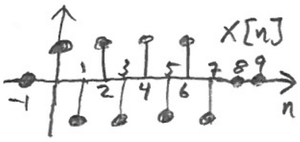
5.1-1 $\quad X[z] = \sum_{n=0}^{7} (-1)^n z^{-n} = \sum_{n=0}^{7} (-z^{-1})^n = \frac{1-(-z^{-1})^8}{1-(-z^{-1})}$



$$= \frac{z^8-1}{z^8+z^7} = \frac{z^8-1}{z^7(z+1)}$$

$x[n]=0$ för $n<0$ $\Rightarrow$ Konvergensomr. för $X[z]$ är $|z|>R_0$

Nämnarpolynomet hos $X[z]$ är noll, dvs. $X[z]=\infty$, för $z=0$ och $z=-1$ $\Rightarrow$ Det verkar som om konvergensområdet är $|z|>1$. Dock är $z=-1$ även ett nollställe hos täljarpolynomet (inses lätt):

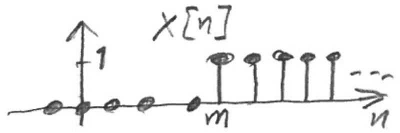
$z^8-1=(z+1)(z^7-z^6+z^5-z^4+z^3-z^2+z-1)$

(kan visas med polynomdivision $\frac{z^8-1}{z+1}$) $\Longrightarrow$

$$X[z] = \frac{z^7-z^6+z^5-z^4+z^3-z^2+z-1}{z^7}\ ; \quad |z|>0 \quad (R_0=0)$$

5.1-2

a)



$$\underline{X[z]} = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n} = \sum_{n=m}^{\infty} 1\cdot z^{-n}$$

$$= /k=n-m/ = \sum_{k=0}^{\infty} z^{-(k+m)} = z^{-m}\sum_{k=0}^{\infty}(z^{-1})^k = \Big/ \begin{matrix}|z^{-1}|<1 \\ \Rightarrow |z|>1\end{matrix} \Big/$$

$$= z^{-m}\frac{1}{1-z^{-1}} = \underline{\frac{z}{z^m(z-1)}}\ ; \ \underline{\text{Konv.omr. } |z|>1}$$

b) $x[n] = \gamma^n \cdot \sin(n\pi)u[n] = 0$ ty $\sin(n\pi)=0 \ \forall n$

$\Rightarrow X[z]=0$

c) $x[n] = \gamma^n\cos(n\pi)u[n] = \gamma^n(-1)^n u[n] = (-\gamma)^n\cdot u[n]$

$$\Longrightarrow \underline{X[z]} = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n} = \sum_{n=0}^{\infty}(-\gamma)^n z^{-n} = \sum_{n=0}^{\infty}(-\gamma z^{-1})^n$$

$$= \Big/ \begin{matrix}|-\gamma z^{-1}|<1 \\ \Rightarrow |z|>|\gamma|\end{matrix} \Big/ = \frac{1}{1-(-\gamma z^{-1})} = \underline{\frac{z}{z+\gamma}}\ ; \ \underline{|z|>|\gamma|}$$

e) $x[n] = \gamma^n \cos\left(\frac{\pi n}{2}\right) u[n] = \gamma^n \cdot \frac{e^{j\frac{\pi n}{2}} + e^{-j\frac{\pi n}{2}}}{2} \cdot u[n]$

$= / e^{\pm j \frac{\pi n}{2}} = \left(e^{\pm j \frac{\pi}{2}}\right)^n = (\pm j)^n / = \frac{1}{2}\left((\gamma j)^n + (-\gamma j)^n\right) u[n]$

$\Rightarrow \underline{\underline{X[z]}} = \sum_{n=0}^{\infty} \frac{1}{2}\left((\gamma j)^n + (-\gamma j)^n\right) z^{-n} =$

$= \frac{1}{2} \sum_{n=0}^{\infty} (\gamma j z^{-1})^n + \frac{1}{2} \sum_{n=0}^{\infty} (-\gamma j z^{-1})^n$

$= / |\pm \gamma j z^{-1}| < 1 \Rightarrow |z| > |\gamma| /$

$= \frac{1}{2} \cdot \frac{1}{1 - \gamma j z^{-1}} + \frac{1}{2} \cdot \frac{1}{1 - (-\gamma j z^{-1})}$

$= \frac{1}{2}\left(\frac{z}{z - \gamma j} + \frac{z}{z + \gamma j}\right) = \underline{\underline{\frac{z^2}{z^2 + \gamma^2}}} \; ; \underline{|z| > |\gamma|}$

f) $x[n] = \sum_{k=0}^{\infty} 2^{2k} \delta[n - 2k] \Rightarrow$

$\underline{\underline{X[z]}} = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} \left(\sum_{k=0}^{\infty} 2^{2k} \delta[n - 2k] \right) z^{-n} =$

$= \sum_{k=0}^{\infty} 2^{2k} \cdot \underbrace{\sum_{n=\infty}^{\infty} \delta[n - 2k] \cdot z^{-n}}_{= z^{-n}|_{n=2k} = z^{-2k}} =$

$= \sum_{k=0}^{\infty} (4 z^{-2})^k = / |4 z^{-2}| < 1 \Rightarrow |z^2| > 4 /$

$= \frac{1}{1 - 4z^{-2}} = \underline{\underline{\frac{z^2}{z^2 - 4}}} \; ; \underline{|z| > 2}$

g) $x[n] = \gamma^{n-1} u[n-1] \Rightarrow$

$$\underline{X[z]} = \sum_{n=1}^{\infty} \gamma^{n-1} \cdot z^{-n} = \gamma^{-1} \sum_{n=1}^{\infty} (\gamma z^{-1})^n =$$

$$= // |\gamma z^{-1}| < 1 \Rightarrow |z| > |\gamma| // = \gamma^{-1} (\gamma z^{-1})^1 \cdot \frac{1}{1 - \gamma z^{-1}}$$

$$= \underline{\frac{1}{z - \gamma}} \quad ; \text{ Konvergensområde } \underline{|z| > |\gamma|}$$

5.1-3

$$\sum_{n=0}^{\infty} n \cdot \left(\frac{-3}{2}\right)^{-n} = \sum_{n=0}^{\infty} x[n] \cdot z^{-n} \Big|_{z = \frac{-3}{2}} =$$

$$= X[z] \Big|_{z = \frac{-3}{2}} \quad \text{för } x[n] = n \cdot u[n]$$

Tab. 10:6 med $\gamma = 1 \Rightarrow X[z] = \frac{z}{(z-1)^2} \; ; |z| > 1$

$\Rightarrow$ Den sökta summan är $\frac{-3/2}{(-3/2 - 1)^2} = \underline{\frac{-6}{25}}$

5.1-4

a) $x[n] = u[n] - u[n-2] = \delta[n] + \delta[n-1]$

Tab 10:1 & 10:2 $\Rightarrow \underline{X[z]} = 1 + z^{-1} = \underline{\frac{z+1}{z}} \; ; \underline{|z| > 0}$ ty

b) $x[n] = \gamma^{n-2} u[n-2] = \gamma^{-2} (\gamma^n u[n] - \gamma^0 \delta[n] - \gamma^1 \delta[n-1])$

Tab. 10:4, 10:1 & 10:2 $\Rightarrow$

$$\underline{X[z]} = \gamma^{-2} \Big(\underbrace{\frac{z}{z-\gamma}}_{|z|>|\gamma|} - 1 - \underbrace{\gamma z^{-1}}_{|z|>0} \Big) = \underline{\frac{1}{(z-\gamma)z}} \quad ; \underline{|z| > |\gamma|}$$

c) $x[n] = 2^{n+1}u[n-1] + e^{n-1}u[n]$

$= 4 \cdot 2^{n-1}u[n-1] + e^{-1} \cdot e^{n}u[n]$

Tab. 10:5 & 10:4 $\Rightarrow$

$$\underline{X[z]} = 4 \cdot \underbrace{\frac{1}{z-2}}_{(|z|>2)} + e^{-1} \cdot \underbrace{\frac{z}{z-e}}_{(|z|>e)} = \underline{\frac{4(z-e)+e^{-1}z(z-2)}{(z-2)(z-e)}} \; ; \; \underline{|z|>e}$$

5.1-5

a) $$X[z] = \frac{z(z-4)}{z^2-5z+6} \Rightarrow \left(z^2-5z+6 = (z-\tfrac{5}{2})^2 - (\tfrac{1}{2})^2 \right) \Rightarrow$$

$$\frac{X[z]}{z} = \frac{z-4}{(z-2)(z-3)} = \frac{2}{z-2} - \frac{1}{z-3} \Rightarrow$$

$$X[z] = 2 \cdot \underbrace{\frac{z}{z-2}}_{|z|>2} - \underbrace{\frac{z}{z-3}}_{|z|>3}$$

← Ty enkelsidig transform $X[z]$

Tab. 10:4 $\Rightarrow \underline{x[n] = (\underbrace{2 \cdot 2^n}_{2^{n+1}} - 3^n)u[n]}$

b) $$\frac{X[z]}{z} = \frac{z-4}{z(z-2)(z-3)} = \frac{-2/3}{z} + \frac{1}{z-2} + \frac{-1/3}{z-3} \Rightarrow$$

$$X[z] = -\frac{2}{3} + \underbrace{\frac{z}{z-2}}_{|z|>2} - \frac{1}{3}\underbrace{\frac{z}{z-3}}_{|z|>3}$$

Tab. 10:1 & 10:4 $\Rightarrow$

$$\underline{x[n] = -\frac{2}{3}\delta[n] + (2^n - \underbrace{\tfrac{1}{3}3^n}_{=3^{n-1}})u[n]}$$

c) $$\frac{X[z]}{z} = \frac{e^{-2}-2}{(z-e^{-2})(z-2)} = \frac{1}{z-e^{-2}} - \frac{1}{z-2} \Rightarrow$$

$$X[z] = \underbrace{\frac{z}{z-e^{-2}}}_{|z|>e^{-2}} - \underbrace{\frac{z}{z-2}}_{|z|>2}$$

Tab. 10:4 $\Rightarrow$ $\underline{\underline{x[n] = \left(e^{-2n} - 2^n\right) u[n]}}$

d) $$X[z] = \frac{z^2-2z+1}{z^3} = z^{-1} - 2z^{-2} + z^{-3}$$

Tab. 10:2 $\Rightarrow$ $\underline{\underline{x[n] = \delta[n-1] - 2\delta[n-2] + \delta[n-3]}}$

f) $$\frac{X[z]}{z} = \frac{-5z+22}{(z+1)(z-2)^2} = \frac{3}{z+1} + \frac{-3}{z-2} + \frac{4}{(z-2)^2}$$

$$\Rightarrow X[z] = 3 \cdot \underbrace{\frac{z}{z+1}}_{|z|>1} - 3\underbrace{\frac{z}{z-2}}_{|z|>2} + 4\underbrace{\frac{z}{(z-2)^2}}_{|z|>2}$$

Tab. 10:4 & 10:6 $\Big\} \Rightarrow$ $\underline{\underline{x[n] = \left(3(-1)^n - 3\cdot 2^n + \underbrace{2n\cdot 2^n}_{=2^{n+1}\cdot n\cdot u[n]}\right) u[n]}}$

5.1-6 a) Polynomdivision:

$$\begin{array}{r|l} & 2 - z^{-1} + 4z^{-2} - \dots \\ \hline z^3+7z^2+2z+1 & 2z^3+13z^2+z \\ & -(2z^3+14z^2+4z+2) \\ \hline & -z^2-3z-2 \\ & -(-z^2-7z-2-z^{-1}) \\ \hline & 4z+z^{-1} \\ & -(4z+28+8z^{-1}+4z^{-2}) \\ \hline & -28-7z^{-1}-4z^{-2} \end{array}$$

$$\Rightarrow X[z] = 2\cdot z^{-0} - 1\cdot z^{-1} + 4\cdot z^{-2} + \dots$$

$\Rightarrow x[0]=2$, $x[1]=-1$, $x[2]=4$

(Erhålls från $X[z] = \sum_{n=0}^{\infty} x[n]z^{-n}$, ty enkelsidig transform)

5.2-1

- Ber. enligt def: $X[z] = \sum_{n=0}^{\infty} x[n] z^{-1} = \sum_{n=0}^{m-1} \overset{x[n]=1 \text{ i intervallet}}{(1 \cdot z^{-1})^n} = \frac{1-(z^{-1})^m}{1-z^{-1}}$

- $x[n] = u[n] - u[n-m]$

Tabell 10:3 $\Rightarrow$ $u[n] \Leftrightarrow \frac{z}{z-1}$

Tabell 9:5 $\Rightarrow$ $x[n-m]u[n-m] \Leftrightarrow z^{-m} \cdot X[z]$

$\Rightarrow X[z] = \frac{z}{z-1} - z^{-m} \cdot \frac{z}{z-1} = \frac{1-z^{-m}}{1-z^{-1}}$

5.2-7

a) $\mathcal{Z}_{II}\{(-1)^n x[n]\} = \sum_{n=-\infty}^{\infty} (-1)^n x[n] z^{-n} = \sum_{n=-\infty}^{\infty} x[n](-z)^{-n} = X[-z]$

b) Teb. 10:4 $\Rightarrow$ $\gamma^n u[n] \Leftrightarrow \frac{z}{z-\gamma}$ $|z| > |\gamma|$

$\Rightarrow \underline{(-\gamma)^n u[z]} = (-1)^n \gamma^n u[n] \underset{\text{Teb. 10:4 \& uppg. a}}{\Leftrightarrow} \frac{-z}{-z-\gamma} = \underline{\frac{z}{z+\gamma}}$

$|z| > |\gamma|$

c)

(i) $2^{n-1}u[n] = \frac{1}{2} \cdot 2^n u[n] \overset{\text{Teb 10:4}}{\Leftrightarrow} \frac{1}{2} \cdot \frac{z}{z-2}$ $|z| > 2$

$(-2)^{n-1}u[n] = \frac{-1}{2}(-1)^n 2^n u[n] \overset{(b)}{\Leftrightarrow} \frac{-1}{2} \cdot \frac{-2}{-z-2} = \frac{-1}{2} \frac{z}{z+2}$

$|z| > 2$

$\Rightarrow$

$\underline{(2^{n-1} - (-2)^{n-1})u[n]} \Leftrightarrow \frac{1}{2}\left(\frac{z}{z-2} + \frac{z}{z+2}\right) = \underline{\frac{z^2}{z^2-4}; |z| > 2}$

(ii) $\underline{\gamma^n \cdot \cos(\pi n) u[n]} = (-1)^n \cdot \gamma^n u[n] \overset{(b)}{\Leftrightarrow} \underline{\frac{z}{z+\gamma}; |z| > |\gamma|}$

5.9-1

a) $x[n] = \underbrace{0{,}8^n u[n]}_{=x_1[n]} + \underbrace{2^n u[-n-1]}_{=x_2[n]}$

Tab. 10:4 (och mänga av uppgifternas lösningarna som reden lösts) $\Rightarrow$ $X_1[z] = \dfrac{z}{z-0{,}8}$; $|z|>0{,}8$

$$X_2[z] = \sum_{n=-\infty}^{\infty} x_2[n] z^{-n} = \sum_{n=-\infty}^{-1} 2^n \cdot z^{-n} = /n=-k-1/$$

$$= \sum_{k=0}^{\infty} (2^{-1} z)^{k+1} = \frac{z}{2} \sum_{k=0}^{\infty} \left(\frac{z}{2}\right)^k = / \left|\frac{z}{2}\right| < 1 /$$

$$= \frac{z}{2} \cdot \frac{1}{1 - z/2} = \frac{-z}{z-2} \; ; \quad |z| < 2$$

[Erhålls även direkt från Tab. 10:13]

$$\therefore \; X[z] = X_1[z] + X_2[z] = \underbrace{\frac{z}{z-0{,}8}}_{|z|>0{,}8} - \underbrace{\frac{z}{z-2}}_{|z|<2} = \frac{-1{,}2z}{(z-0{,}8)(z-2)}$$

$0{,}8<|z|<2$

b) $x[n] = \underbrace{2^n u[n]}_{=x_1[n]} - \underbrace{3^n u[-n-1]}_{=x_2[n]}$

På motsvarande sätt som i uppgift a:

$$\left.\begin{aligned} X_1[z] &= \frac{z}{z-2} \; ; \quad |z|>2 \\ X_2[z] &= \frac{-z}{z-3} \; ; \quad |z|<3 \end{aligned}\right\} \Rightarrow$$

[$X_1[z]$ & $X_2[z]$ kan även erhålles från Tab. 10:4 resp. Tab. 10:13]

$$\Rightarrow X[z] = \frac{z}{z-2} - \frac{-z}{z-3} =$$

$$= \frac{z(2z-5)}{(z-2)(z-3)} \; ; \quad 2<|z|<3$$

f) $x[n] = \underbrace{0{,}8^n u[n]}_{=x_1[n]} + \underbrace{3 \cdot 0{,}4^n u[-n-1]}_{=x_2[n]}$

På motsvarande sätt som i uppgift a):

$$\left.\begin{aligned} X_1[z] &= \frac{z}{z-0{,}8} \;;\; |z| > 0{,}8 \\ X_2[z] &= 3 \cdot \frac{-z}{z-0{,}4} \;;\; |z| < 0{,}4 \end{aligned}\right\} \Rightarrow$$

$\Rightarrow$ Konvergensområdena överlapper inte, dvs. $X_1[z] + X_2[z]$ har inget sammanhängande konvergensområde, vilket innebär att $X[z]$ $\nexists$

g) $x[n] = 0{,}5^{|n|} = \underbrace{0{,}5^n \cdot u[n]}_{=x_1[n]} + \underbrace{0{,}5^{-n} \cdot u[-n-1]}_{=x_2[n]}$

På motsvarande sätt som i uppgift e):

$$\left.\begin{aligned} X_1[z] &= \frac{z}{z-0{,}5} \;;\; |z| > 0{,}5 \\ X_2[z] &= \frac{-z}{z-(0{,}5^{-1})} = \frac{-z}{z-2} \;;\; |z| < 2 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \underline{\underline{X[z]}} = X_1[z] + X_2[z] = \frac{z}{z-0{,}5} + \frac{-z}{z-2}$$

$$= \underline{\underline{\frac{-1{,}5z}{(z-0{,}5)(z-2)}}} \;;\; \underline{0{,}5 < |z| < 2}$$

5.9-2 $\frac{X[z]}{z} = \frac{e^{-2}-2}{(z-e^{-2})(z-2)} = \frac{1}{z-e^{-2}} + \frac{-1}{z-2}$

$$\Rightarrow X[z] = \underbrace{\frac{z}{z-e^{-2}}}_{=X_1[z]} - \underbrace{\frac{z}{z-2}}_{=X_2[z]} \Rightarrow x[n] = x_1[n] - x_2[n]$$

$$\begin{cases} \text{Om } |z| > e^{-2} \Rightarrow x_1[n] = e^{-2n}u[n] \\ \text{Om } |z| < e^{-2} \Rightarrow x_1[n] = -e^{-2n}u[-n-1] \end{cases}$$

$$\gamma^n u[-n-1] \Leftarrow\Rightarrow \frac{-z}{z-\gamma} \; ; |z| < |\gamma|$$

$$\begin{cases} \text{Om } |z| < 2 \Rightarrow x_2[n] = -2^n u[-n-1] \\ \text{Om } |z| > 2 \Rightarrow x_2[n] = 2^n u[n] \end{cases}$$

a) $|z| > 2 \Rightarrow x[n] = (e^{-2n} - 2^n)u[n]$

b) $e^{-2} < |z| < 2 \Rightarrow x[n] = e^{-2n}u[n] + 2^n u[-n-1]$

c) $|z| < e^{-2} \Rightarrow x[n] = (-e^{-2n} + 2^n)u[-n-1]$

5.9-3 Här visar det sig vara enklare att partialbråksuppdela $X[z]$ än $\frac{X[z]}{z}$ (inses efter test):

$$X[z] = \frac{1/2}{(z+1)(z+\frac{1}{2})^2} = \frac{2}{z+1} + \frac{-2}{z+\frac{1}{2}} + \frac{1}{(z+\frac{1}{2})^2}$$

$$= 2 \cdot \underbrace{\frac{-(-1)}{z-(-1)}}_{|z|<1} - 4 \cdot \underbrace{\frac{-(-1/2)}{z-(-\frac{1}{2})}}_{|z|<1/2} + 2z^{-1} \underbrace{\frac{-(-\frac{1}{2})z}{(z-(\frac{-1}{2}))^2}}_{|z|<1/2}$$

$$\Rightarrow x[n] = 2(-1)^n u[-n-1] - 4 \cdot (\tfrac{-1}{2})^n u[-n-1] + 2(n-1)(\tfrac{-1}{2})^{n-1} u[-(n-1)-1]$$

$$= 2(-1)^n u[-n-1] - 4n(\tfrac{-1}{2})^n u[-n] + 4\delta[n]$$