Vektorer	Funktioner
$\bar{a} = (a_1, a_2 \dots a_N)$ $\bar{b} = (b_1, b_2, \dots b_N)$	$a(t)$, $b(t)$ i ett tidsintervall av längd T (T_0 om periodisk)
Inre produkt $\langle \bar{a}, \bar{b} \rangle = \sum_k a_k \cdot b_k^*$ $\bar{a} \perp \bar{b} \iff \langle \bar{a}, \bar{b} \rangle = 0$	**Inre produkt** $\langle a, b \rangle = \int_T a(t) \cdot b^*(t)\, dt$ $a(t) \perp b(t) \iff \langle a, b \rangle = 0$
$\bar{x} \approx \hat{\bar{x}} = \sum_k c_k \cdot \bar{\Phi}_k$ Om ortogonala basvektorer $\bar{\Phi}_k$ $\Rightarrow \langle \bar{\Phi}_p, \bar{\Phi}_k \rangle = \begin{cases} 0; & p \neq k \\ \lVert \bar{\Phi}_k \rVert^2; & p = k \end{cases}$ $\Rightarrow \quad c_k = \frac{1}{\lVert \bar{\Phi}_k \rVert^2} \cdot \langle \bar{x}, \bar{\Phi}_k \rangle$	$x(t) \approx \hat{x}(t) = \sum_k c_k \cdot \Phi_k(t)$ Om ortogonala basfunktioner $\Phi_k(t)$ $\langle \Phi_p, \Phi_k \rangle = \begin{cases} 0; & p \neq k \\ \lVert \Phi_k \rVert^2; & p = k \end{cases}$ $\lVert \Phi_k \rVert^2 = \int_T \lvert \Phi_k(t) \rvert^2 dt$ $\Rightarrow \quad c_k = \frac{1}{\lVert \Phi_k \rVert^2} \cdot \langle x, \Phi_k \rangle$

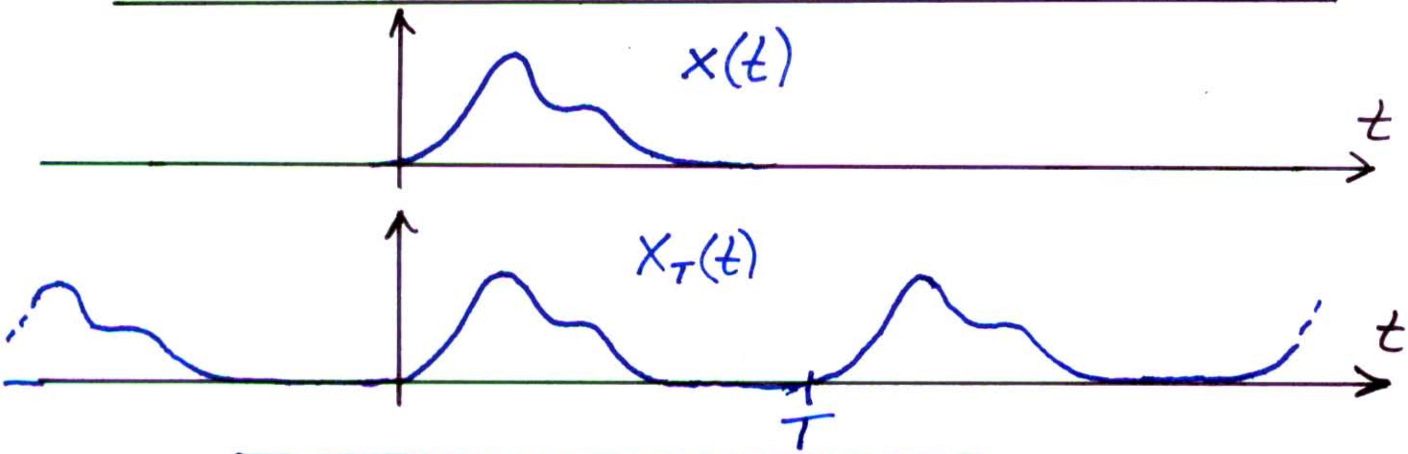
T_0 - periodisk signal $x(t)$ med $\Phi_k(t) = e^{jk\omega_0 t}$; $\omega_0 = \frac{2\pi}{T_0}$

$$\Rightarrow \quad x(t) = \hat{x}(t) = \sum_{k=-\infty}^{\infty} c_k \cdot e^{jk\omega_0 t} \qquad (T = T_0)$$

samt $\quad \Phi_p \perp \Phi_k \iff \langle \Phi_p, \Phi_k \rangle = \begin{cases} 0; & p \neq k \\ \lVert \Phi_k \rVert^2 = T_0; & p = k \end{cases}$

$$\Rightarrow \quad c_k = \frac{1}{\lVert \Phi_k \rVert^2} \langle x, \Phi_k \rangle = \frac{1}{T_0} \int_{T_0} x(t) \cdot e^{-jk\omega_0 t} dt$$

ÖVERGÅNG, FOURIERSERIER → FOURIERTRANSFORM



$$\boxed{x(t) = \lim_{T\to\infty} x_T(t)\ ;\ |t|<\infty}$$

Frekvensbeskrivning:

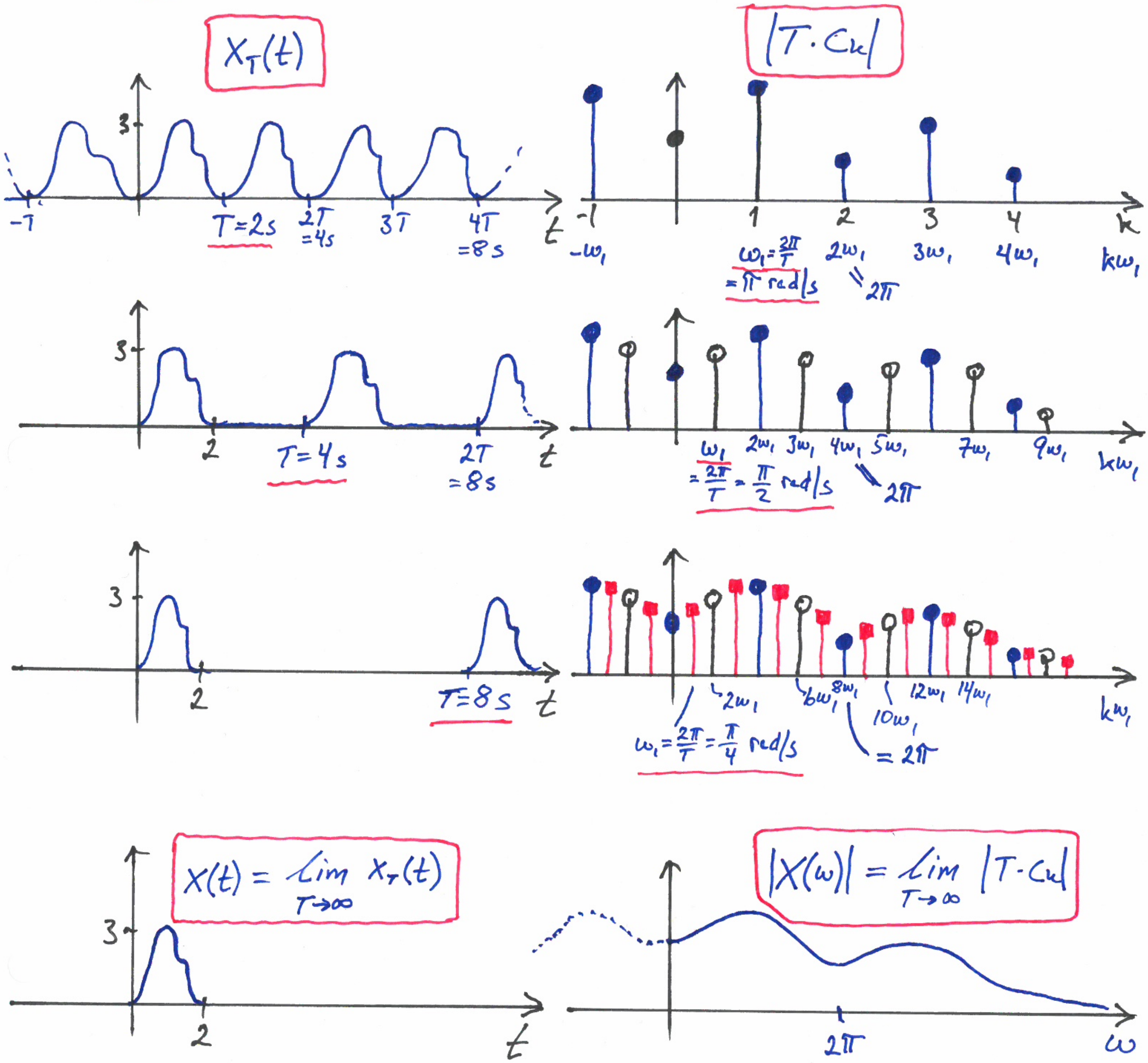
$$c_k = \frac{1}{T}\int_T x_T(t)\cdot e^{-jk\omega_1 t}\,dt\ , \quad \text{där} \quad x_T(t)=\sum_{k=-\infty}^{\infty} c_k\cdot e^{jk\omega_1 t}$$

$$\underline{\text{Låt } T\to\infty} \quad \Rightarrow \quad \begin{cases} \omega_1 = \frac{2\pi}{T}\ (=\Delta\omega) \longrightarrow d\omega \\ k\cdot\omega_1 \longrightarrow \omega \end{cases}$$

$$\Rightarrow \begin{cases} \underline{\underline{T\cdot c_k}} = \int_{-T/2}^{T/2} x_T(t)\, e^{-jk\omega_1 t}\,dt \longrightarrow \underline{\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,dt \triangleq X(\omega)} \\ \underline{\underline{x_T(t)}} = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} T\cdot c_k\cdot e^{jk\omega_1 t}\cdot\frac{2\pi}{T} \longrightarrow \underline{\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\,d\omega = x(t)} \end{cases}$$

$\therefore$ Fourierserieutveckling av $x_T(t)$ $\xrightarrow{T\to\infty}$ Fouriertransform av $x(t)$

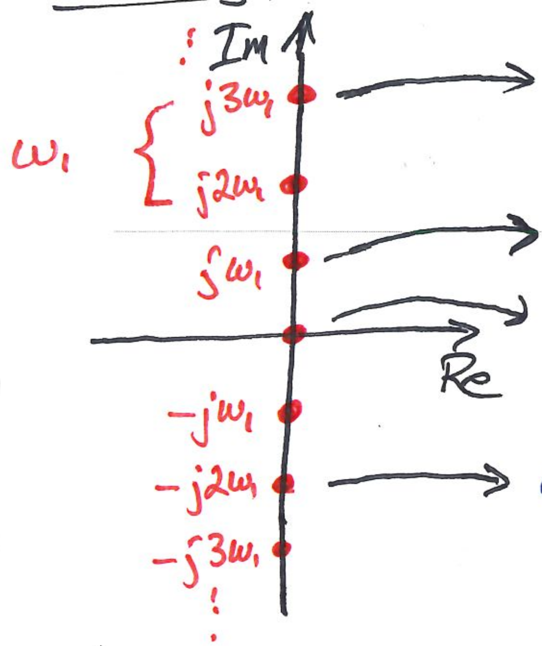
Fourierserie ⟶ Fouriertransform



T-periodisk signal $x_T(t) = \sum_{k=-\infty}^{\infty} c_k \cdot e^{jk\omega_1 t}$:

Tolkning:

$\omega_1 = \frac{2\pi}{T}$ rad/s

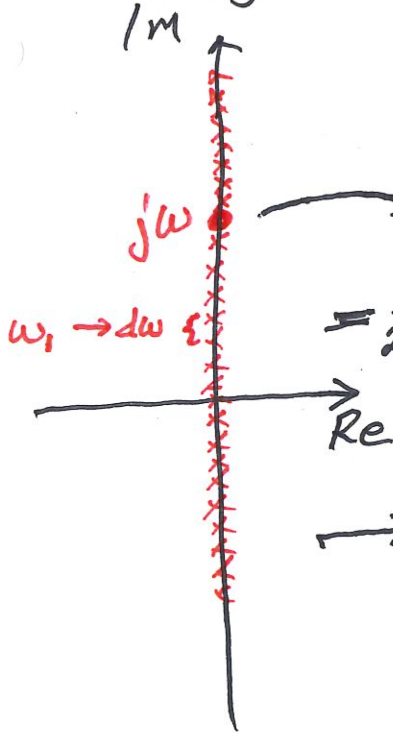


$$\left.\begin{array}{l} c_3 \cdot e^{j3\omega_1 t} \\ c_1 \cdot e^{j\omega_1 t} \\ c_0 \cdot e^{0t} \\ c_{-2} \cdot e^{-j2\omega_1 t} \end{array}\right\} \sum \Rightarrow x_T(t)$$

Ikke-periodisk signal,

$$x(t) = \lim_{T\to\infty} x_T(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

Tolkning:



$$T \to \infty \Rightarrow \left\{\begin{array}{l} \omega_1 = \frac{2\pi}{T} \to d\omega \to 0 \\ k\omega_1 \to \omega \\ \sum \to \int \\ c_k = \frac{1}{T}\int_T \to 0 \end{array}\right.$$

$\Rightarrow T \cdot c_k \Rightarrow$ begrenset

$$\text{"}c_k\text{"} \cdot e^{j\omega t}$$

$$= \frac{1}{2\pi} \cdot \underbrace{T \cdot c_k}_{\downarrow} \cdot e^{j\omega t} \cdot \underbrace{\frac{2\pi}{T}}_{\downarrow}$$

$$\to \frac{1}{2\pi} \cdot X(\omega) \cdot e^{j\omega t}\, d\omega \quad \overset{\sum}{\Longrightarrow} \quad x(t)$$