Fouriertransformen till tidsdiskret signal

- Om ***jω*-axeln** ligger i konvergensområdet för $X(s)$

$$\Rightarrow \quad X(\omega) = X(s)\Big|_{s=j\omega} = X(j\omega) \qquad \left(\mathcal{F}\{x(t)\} = \mathcal{L}\{x(t)\}\Big|_{s=j\omega}\right)$$

- Om **enhetscirkeln** ($|z|=1$) ligger i konvergensområdet för $X[z]$

$$\Rightarrow \quad X[\Omega] = X[z]\Big|_{z=e^{j\Omega}} = X\left[e^{j\Omega}\right] \qquad \left(\mathcal{F}\{x[n]\} = \mathcal{Z}\{x[n]\}\Big|_{z=e^{j\Omega}}\right)$$

Fouriertransformen till $x[n]$ (*Eng*: *"DTFT, Discrete-Time Fourier Transform"*):

$$X[\Omega] = \sum_{n=-\infty}^{\infty} x[n]z^{-n}\Bigg|_{z=e^{j\Omega}} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

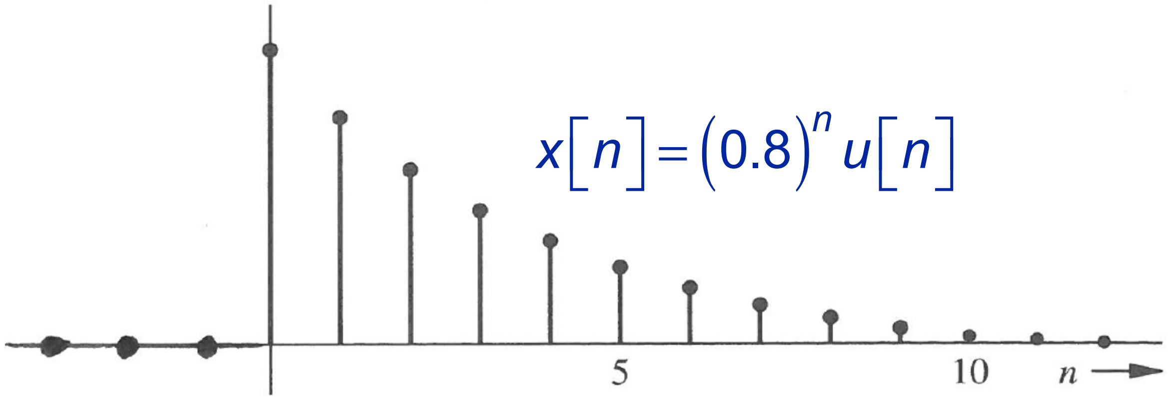
Existensvillkor:

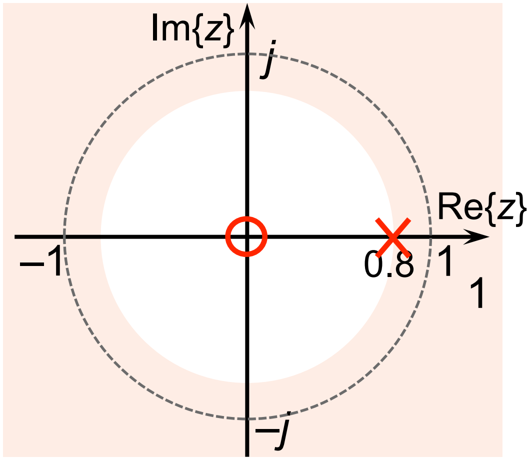
$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

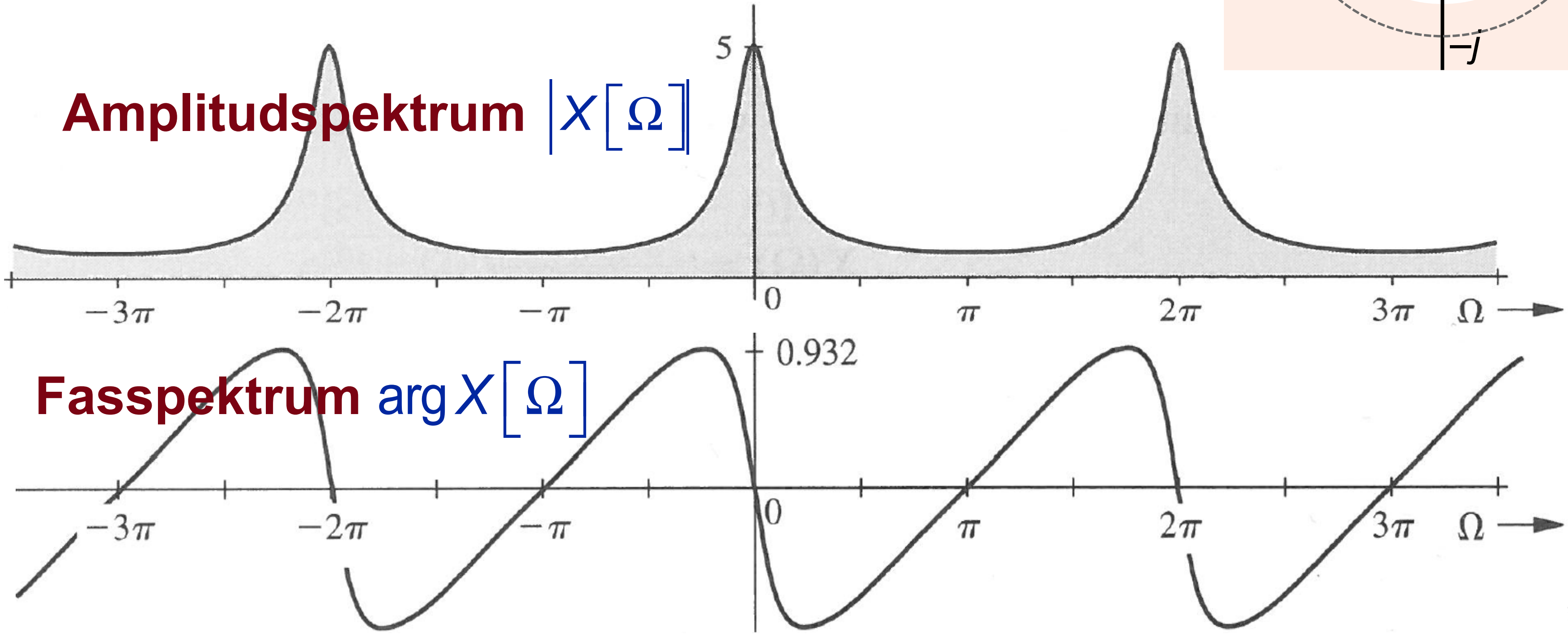
Boken: $X(\Omega) = \text{DTFT}\{x[n]\}$

$$\mathcal{F}\left\{(0.8)^n u[n]\right\} = \frac{e^{j\Omega}}{e^{j\Omega} - 0.8}$$

$$X[z] = \frac{z}{z-0.8}, \quad |z| > 0.8$$







Några centrala fouriertransformpar

$\mathcal{F}\{x[n]\}$ är 2π-periodisk!

$$\delta[n] \Leftrightarrow 1$$

$$\delta[n-k] \Leftrightarrow e^{-jk\Omega}$$

$$1 \Leftrightarrow 2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - k \cdot 2\pi)$$

$$\gamma^n u[n] \Leftrightarrow \frac{e^{j\Omega}}{e^{j\Omega}-\gamma}; \quad |\gamma| < 1$$

$$-\gamma^n u[-n-1] \Leftrightarrow \frac{e^{j\Omega}}{e^{j\Omega}-\gamma}; \quad |\gamma| > 1$$

$$u[n] \Leftrightarrow \mathrm{vp}\left\{\frac{e^{j\Omega}}{e^{j\Omega}-1}\right\} + \pi \sum_{k=-\infty}^{\infty} \delta(\Omega - k \cdot 2\pi)$$

$$\cos(\Omega_0 n) \Leftrightarrow \pi \sum_{k=-\infty}^{\infty} \left(\delta(\Omega + \Omega_0 - k \cdot 2\pi) + \delta(\Omega - \Omega_0 - k \cdot 2\pi)\right)$$

$$\cos(\Omega_0 n) u[n] \Leftrightarrow \mathrm{vp}\left\{\frac{e^{j2\Omega} - e^{j\Omega}\cos\Omega_0}{e^{j2\Omega} - 2e^{j\Omega}\cos\Omega_0 + 1}\right\} + \frac{\pi}{2} \sum_{k=-\infty}^{\infty} \left(\delta(\Omega + \Omega_0 - k \cdot 2\pi) + \delta(\Omega - \Omega_0 - k \cdot 2\pi)\right)$$

Invers fouriertransform

(*IDTFT, Inverse Discrete-Time Fourier Transform*)

Inversa fouriertransformen till $X[\Omega]$ ($X(\Omega)$):

$$\mathcal{F}^{-1}\{X[\Omega]\} = x[n] = \frac{1}{2\pi}\int_{2\pi} X[\Omega]e^{j\Omega n}\,d\Omega$$

Boken:

$$x[n] = \text{IDTFT}\{X(\Omega)\}$$

Erhålls från: $x[n] = \mathcal{Z}^{-1}\{X[z]\} = \frac{1}{2\pi j}\oint_C X[z]z^{n-1}dz \quad = \frac{1}{2\pi}\oint_C X[z]z^n\frac{1}{jz}dz$

$$= \Bigg/ \; z = e^{j\Omega} \Rightarrow \frac{dz}{d\Omega} = je^{j\Omega} \Rightarrow \frac{1}{jz}dz = d\Omega$$

$$\Rightarrow X\left[e^{j\Omega}\right] = X[\Omega] \;\; \&$$

$C =$ enhetscirkeln; $\Omega : \alpha \to \alpha + 2\pi$ $\Bigg/$

$$= \frac{1}{2\pi}\int_{2\pi} X[\Omega]e^{j\Omega n}\,d\Omega$$

Centrala fouriertransformegenskaper

Tidsskiftning: $x[n-n_0] \Leftrightarrow X[\Omega]e^{-j\Omega n_0}$

Frekvensskiftning: $x[n]e^{j\Omega_0 n} \Leftrightarrow X[\Omega-\Omega_0]$

spec.fall: $x[n](-1)^n \Leftrightarrow X[\Omega-\pi]$

Spegling: $x[-n] \Leftrightarrow X[-\Omega]$

Mult. med n: $n \cdot x[n] \Leftrightarrow j\frac{dX[\Omega]}{d\Omega}$

Symmetri, $x[n]$ reell: $X[-\Omega] = X^*[\Omega]$