5.3-3. (a) The system equation in delay form is

$$2y[n] - 3y[n-1] + y[n-2] = 4x[n] - 3x[n-1]$$

Also

$$y[n] \iff Y[z] \quad y[n-1] \iff \frac{1}{z}Y[z] \quad y[n-2] \iff \frac{1}{z^2}Y[z] + 1$$
$$x[n] \iff X[z] = \frac{z}{z-0.25} \quad x[n-1] \iff \frac{1}{z-0.25}$$

The z-transform of the equation is

$$2Y[z] - \frac{3}{z}Y[z] + \frac{1}{z^2}Y[z] + 1 = \frac{4z}{z-0.25} - \frac{3}{z-0.25} = \frac{4z-3}{z-0.25}$$

or

$$\left(2 - \frac{3}{z} + \frac{1}{z^2}\right)Y[z] = -1 + \frac{4z-3}{z-0.25} = \frac{3z-2.75}{z-0.25}$$

and

$$\begin{aligned}
\frac{Y[z]}{z} &= \frac{z(3z-2.75)}{(2z^2-3z+1)(z-0.25)} \\
&= \frac{z(3z-2.75)}{2(z-0.5)(z-1)(z-0.25)} \\
&= \frac{5/2}{z-1/2} + \frac{1/3}{z-1} - \frac{4/3}{z-0.25} \\
y[n] &= \left[\frac{1}{3} + \frac{5}{2}(0.5)^n - \frac{4}{3}(0.25)^n\right]u[n] \\
&= \left[\frac{1}{3} + \frac{5}{2}(2)^{-n} - \frac{4}{3}(4)^{-n}\right]u[n]
\end{aligned}$$

Kausalt system förutsätts $\Rightarrow$ Termerna har konvergensområde $|z| > 0.5$, $|z| > 1$ resp. $|z| > 0.25$

(b) From part (a), we have

$$\left(2 - \frac{3}{z} + \frac{1}{z^2}\right)Y[z] = \underbrace{-1}_{I.C.\ term} + \underbrace{\frac{4z-3}{z-0.25}}_{input\ term}$$

$$\frac{2z^2-3z+1}{z^2}Y[z] = -1 + \frac{4z-3}{z-0.25}$$

and

$$\begin{aligned}
\frac{Y[z]}{z} &= \underbrace{\frac{-z}{2(z-0.5)(z-1)}}_{z-i} + \underbrace{\frac{z(4z-3)}{2(z-0.5)(z-1)(z-0.25)}}_{z-s} \\
&= \frac{0.5}{z-0.5} - \frac{1}{z-1} + \frac{2}{z-0.5} + \frac{4}{3}\frac{1}{z-1} - \frac{4}{3}\frac{1}{z-0.25}
\end{aligned}$$

and

$$Y[z] = 0.5\frac{z}{z-0.5} - \frac{z}{z-1} + 2\frac{z}{z-0.5} + \frac{4}{3}\frac{z}{z-1} - \frac{4}{3}\frac{z}{z-0.25}$$

and

Kausalt system ⇒ $Y[z]$-termerna har konvergensområde $|z| > 0.5$, $|z| > 1$ resp. $|z| > 0.25$

$$y[n] = \underbrace{\left[\frac{1}{2}(0.5)^n - 1\right]u[n]}_{y_{zi}} + \underbrace{\left[2(0.5)^n + \frac{4}{3} - \frac{4}{3}(0.25)^n\right]u[n]}_{y_{zs}}$$

$$= \underbrace{\left[2.5(0.5)^n - \frac{4}{3}(0.5)^n\right]u[n]}_{y_{tr}} + \underbrace{\frac{1}{3}u[n]}_{y_{ss}}$$

(c) → y_{tr}, y_{ss}

5.3-9. In transform domain, $H(z) = z^{-1}\frac{2z/3}{z-1/3}$ and $Y(z) = z^{-1}\frac{-2z}{z+2}$. Since $Y(z) = H(z)X(z)$, we know $X(z) = Y(z)/H(z) = \frac{z^{-1}\frac{-2z}{z+2}}{z^{-1}\frac{2z/3}{z-1/3}}$. Thus, $X(z) = -3\frac{z-1/3}{z+2}$. Using tables, $x[n] = -3\left((-2)^n u[n] - \frac{1}{3}(-2)^{n-1}u[n-1]\right) = -3\left(-2(-2)^{n-1}u[n] - \frac{1}{3}(-2)^{n-1}u[n-1]\right)$ or

$$x[n] = -3\delta[n] + 7(-2)^{n-1}u[n-1].$$

5.3-11. (a) Note, $h_1[n] = (-1 + (0.5)^n)u[n] = -(1)^n u[n] + (1/2)^n u[n]$. Thus, two real poles are evident (in $H_1[z]$) at $z = 1$ and $z = 1/2$. Since $h[n]$ is not absolutely summable, the system is not BIBO stable. Thought of another way, the pole on the unit-circle makes the system marginally stable, at best. Marginally stable systems are not BIBO stable.

(b) Notice, $h_2[n] = (j)^n(u[n] - u[n-10])$ is a finite duration, causal signal. Thus, $H_2[z]$ has no poles (other than at zero). Since $h_2[n]$ is absolutely summable, the system is BIBO stable.

(b) From the given $H[z]$, we can write

$$(z^2 - 0.6z - 0.16)Y[z] = zX[z]$$

Hence, the corresponding difference equation of the system is

$$y[n+2] - 0.6y[n+1] - 0.16y[n] = x[n+1]$$

or

$$y[n] - 0.6y[n-1] - 0.16y[n-2] = x[n-1]$$

5.3-18. (a)

$$x[n] = ee^n u[n] \qquad X[z] = \frac{ez}{z-e}$$

$$Y[z] = X[z]H[z] = \frac{ez^2}{(z-e)(z+0.2)(z-0.8)}$$

Therefore

$$\begin{aligned}
\frac{Y[z]}{z} &= \frac{ez}{(z-e)(z+0.2)(z-0.8)} = \frac{1.32}{z-e} - \frac{0.186}{z+0.2} - \frac{1.13}{z-0.8} \\
Y[z] &= 1.32\frac{z}{z-e} - 0.186\frac{z}{z+0.2} - 1.13\frac{z}{z-0.8} \\
y[n] &= [1.32(e)^n - 0.186(-0.2)^n - 1.13(0.8)^n]\, u[n]
\end{aligned}$$

(b) From the given $H[z]$, we can write

$$(z^2 - 0.6z - 0.16)Y[z] = zX[z]$$

Hence, the corresponding difference equation of the system is

$$y[n+2] - 0.6y[n+1] - 0.16y[n] = x[n+1]$$

or

$$y[n] - 0.6y[n-1] - 0.16y[n-2] = x[n-1]$$

5.3-20. All cases use the same transfer function. From the given $H[z]$ (after dividing the numerator and the denominator by 6), we can write

$$\left(z^2 - \frac{5}{6}z + \frac{1}{6}\right)Y[z] = (5z-1)X[z]$$

Hence, the corresponding difference equation of the system is

$$y[n+2] - \frac{5}{6}y[n+1] + \frac{1}{6}y[n] = 5x[n+1] - x[n]$$

or

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 5x[n-1] - x[n-2]$$

(a) $x[n] = 4^{-n}u[n] = (\frac{1}{4})^n u[n]$ so that $X[z] = \frac{z}{z-\frac{1}{4}}$, and

$$Y[z] = X[z]\,H[z] = \frac{6z(5z-1)}{(z-\frac{1}{4})(6z^2-5z+1)} = \frac{z(5z-1)}{(z-\frac{1}{4})(z-\frac{1}{3})(z-\frac{1}{2})}$$

Therefore

$$\begin{aligned}
\frac{Y[z]}{z} &= \frac{5z-1}{(z-\frac{1}{4})(z-\frac{1}{3})(z-\frac{1}{2})} = \frac{12}{z-\frac{1}{4}} - \frac{48}{z-\frac{1}{3}} + \frac{36}{z-\frac{1}{2}} \\
Y[z] &= 12\frac{z}{z-\frac{1}{4}} - 48\frac{z}{z-\frac{1}{3}} + 36\frac{z}{z-\frac{1}{2}} \\
y[n] &= \left[12(\frac{1}{4})^n - 48(\frac{1}{3})^n + 36(\frac{1}{2})^n\right] u[n] \\
&= 12\left[4^{-n} - 4(3)^{-n} + 3(2)^{-n}\right] u[n]
\end{aligned}$$

Kausalt system $\Rightarrow$ $Y[z]$-termerna har konvergensområde $|z| > \frac{1}{4}$, $|z| > \frac{1}{3}$, $|z| > \frac{1}{2}$

(b) Here the input is $4^{-(n-2)}u[n-2]$ which is identical to the input in part (a) delayed by 2 units. Therefore the response will be the output in part (a) delayed by 2 units (time-invariance property). Therefore

$$y[n] = 12\left[4^{-(n-2)} - 4(3)^{-(n-2)} + 3(2)^{-(n-2)}\right]u[n-2]$$

(c) Here the input can be expressed as

$$x[n] = 4^{-(n-2)}u[n] = 16(4)^{-n}u[n]$$

This input is 16 times the input in part (a). Therefore the response will be 16 times the output in part (a) (linearity property). Therefore

$$y[n] = 192\left[4^{-n} - 4(3)^{-n} + 3(2)^{-n}\right]u[n]$$

(d) Here the input can be expressed as

$$x[n] = 4^{-n}u[n-2] = \frac{1}{16}(4)^{-(n-2)}u[n-2]$$

This input is $\frac{1}{16}$ times the input in part (b). Therefore the response will be $\frac{1}{16}$ times the output in part (b). Therefore

$$y[n] = \frac{3}{4}\left[4^{-(n-2)} - 4(3)^{-(n-2)} + 3(2)^{-(n-2)}\right]u[n-2]$$

5.3-23. (a)

$$H[z] = \frac{z^2+3z+3}{z^2+3z+2} = \frac{z^2+3z+3}{(z+1)(z+2)}$$

Therefore

$$\frac{H[z]}{z} = \frac{z^2+3z+3}{z(z+1)(z+2)} = \frac{3/2}{z} - \frac{1}{z+1} + \frac{1/2}{z+2}$$

Kausalt system $\Rightarrow$ $H[z]$-termerna har konvergensområde alla z, $|z|>1$ resp. $|z|>2$

$$H[z] = \frac{3}{2} - \frac{z}{z+1} + \frac{1}{2}\frac{z}{z+2}$$

$$h[n] = \left[\frac{3}{2}\delta[n] - (-1)^n + \frac{1}{2}(-2)^n\right]u[n]$$

(b)

$$H[z] = \frac{2z^2 - z}{z^2 + 2z + 1} = \frac{z(2z-1)}{(z+1)^2}$$

Therefore

Kausalt system $\Rightarrow$ $H[z]$ har konvergens-område $|z| > 1$

$$\begin{aligned} \frac{H[z]}{z} &= \frac{2z-1}{(z+1)^2} = \frac{2}{z+1} - \frac{3}{(z+1)^2} \\ H[z] &= 2\left(\frac{z}{z+1}\right) - 3\frac{z}{(z+1)^2} \\ h[n] &= [2(-1)^n + 3n(-1)^n]\, u[n] = (2+3n)(-1)^n u[n] \end{aligned}$$

(c)

$$H[z] = \frac{z^2+2z}{z^2 - z + 0{,}5} = \frac{z^2+2z}{\left(z-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$\Rightarrow$ Komplexkonjugerat polpar i $\gamma = \frac{1}{2} \pm j\frac{1}{2} = \frac{1}{\sqrt{2}} e^{\pm j\frac{\pi}{4}}$

Dvs. använd transformpar 11a & 11b i Tab. 5.1, sid 498:
Här är $|\gamma| = \frac{1}{\sqrt{2}}$ och $\beta = \frac{\pi}{4}$ rad

$$\Rightarrow H[z] = \frac{z(z+2)}{z^2 - 2z + \frac{1}{2}} = \frac{z\left(z - \frac{1}{\sqrt{2}} \cdot \cos\frac{\pi}{4}\right)}{z^2 - \left(2 \cdot \frac{1}{\sqrt{2}} \cos\frac{\pi}{4}\right) \cdot z + \left(\frac{1}{\sqrt{2}}\right)^2} + $$

$$+ 5 \cdot \frac{z \cdot \frac{1}{\sqrt{2}} \cdot \sin\frac{\pi}{4}}{z^2 - \left(2 \cdot \frac{1}{\sqrt{2}} \cos\frac{\pi}{4}\right) z + \left(\frac{1}{\sqrt{2}}\right)^2}$$

(Kausalt system $\Rightarrow$ $H[z]$ har konvergens-område $|z| > |\gamma| = \frac{1}{\sqrt{2}}$)

$$\Rightarrow h[n] = \left(\frac{1}{\sqrt{2}}\right)^n \cos\left(\frac{\pi}{4} n\right) \cdot u[n] + 5\left(\frac{1}{\sqrt{2}}\right)^n \sin\left(\frac{\pi}{4} n\right) \cdot u[n]$$

5.3-25. (a) Noting that $H(z) = z^{-3}\frac{z}{z-1}$, $H^{-1}(z) = \frac{1}{H(z)} = \frac{z-1}{z^{-2}} = z^3 - z^2$. Thus,

$$h^{-1}[n] = \delta[n+3] - \delta[n+2].$$

(b) Since $h^{-1}[n]$ is absolutely summable, the system inverse is stable. However, $h^{-1}[n] \neq 0$ for $n < 0$ so the system is not causal.

(c) For systems that have time as the independent variable, it is only possible to realize causal systems. Shifting $h^{-1}[n]$ by three makes it causal and therefore realizable. That is, implement $h^{-1}_{\text{causal}}[n] = h^{-1}[n-3] = \delta[n] - \delta[n-1]$, as shown in S5.3-25c. Within a delay factor, this implementation functions as the system inverse.

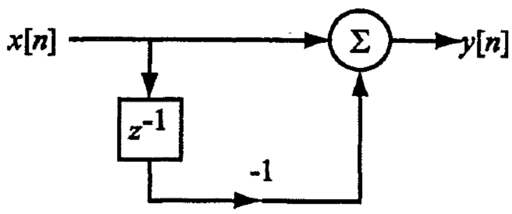


Figure S5.3-25c: Block realization of $h^{-1}_{\text{causal}}[n]$.

5.5-1. (a)

$$H[z] = \frac{1}{z-0.4} \quad \text{and} \quad H\left[e^{j\Omega}\right] = \frac{1}{e^{j\Omega} - 0.4} = \frac{1}{\cos\Omega - 0.4 + j\sin\Omega}$$

$$\left|H\left[e^{j\Omega}\right]\right|^2 = HH^* = \frac{1}{(e^{j\Omega} - 0.4)(e^{-j\Omega} - 0.4)} = \frac{1}{1.16 - 0.8\cos\Omega}$$

(Kausalt system enl. fig $\Rightarrow H[z]$ har konvergensområde $|z| > 0.4$, $\Rightarrow H[\Omega] = H[z]\big|_{z=e^{j\Omega}} = H\left[e^{j\Omega}\right]$)

$$\left|H\left[e^{j\Omega}\right]\right| = \frac{1}{\sqrt{1.16 - 0.8\cos\Omega}}$$

$$\angle H\left[e^{j\Omega}\right] = -\tan^{-1}\frac{\sin\Omega}{\cos\Omega - 0.4}$$

(b)

Konvergensområde $|z| > 0.4$ även här,
$\Rightarrow$ enhetscirkeln i konv.området
$\Rightarrow H[\Omega] = H[z]\big|_{z=e^{j\Omega}} = H\left[e^{j\Omega}\right]$

$$H[z] = \frac{z}{z-0.4} = \frac{1}{1-0.4z^{-1}}$$

$$H\left[e^{j\Omega}\right] = \frac{1}{1-0.4e^{-j\Omega}} = \frac{1}{1 - 0.4\cos\Omega - j\sin\Omega}$$

0,4

Therefore

$$\left|H\left[e^{j\Omega}\right]\right| = \sqrt{HH^*} = \sqrt{\frac{1}{1-0.4e^{-j\Omega}}\frac{1}{1-0.4e^{j\Omega}}} = \frac{1}{\sqrt{1.16 - 0.8\cos\Omega}}$$

and

$$\angle H\left[e^{j\Omega}\right] = -\tan^{-1}\left(\frac{0.4\sin\Omega}{1-0.4\cos\Omega}\right)$$

(c)

$$H[z] = \frac{3z^2+1.8z}{z^2-z+0.16}$$

and $$H\left[e^{j\Omega}\right] = \frac{3e^{2j\Omega}+1.8e^{j\Omega}}{e^{2j\Omega}-e^{j\Omega}+0.16} = \frac{(3\cos 2\Omega+1.8\cos\Omega)+j(3\sin 2\Omega+1.8\sin\Omega)}{(\cos 2\Omega-\cos\Omega+0.16)+j(\sin 2\Omega-\sin\Omega)}$$

$$\begin{aligned}\left|H\left[e^{j\Omega}\right]\right|^2 &= \left[\frac{3e^{2j\Omega}+1.8e^{j\Omega}}{e^{2j\Omega}-e^{j\Omega}+0.16}\right]\left[\frac{3e^{-2j\Omega}+1.8e^{-j\Omega}}{e^{-2j\Omega}-e^{-j\Omega}+0.16}\right] \\ &= \frac{12.24+10.8\cos\Omega}{2.0256-2.32\cos\Omega+0.32\cos 2\Omega}\end{aligned}$$

Therefore $$\left|H\left[e^{j\Omega}\right]\right| = \left[\frac{12.24+10.8\cos\Omega}{2.0256-2.32\cos\Omega+0.32\cos 2\Omega}\right]^{1/2}$$

and

$$\angle H\left[e^{j\Omega}\right] = \tan^{-1}\left(\frac{3\sin 2\Omega+1.8\sin\Omega}{3\cos 2\Omega+1.8\cos\Omega}\right) - \tan^{-1}\left(\frac{\sin 2\Omega-\sin\Omega}{\cos 2\Omega-\cos\Omega+0.16}\right)$$

Tips, 5.5-1(c): Ansätt hjälpstorheter efter de två summatorerna till vänster!
Sätt sedan upp uttrycken för hjälpstorheternas z-transformer och för *Y*[*z*].

5.5-3. The advance operator form of this equation is

$$E^4y[n] = \frac{1}{5}\left[E^4+E^3+E^2+E+1\right]$$

and

$$H[z] = \frac{1}{5}\left[\frac{z^4+z^3+z^2+z+1}{z^4}\right]$$

$$\begin{aligned}H\left[e^{j\Omega}\right] &= \frac{1}{5}\left[\frac{e^{j4\Omega}+e^{j3\Omega}+e^{j2\Omega}+e^{j\Omega}+1}{e^{j4\Omega}}\right] \\ &= \frac{1}{5}e^{-j2\Omega}\left[e^{j2\Omega}+e^{j\Omega}+1+e^{-j\Omega}+e^{-j2\Omega}\right] \\ &= \frac{1}{5}e^{-j2\Omega}\left[1+2\cos\Omega+2\cos 2\Omega\right]\end{aligned}$$

(Kausalt system $\Rightarrow H[z]$ har konvergensområde $|z|>0$
$\Rightarrow$ enhetscirkeln i konv.området
$\Rightarrow H[\Omega] = H[z]\big|_{z=e^{j\Omega}} = H\left[e^{j\Omega}\right]$)

5.5-4. (a) The z-transform of the two equations yield:

$$\text{(i).}\ \left(1+\frac{0.9}{z}\right)Y[z]=X[z]$$

$$\text{(ii).}\ \left(1-\frac{0.9}{z}\right)Y[z]=X[z]$$

Hence the transfer functions of these filters are

$$\text{(i).}H[z]=\frac{z}{z+0.9}\qquad \text{(ii).}H[z]=\frac{z}{z-0.9}$$

Consider the first system.

$\left(\begin{array}{l}\text{Båda systemfkn:erna har konvergensområde } |z|>0.9\Rightarrow\\ |z|=1 \text{ i konv.området } \Rightarrow H[\Omega]=H[z]\big|_{z=e^{j\Omega}}=H\left[e^{j\Omega}\right]\end{array}\right)$

(i)

$$H\left[e^{j\Omega}\right]=\frac{e^{j\Omega}}{e^{j\Omega}+0.9}=\frac{1}{1+0.9e^{-j\Omega}}=\frac{1}{1+0.9\cos\Omega-j0.9\sin\Omega}$$

$$\left|H\left[e^{j\Omega}\right]\right|=\frac{1}{\sqrt{1.81+1.8\cos\Omega}},\ \angle H\left[e^{j\Omega}\right]=-\tan^{-1}\left[\frac{-0.9\sin\Omega}{1+0.9\cos\Omega}\right]$$

(ii)

$$H\left[e^{j\Omega}\right]=\frac{e^{j\Omega}}{e^{j\Omega}-0.9}=\frac{1}{1-0.9e^{-j\Omega}}=\frac{1}{1-0.9\cos\Omega+j0.9\sin\Omega}$$

$$\left|H\left[e^{j\Omega}\right]\right|=\frac{1}{\sqrt{1.81-1.8\cos\Omega}},\ \angle H\left[e^{j\Omega}\right]=-\tan^{-1}\left[\frac{0.9\sin\Omega}{1-0.9\cos\Omega}\right]$$

Filter(i) has a zero at the origin and a pole at -0.9. Because the only pole is near $\Omega=\pi(z=-1)$, this is a highpass filter, as verified from the frequency response shown in Figure S5.5-4a.

Filter(ii) has a zero at the origin and a pole at 0.9. Because the only pole is near $\Omega=0(z=1)$, this is a lowpass filter, as verified from the frequency response shown in Figure S5.5-4b.

(b) (i) For $\Omega=0.01\pi$

$$\left|H\left[e^{j0.01\pi}\right]\right|=\frac{1}{\sqrt{1.81+1.8\cos 0.01\pi}}=0.5966$$

For $\Omega = 0.99\pi$

$$\left|H\left[e^{j0.99\pi}\right]\right| = \frac{1}{\sqrt{1.81 + 1.8\cos 0.99\pi}} = 9.58$$

(ii) For $\Omega = 0.01\pi$

$$\left|H\left[e^{j0.01\pi}\right]\right| = \frac{1}{\sqrt{1.81 - 1.8\cos 0.01\pi}} = 9.58$$

For $\Omega = 0.99\pi$

$$\left|H\left[e^{j0.99\pi}\right]\right| = \frac{1}{\sqrt{1.81 - 1.8\cos 0.99\pi}} = 0.5966$$

Filter(i) gain at Ω_0 is

$$|H| = \frac{1}{\sqrt{1.81 + 1.8\cos\Omega_0}}$$

Filter(ii) gain at $\pi - \Omega_0$ is

$$|H| = \frac{1}{\sqrt{1.81 - 1.8\cos(\pi - \Omega_0)}} = \frac{1}{\sqrt{1.81 + 1.8\cos(\Omega_0)}}$$

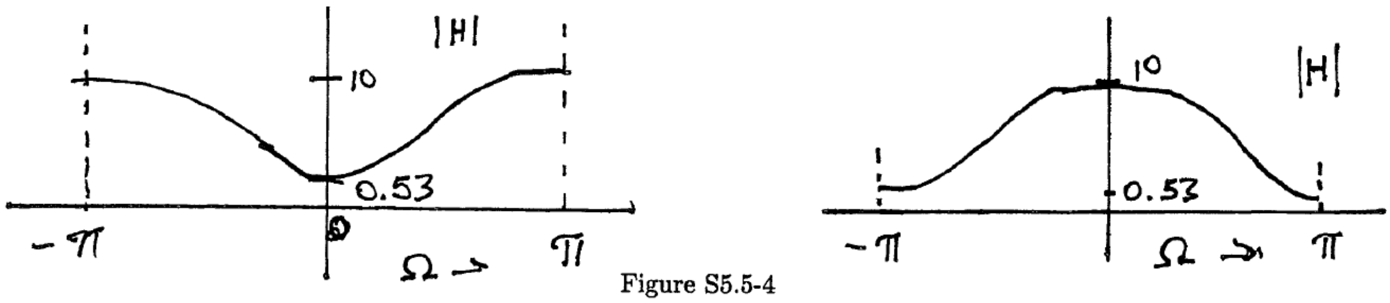


Figure S5.5-4

5.5-5.

$$H[z] = \frac{z+0.8}{z-0.5}$$

(Kausalt system $\Rightarrow H[z]$ har konvergensområde $|z| > 0.5$ $\Rightarrow$ enhetscirkeln i konv.området $\Rightarrow H[\Omega] = H[z]\big|_{z=e^{j\Omega}} = H\left[e^{j\Omega}\right]$)

(a)

$$H\left[e^{j\Omega}\right] = \frac{e^{j\Omega}+0.8}{e^{j\Omega}-0.5} = \frac{(\cos\Omega+0.8)+j\sin\Omega}{(\cos\Omega-0.5)+j\sin\Omega}$$

$$\left|H\left[e^{j\Omega}\right]\right|^2 = H\left[e^{j\Omega}\right]H\left[e^{-j\Omega}\right] = \frac{(e^{j\Omega}+0.8)(e^{-j\Omega}+0.8)}{(e^{j\Omega}-0.5)(e^{-j\Omega}-0.5)} = \frac{1.64+1.6\cos\Omega}{1.25-\cos\Omega}$$

$$\angle H\left[e^{j\Omega}\right] = \tan^{-1}\left(\frac{\sin\Omega}{\cos\Omega+0.8}\right) - \tan^{-1}\left(\frac{\sin\Omega}{\cos\Omega-0.5}\right)$$

(b) $\Omega = 0.5$

$$|H\left[e^{j0.5}\right]|^2 = \frac{1.64+1.6\cos(0.5)}{1.25-\cos(0.5)} = 8.174$$

$$|H\left[e^{j0.5}\right]| = 2.86$$

$$\angle H\left[e^{j0.5}\right] = 0.2784 - 0.9037 = -0.6253\,\text{rad}$$

Therefore

$$y[n] = 2.86\cos(0.5n - \frac{\pi}{3} - 0.6253) = 2.86\cos(0.5n - 1.6725)$$

5.6-1. Figure S5.6-1 shows a rough sketch of the amplitude and phase response of this filter. For the case (a), the poles are in the vicinity of $\Omega = \frac{\pi}{4}$. Therefore, the gain $|H[e^{j\Omega}]|$ is high in the vicinity of $\Omega = \pi/4$. In the case (b), the poles are in the vicinity of $\Omega = \pi$. therefore, the gain $|H[e^{j\Omega}]|$ is high in the vicinity of $\Omega = \pi$. For case (a), the phases of the two poles are equal and opposite at $\Omega = 0$. Hence $\angle H[e^{j\Omega}]$ starts at 0

(for $\Omega = 0$). As Ω increases, the angle due to both poles increase. Hence, $\angle H[e^{j\Omega}]$ increases in negative direction until it reaches the value -2π at $\Omega = \pi$. For case (b), similar behavior is observed. Note that angle -2π is the same as 0.

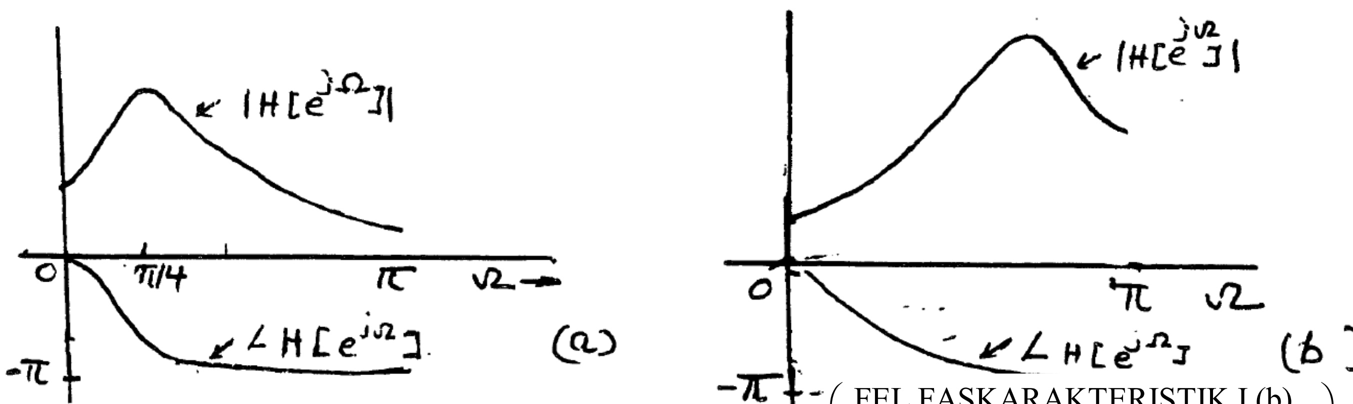


Figure S5.6-1

FEL FASKARAKTERISTIK I (b)
Den blir först positiv för att sedan gå tillbaka mot/till noll då $\Omega \to \pi$.
Notera att det är <u>2 nollställen</u> i origo i figur (b) i boken!

5.6-2. The two systems are very similar and have identical steady-state characteristics. There is an important difference, however, between the two systems. The system $y[n] - y[n-1] = x[n] - x[n-1]$ is first-order and can support an initial condition; the system $y[n] = x[n]$ is zero-order and cannot support an initial condition. If the initial condition of the first system is non-zero, the output of the two systems can be quite different.

5.6-3. (a) From the magnitude response plot, it is clear that this is a ~~low~~ high pass filter. Low frequencies near $\Omega = 0$ are attenuated, and high frequencies near $\Omega = \pm\pi$ are passed with unity gain.

(b) From the magnitude and phase response plots, $H(e^{j\pi/2}) = \frac{1}{\sqrt{2}} e^{j\frac{3\pi}{4}}$. Thus, the output to $x_1[n] = 2\sin(\frac{\pi}{2}n + \frac{\pi}{4})$ is

$$y_1[n] = \sqrt{2}\sin(\frac{\pi}{2}n + \pi) = -\sqrt{2}\sin(\frac{\pi}{2}n).$$

(c) Notice, $H(e^{j7\pi/4}) = H(e^{-j\pi/4})$. From the magnitude and phase response plots, $H(e^{-j\pi/4}) \approx 0.071e^{j2.43}$. Thus, the output to $x_2[n] = \cos(\frac{7\pi}{4}n)$ is

$$y_2[n] = 0.071\cos(\frac{7\pi}{4}n + 2.43).$$

5.6-4. Refer to Figure S5.6-4 and the solution to 5.M-1.

5.M-1. Taking the z-transform of $4y[n+2] - y[n] = x[n+2] + x[n]$ yields $Y(z)\left(4z^2 - 1\right) = X(z)\left(z^2 + 1\right)$. Thus, the system function is $H(z) = \frac{Y(z)}{X(z)} = \frac{z^2+1}{4z^2-1} = 0.25\frac{1+z^{-2}}{1-z^{-2}/4}$.

(a) MATLAB is used to create the pole-zero diagram.

```
>> zz = roots([1 0 1]); zp = roots([4 0 -1]);
>> theta = linspace(0,2*pi,201);
>> plot(real(zz),imag(zz),'ko',real(zp),imag(zp),'kx',...
     cos(theta),sin(theta),'k');
>> xlabel('Re(z)'); ylabel('Im(z)'); grid;
>> axis([-1.1 1.1 -1.1 1.1]); axis equal;
```

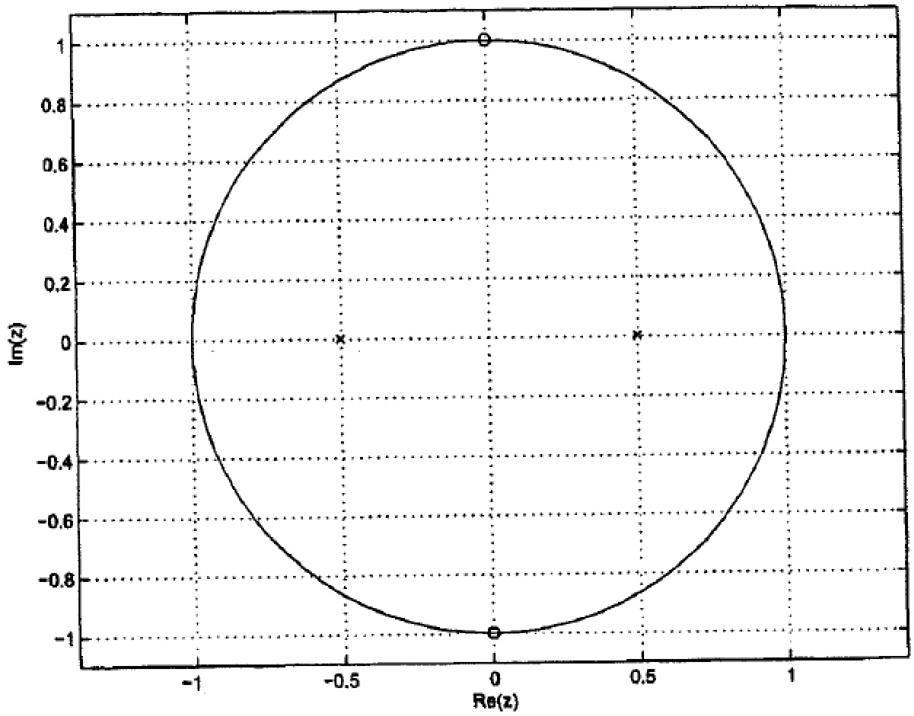


Figure S5.M-1a: Pole-zero diagram for $H(z) = \frac{z^2+1}{4z^2-1}$.

(b) $H(e^{j\Omega}) = \left.\frac{z^2+1}{4z^2-1}\right|_{z=e^{j\Omega}}$. MATLAB is used to plot the magnitude-response $|H(e^{j\Omega})|$.

```
>> Omega = linspace(-pi,pi,501);
>> z = exp(j*Omega); H = (z.^2+1)./(4*z.^2-1);
>> plot(Omega,abs(H),'k'); axis([-pi pi 0 1]); grid;
>> xlabel('\Omega'); ylabel('|H(e^{j\Omega})|');
>> set(gca,'xtick',[-pi:pi/4:pi],'xticklabel',['-p ';...
    '   ';'   ';'   ';' 0 ';'   ';'   ';'   ';' p '],...
    'fontname','symbol');
```

(c) The pole-zero plot of Figure S5.M-1a and the magnitude response plot of Figure S5.M-1b confirm that this is a band-stop system.

(d) Yes, the system is asymptotically stable. Referring to Figure S5.M-1a, all the system poles are within the unit circle. (& inga poler har eliminerats av nollställen)

(e) Yes, the system is real. Since the system is expressed as a constant-coefficient linear difference equation with real coefficients, the impulse response $h[n]$ and system are both real.

(f) For an input of the form $x[n] = \cos(\Omega n)$, the greatest possible amplitude of the output corresponds to the greatest gain shown in the magnitude response plot of Figure S5.M-1b. Thus, 2/3 is the greatest output amplitude given an input of $x[n] = \cos(\Omega n)$. This output amplitude occurs when $\Omega = k\pi$, for any integer k.

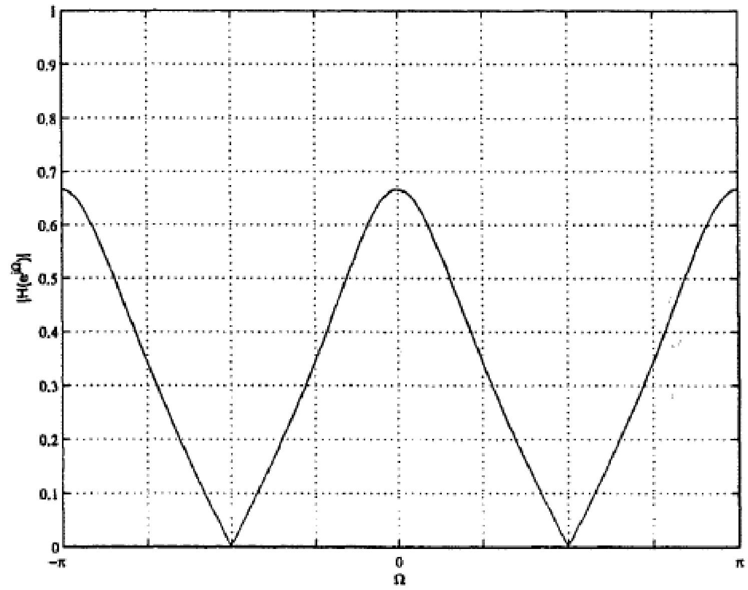
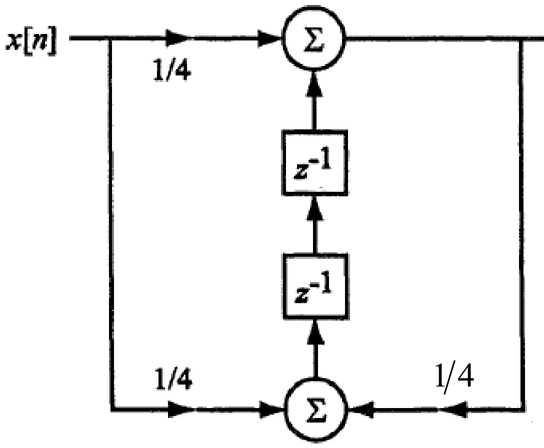


Figure S5.M-1b: Magnitude response plot for $4y[n+2] - y[n] = x[n+2] + x[n]$.



Anm: Lägre komplexitet erhålls om man endast har <u>en</u> multiplicering med $1/4$, vid utsignalen $y[n]$:

$$y[n] = \frac{1}{4}\left(x[n] + x[n-2] + y[n-2]\right)$$

Figure S5.M-1g: TDFII implementation of $y[n] - 0.25y[n-2] = 0.25x[n] + 0.25x[n-2]$.

(g) Inverting $H(z) = \frac{Y(z)}{X(z)} = 0.25\frac{1+z^{-2}}{1-z^{-2}/4}$ provides $y[n] - 0.25y[n-2] = 0.25x[n] + 0.25x[n-2]$, which is a convenient form for implementation. Figure S5.M-1g illustrates a TDFII implementation of the system.

5.9-4. (a) The three poles satisfy $z^3 = \frac{27}{8}$, or $z = 3/2e^{j2\pi k/3}$ for $k = (0, 1, 2)$. There are two finite zeros at $z = 0$ and $z = 1/2$ as well as a zero at infinity. MATLAB is used to create the corresponding pole-zero plot.

```
>> k = [0:2]; zp = 3/2*exp(j*2*pi*k/3); zz = [0,1/2];
>> plot(real(zz),imag(zz),'ko',real(zp),imag(zp),'kx');
>> xlabel('Re(z)'); ylabel('Im(z)');
>> axis([-1.5 1.5 -1.5 1.5]); axis equal; grid;
```

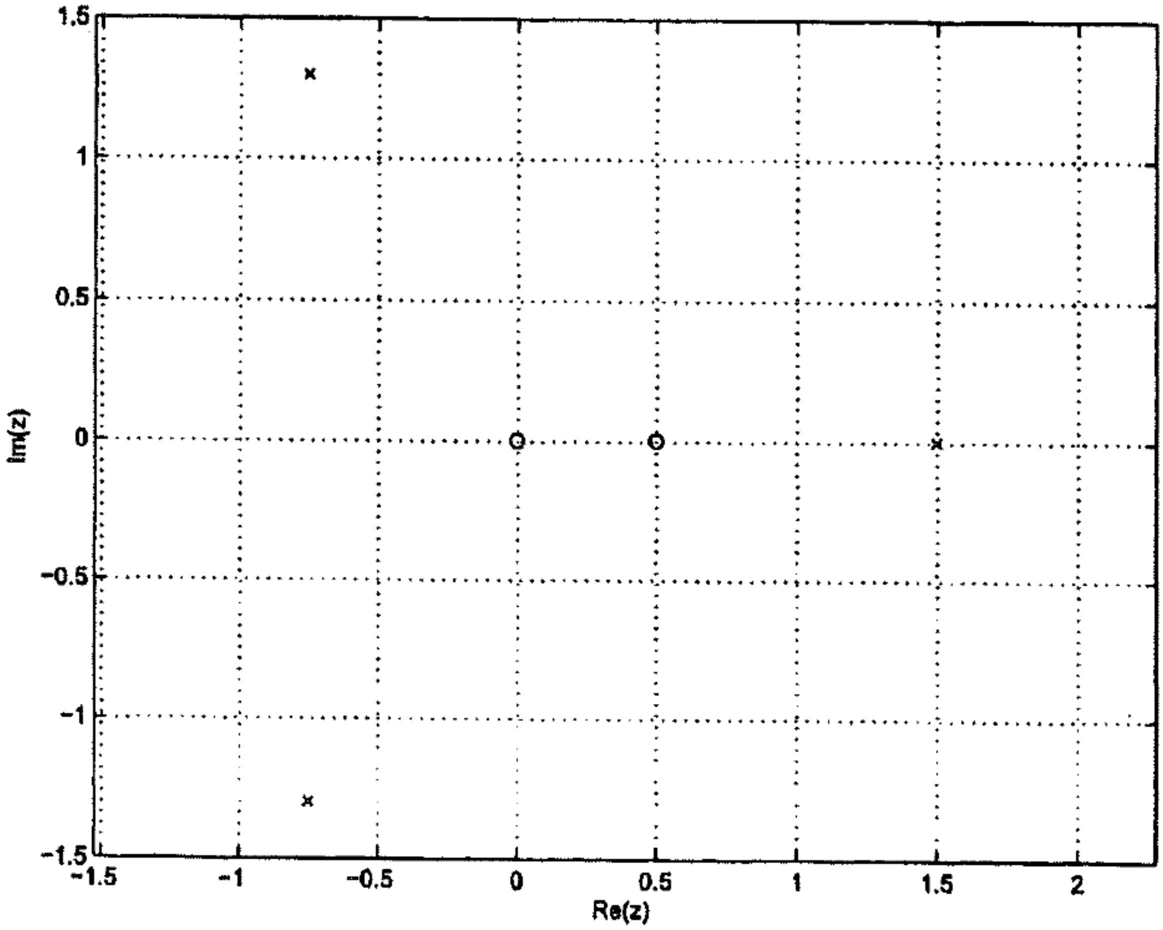


Figure S5.9-4a: Pole-zero plot for $H(z) = \frac{z(z-\frac{1}{2})}{(z^3-\frac{27}{8})}$.

There are two possible regions of convergence, both of which exclude the three system poles: $|z| < 3/2$ or $|z| > 3/2$.

(b) The poles and zeros of $H^{-1}(z)$ are just the zeros and poles, respectively, of $H(z)$. Thus, the three zeros of $H(z)$ satisfy $z^3 = \frac{27}{8}$, or $z = 3/2e^{j2\pi k/3}$ for $k = (0, 1, 2)$. There are two finite poles at $z = 0$ and $z = 1/2$ as well as a pole at infinity. MATLAB is used to create the corresponding pole-zero plot.

```
>> k = [0:2]; zz = 3/2*exp(j*2*pi*k/3); zp = [0,1/2];
>> plot(real(zz),imag(zz),'ko',real(zp),imag(zp),'kx');
>> xlabel('Re(z)'); ylabel('Im(z)');
>> axis([-1.5 1.5 -1.5 1.5]); axis equal; grid;
```

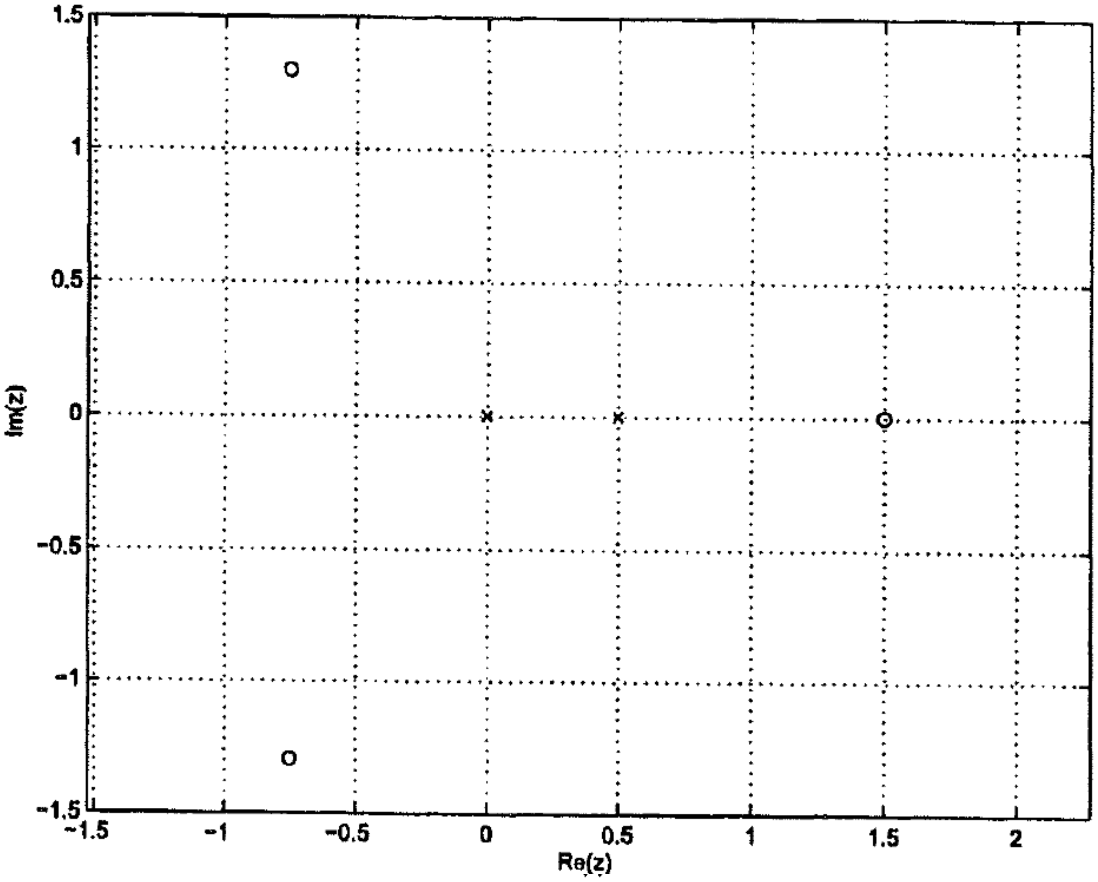


Figure S5.9-4b: Pole-zero plot for $H^{-1}(z) = \frac{(z^3 - \frac{27}{8})}{z(z-\frac{1}{2})}$

There are two possible regions of convergence, both of which exclude the three system poles: $0 < |z| < 1/2$ or $1/2 < |z| < \infty$.

5.9-13.
(b)

$$X[z] = \frac{-z}{z-2} \qquad |z| < 2$$
$$H[z] = \frac{z}{(z+0.2)(z-0.8)} \qquad |z| > 0.8$$

$$Y[z] = \frac{-z^2}{(z+0.2)(z-0.8)(z-2)} \qquad 0.8 < |z| < 2$$

and

$$\frac{Y[z]}{z} = \frac{-z}{(z+0.2)(z-0.8)(z-2)} = \frac{1/11}{z+0.2} + \frac{2/3}{z-0.8} - \frac{0.758}{z-2}$$

Therefore $Y[z] = \frac{1}{11}\frac{z}{z+0.2} + \frac{2}{3}\frac{z}{z-0.8} - 0.758\frac{z}{z-2} \quad 0.8 < |z| < 2$

and $y[n] = \left[\frac{1}{11}(-0.2)^n + \frac{2}{3}(0.8)^n\right] u[n] + 0.758(2)^n u[-(n+1)]$

5.9-14.

$$x[n] = \underbrace{2^n u[n]}_{x_1[n]} + \underbrace{u[-(n+1)]}_{x_2[n]}$$

$$\begin{aligned} X_1[z] &= \frac{z}{z-2} \qquad |z| > 2 \\ X_2[z] &= \frac{-z}{z-1} \qquad |z| < 1 \end{aligned}$$

There is no region of convergence common to $X_1[z]$ and $X_2[z]$

$$H[z] = \frac{z}{(z+0.2)(z-0.8)}$$

The region of convergence of $H[z]$ is $|z| > 0.8$ (assuming a causal system). We should find the response to $x_1[n]$ and $x_2[n]$ separately.

$$Y_1[z] = \frac{z^2}{(z-2)(z+0.2)(z-0.8)} \qquad |z| > 2$$

The modified partial fractions of $Y[z]$ yield

$$Y_1[z] = -\frac{1}{11}\frac{z}{z+0.2} - \frac{2}{3}\frac{z}{z-0.8} + 0.758\frac{z}{z-2}$$

and

$$y_1[n] = \left[-\frac{1}{11}(-0.2)^n - \frac{2}{3}(0.8)^n + 0.758(2)^n\right] u[n]$$

Similarly

$$Y_2[z] = \frac{-25}{6}\frac{z}{z-1} + \frac{1}{6}\frac{z}{z+0.2} + 4\frac{z}{z-0.8} \qquad 0.8 < |z| < 1$$

and

$$y_2[n] = \left[\frac{1}{6}(-0.2)^n + 4(0.8)^n\right] u[n] + \frac{25}{6}u[-(n+1)]$$

and

$$y[n] = y_1[n] + y_2[n] = \left[\frac{5}{66}(-0.2)^n + \frac{10}{3}(0.8)^n + 0.758(2)^n\right] u[n] + \frac{25}{6}u[-(n+1)]$$

5.9-15.

$$X[z] = \frac{-z}{z-e^2} \qquad |z| < e^2$$

e^{-2}

and

$$H[z] = \frac{z}{(z+0.2)(z-0.8)} \qquad |z| > 0.8$$

No common region of convergence for $X[z]$ and $H[z]$ exists. Hence

$$y[n] = \infty$$