.

3.4-13. Using the definition of the ramp function, the system expression is rewritten as $y[n] = nx[n]u[n]$.

(a) No, the system is not BIBO stable. For example, if the input is a unit step $x[n] = u[n]$, then the output is a ramp function $y[n] = r[n]$, which grows unbounded with time.

(b) Yes, the system is linear. Let $y_1[n] = nx_1[n]u[n]$ and $y_2[n] = nx_2[n]u[n]$. Applying $ax_1[n] + bx_2[n]$ to the system yields $y[n] = n\left(ax_1[n] + bx_2[n]\right)u[n] = anx_1[n]u[n] + bnx_2[n]u[n] = ay_1[n] + by_2[n]$.

(c) Yes, the system is memoryless. The current output only depends on the current input multiplied by a known (time-varying) scale factor.

(d) Yes, the system is causal. All memoryless systems are causal. The output does not depend on future values of the input or output.

(e) No, the system is not time-invariant. For example, applying $x_1[n] = \delta[n]$ yields the output $y_1[n] = n\delta[n]u[n] = 0$. Applying $x_2[n] = \delta[n-1]$ yields the output $y_2[n] = n\delta[n-1]u[n] = \delta[n-1]$. Note, $x_2[n] = x_1[n-1]$ but $y_2[n] \neq y_1[n-1]$. Shifting the input does not produce a corresponding shift in the output.

3.8-1.

$$\begin{aligned} y[n] &= (-2)^n u[n-1] * e^{-n} u[n+1] \\ &= \sum_{m=-\infty}^{\infty} (-2)^m u[m-1] e^{-(n-m)} u[n-m-1] \end{aligned}$$

However, $u[m-1] = 0$ for $m < 1$ and $u[n-m+1] = 0$ for $m > n+1$. Hence the summation limits may be restricted for $1 \leq m \leq n+1$.

$$\begin{aligned} y[n] = e^{-n} \sum_{m=1}^{n+1} (-2e)^m &= e^{-n} \left[\frac{(-2e)^{n+2} + 2e}{-2e-1} \right] \\ &= \frac{2e^2}{2e+1} \left[(-2)^{n+1} - e^{-(n+1)} \right] u[n] \end{aligned}$$

We can also obtain this answer by using the convolution Table and the shift property of convolution. If we advance impulse response $h[n]$ by one unit and delay the input by one unit, the convolution remains unchanged according to the shift property. Hence we should obtain the convolution by using

$$h[n] = (-2)^{n+1} u[n] \qquad \text{and} \qquad x[n] = e^{-(n-1)} u[n]$$

Thus the desired convolution is given by

$$\begin{aligned} y[n] &= (-2)^{n+1} u[n] * e^{-(n-1)} u[n] \\ &= -2e \left\{ (-2)^n u[n] * e^{-n} u[n] \right\} \end{aligned}$$

From the convolution Table, we obtain

$$\begin{aligned} y[n] &= -2e \left[\frac{(-2)^{n+1} - e^{-(n+1)}}{-1 - e^{-1}} \right] u[n] \\ &= \frac{2e^2}{2e+1} \left[(-2)^{n+1} - e^{-(n+1)} \right] u[n] \end{aligned}$$

which confirms earlier result.

Kommentar 1: $u[n]$ läggs till i $y[n]$ eftersom summan gäller då övre summationsgränsen är större eller lika med undre summationsgränsen, dvs. $n+1 \geq 1$, dvs. $n \geq 0$.

Kommentar 2: Rita alltid $x[m]$ och $h[n-m]$ (eller, som använts i denna uppgift, $x[n-m]$ och $h[m]$)! Då blir det tydligt vilka summationsgränser det blir för olika intervall för n.

OBS: Använd <u>inte</u> faltningstabellen på sid. 291 i boken, som föreslås i den andra delen av lösningen ovan!
Rita i stället upp hjälpfigurer för varje intervall på *n* där funktionerna överlappar på olika sätt och lös faltningssumman för hand för dessa olika fall.

Gör så för <u>alla</u> faltningsuppgifter av standardkaraktär!

3.8-6. #1

$$\delta[n-k] * x[n] = x[n-k]$$

$$\delta[n-k] * x[n] = \sum_{m=0}^{\infty} x[m]\delta[n-m-k]$$

$\delta[n-m-k] = 1$ for $m = n-k$ and is zero for all other values of m. Hence the right-side sum is given by $x[n-k]$.

#2

Rita hjälpgrafer!

$$\gamma^n u[n] * u[n] = \sum_{m=0}^{n} \gamma^m u[n-m]$$

Because $u[n-m] = 1$ for all $0 \leq m \leq n$, we have

$$\gamma^n u[n] * u[n] = \sum_{m=0}^{n} \gamma^m = \frac{\gamma^{n+1}-1}{\gamma-1} u[n] \qquad \gamma \neq 1$$

We multiply the result with $u[n]$ because the convolution is zero for $n < 0$.

#3

Rita hjälpgrafer!

$$u[n] * u[n] = \sum_{m=0}^{n} u[m]u[n-m]$$

Over the range $0 \leq m \leq n$, $u[m] = u[n-m] = 1$. Hence

$$u[n] * u[n] = \sum_{m=0}^{n} 1 = (n+1)u[n]$$

3.8-10. The characteristic root is -2. Therefore

$$y_0[n] = c(-2)^n$$

Setting $n = -1$ and substituting $y[-1] = 10$, yields

$$10 = -\frac{c}{2} \quad \Longrightarrow \quad c = -20$$

Therefore

$$y_0[n] = -20(-2)^n \qquad n \geq 0$$

For this system $h[n]$, the unit impulse response is found in Prob. 3.7-1b to be

$$h[n] = (-2)^n u[n]$$

(I den här uppgiften är dock impulssvaret angivet i uppgiftsbeskrivningen på lektionswebbsidan!)

I texten nedan sägs att *y*[*n*] erhålls från faltningstabellen på sid. 291, men du förväntas att i stället beräkna faltningen själv!

The zero-state response is

$$y[n] = e^{-n}u[n] * (-2)^n u[n]$$

Rita hjälpgrafer!

This is found by using the convolution Table to be

$$\begin{aligned} y[n] &= \frac{e}{2e+1}[e^{-(n+1)} - (-2)^{n+1}]u[n] \\ &= \frac{e}{2e+1}[\frac{1}{e}(e)^{-n} + 2(-2)^n]u[n] \\ &= \left[\frac{1}{2e+1}(e)^{-n} + \frac{2e}{2e+1}(-2)^n\right]u[n] \end{aligned}$$

$$\begin{aligned} \text{Total Response} &= y_0[n] + y[n] \\ &= [-20(-2)^n + \frac{1}{2e+1}(e)^{-n} + \frac{2e}{2e+1}(-2)^n]u[n] \\ &= \frac{1}{2e+1}[-(38e+20)(-2)^n + (e)^{-n}]u[n] \end{aligned}$$

3.8-11. (a)

Rita hjälpgrafer!

$$\begin{aligned} y[n] &= 2^n u[n] * (0.5)^n u[n] \\ &= \frac{2^{n+1} - (0.5)^{n+1}}{2-0.5}u[n] = \frac{2}{3}[2^{n+1} - (0.5)^{n+1}]u[n] \end{aligned}$$

(b)

$$x[n] = 2^{(n-3)}u[n] = 2^{-3}2^n u[n] = \frac{1}{8}2^n u[n]$$

From the result in part (a), it follows that

$$y[n] = \frac{1}{8}\frac{2}{3}[2^{n+1} - (0.5)^{n+1}]u[n] = \frac{1}{12}[2^{n+1} - (0.5)^{n+1}]u[n]$$

(c)

$$x[n] = 2^n u[n-2] = 4\{2^{(n-2)}u[n-2]\}$$

Note that $2^{(n-2)}u[n-2]$ is the same as the input $2^n u[n]$ in part (a) delayed by 2 units. Therefore from the shift property of the convolution, its response will be the same as in part (a) delayed by 2 units. The input here is $4\{2^{(n-2)}u[n-2]\}$. Therefore

$$y[n] = 4\frac{2}{3}[2^{n+1-2} - (0.5)^{n+1-2}]u[n-2] = \frac{8}{3}[2^{n-1} - (0.5)^{n-1}]u[n-2]$$

3.8-12. For $x[n] = u[n]$

$$y[n] = u[n] - 2u[n-1]$$

The highest order difference is one. Hence, this is a first-order system.
This is a nonrecursive system, whose output at any instant depends only on the input, Initial conditions are irrelevant for the finding the response.

3.8-28. (a) MATLAB is used to sketch the function $h[n] = n(u[n-2] - u[n+2])$.

```
>> n = [-5:5]; h = n.*((n>=2)-(n>=-2));
>> stem(n,h,'k'); axis([-5 5 -2.5 2.5]);
>> xlabel('n'); ylabel('h[n]');
```

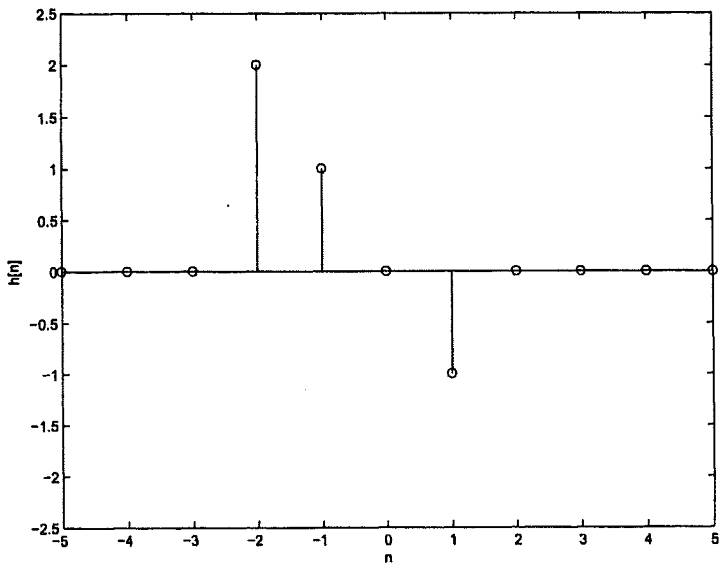


Figure S3.8-28a: Plot of $h[n] = n(u[n-2] - u[n+2])$.

(b) Using Figure S3.8-28a, write $h[n] = 2\delta[n+2] + \delta[n+1] - \delta[n-1]$. A difference equation representation immediately follows from this form,

$$y[n] = 2x[n+2] + x[n+1] - x[n-1].$$

3.10-2 (Vi utgår från att alla systemen är kausala)

a) $\begin{cases} Q[E] = E^2+0{,}6E-0{,}16 = (E-0{,}2)(E+0{,}8) \\ P[E] = E-2 \end{cases}$

$Q[\gamma] = (\gamma-0{,}2)(\gamma+0{,}8)=0 \Rightarrow$ Alla karakteristiska rötter är innanför enhetscirkeln $\Rightarrow$ Systemet är asymptotiskt stabilt

$\Rightarrow$ det är även insignal-utsignal-stabilt (BIBO-stabilt).

b) $y[n+2]+3y[n+1]+2y[n] = x[n+1]+2x[n] \Rightarrow$

$\begin{cases} Q[E] = E^2+3E+2 = (E+2)(E+1) \\ P[E] = E+2 \end{cases}$ (gemensamt nollställe med $Q[E]$)

$Q[\gamma] = (\gamma+2)(\gamma+1)=0 \Rightarrow$ En rot utanför enhetscirkeln (vilket motsvarar en karakteristisk term e^{2t}) $\Rightarrow$ (det kausala) systemet är asymptotiskt instabilt

Eftersom $E=-2$ är gemensamt nollställe hos $Q[E]$ och $P[E]$, så blir systemets externa differensekvationsbeskrivning $(E+1)y[n] = x[n]$, som har karakteristisk ekvation $\gamma+1=0$, vilken har en rot i $\gamma=-1$, dvs. på enhetscirkeln.

$\Rightarrow$ systemet är insignal-utsignal-instabilt.

(om denna rot hade legat innanför enhetscirkeln i stället för på den, så hade systemet varit insignal-utsignal-stabilt)

c) $\begin{cases} Q[E] = (E-1)^2(E+\frac{1}{2}) \\ P[E] = 1 \end{cases}$

$Q[\gamma] = (\gamma-1)^2(\gamma+\frac{1}{2})$ har en dubbelrot på $\gamma=1$

$\Rightarrow$ systemet är asymptotiskt instabilt

$P[E]$ har inget nollställe i $E=1$ (som kancellerar något nollställe hos $Q[E]$ där) $\Rightarrow$ systemet är insignal-utsignal-instabilt

f) $\begin{cases} Q[E] = (E^2-1)(E^2+1) \\ P[E] = 1 \end{cases}$

$Q[\gamma] = (\gamma^2-1)(\gamma^2+1)=0 \Rightarrow$ Rötter i $\gamma=\pm1$ och $\gamma=\pm j$, dvs. enkelrötter på enhetscirkeln $\Rightarrow$ systemet är marginellt stabilt $\Rightarrow$ det är insignal-utsignal-instabilt

(Om $P[E]$ skulle haft 4 nollställen som kancellerar de 4 nollställena på enhetscirkeln hos $Q[E]$, så hade systemet varit insignal-utsignal-stabilt — men så är inte fallet här)

3.10-3. The system $\mathcal{S}_1$ is asymptotically (and BIBO) unstable. The system $\mathcal{S}_2$ is BIBO and asymptotically stable. If we cascade the two systems, the impulse response of the composite system is

$$h[n] = 2^n u[n] * (\delta[n] - 2\delta[n-1]) = 2^n u[n] - 2(2)^{n-1} u[n-1] = \delta[n]$$

The composite system is BIBO stable. However, the system $\mathcal{S}_1$ will burn (or saturate) out because its output contains the signal of the form 2^n.

3.10-4. (a) To be unstable, a causal mode must have magnitude greater than one. That is, at least one characteristic root must be outside the unit circle. By this criteria,

Systems C, D, and H are unstable.

(b) To be real, the characteristic modes need to be either real or in complex-conjugate pairs. By this criteria,

Systems A, B, C, F, H, and I are real.

(c) Oscillatory modes include sinusoids, decaying sinusoids, or exponentially growing sinusoids. Unless the characteristic roots are all real and positive, the corresponding natural mode(s) will exhibit oscillatory behavior. By this criteria,

Systems A, B, C, D, E, F, G, and H have oscillatory natural modes.

(d) To have a mode that decays at a rate of 2^{-n}, at least one characteristic root needs to lie on the circle of radius one-half centered at the origin. By this criteria,

Systems A, C, E, and I have at least one mode that decays by 2^{-n}

(e) For a second-order system with two finite roots to only have one mode, one characteristic root needs to be located at the origin. By this criteria,

Systems E, G, and I have only one mode.

3.10-5. Notice, the system response can be written more simply as $h[n] = \delta[n] + \left(\frac{1}{3}\right)^n u[n-1] = \left(\frac{1}{3}\right)^n u[n]$.

(a) Yes, the system is BIBO-stable since the impulse response function is absolutely summable. That is, $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1-0}{1-1/3} = 3/2 < \infty$.
Yes, the system is causal since $h[n] = 0$ for $n < 0$.

(b) MATLAB is used to plot $x[n]$.

```
>> n = [-5:5]; x = (n>=3)-(n>=-3);
>> stem(n,x,'k'); axis([-5 5 -1.2 1.2]);
>> xlabel('n'); ylabel('x[n]');
```

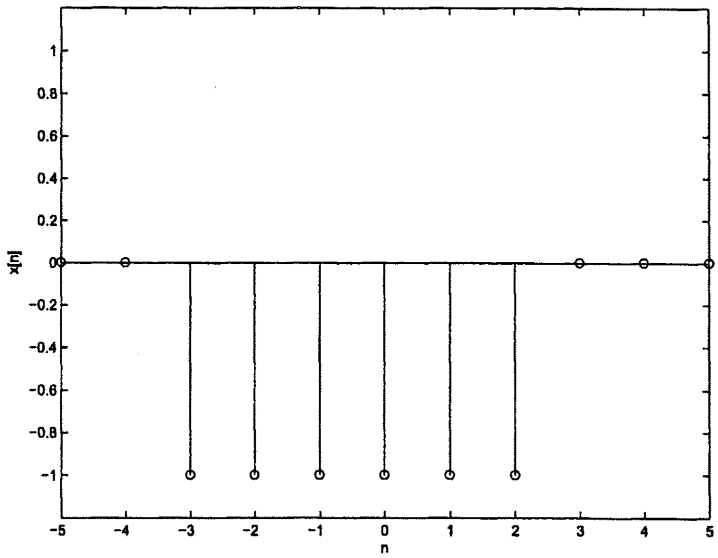


Figure S3.10-5b: Plot of $x[n] = u[n-3] - u[n+3]$.

(c) The zero-state response is computed as $y[n] = x[n] * h[n]$. This convolution involves several regions.
For $(n < -3)$, $y[n] = 0$.
For $(-3 \leq n < 2)$, $y[n] = \sum_{k=-3}^{n} -(1/3)^{n-k} = -(1/3)^n \sum_{k=-3}^{n} 3^k = -(1/3)^n \frac{3^{-3}-3^{n+1}}{1-3} = \frac{3^{-(n+3)}-3}{2}$.
For $(n \geq 2)$, $y[n] = \sum_{k=-3}^{2} -(1/3)^{n-k} = -(1/3)^n \sum_{k=-3}^{2} 3^k = -(1/3)^n \frac{3^{-3}-3^3}{1-3} = -\frac{728}{54}(3)^{-n}$.
Combining yields

$$y[n] = \begin{cases} 0 & n < -3 \\ \frac{3^{-(n+3)}-3}{2} & -3 \leq n < 2 \\ -\frac{728}{54}(3)^{-n} & n \geq 2 \end{cases}.$$

MATLAB is used to plot the result.

```
>> n = [-10:10];
>> y = (3.^(-(n+3))-3)/2.*((n>=-3)&(n<2));
>> y = y+(-3.^(-n)*728/54).*(n>=2);
>> stem(n,y,'k'); axis([-10 10 -2 .5]);
>> xlabel('n'); ylabel('y[n]');
```