(PASSIVA) FILTER

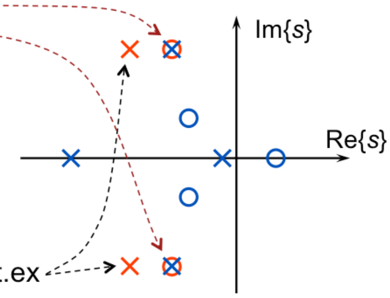
<u>Bortfiltrering/eliminering av enskilda signalkomponenter:</u>

$$Y(s) = X(s) \cdot H(s)$$

$$= \frac{T_X(s)}{(s-p_1)\cancel{(s-p_2)}\cancel{(s-p_3)}(s-p_4)} \cdot \frac{\cancel{(s-p_2)}\cancel{(s-p_3)}}{N_H(s)}$$

"Signalselektiv" filtrering:

De komplexkonjugerade polerna p_2 och p_3 hos $X(s)$ motsvarar en signalkomponent i $x(t)$ som skall <u>helt</u> filtreras bort:
Utförs m.h.a *nollställen* hos $H(s)$!



Krav för
stabilt system: #poler ≥ #nollställen $\Longrightarrow$ t.ex

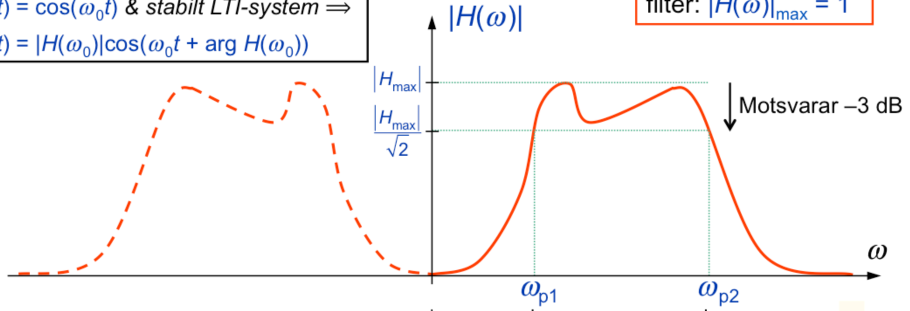
Frekvensselektiva filter

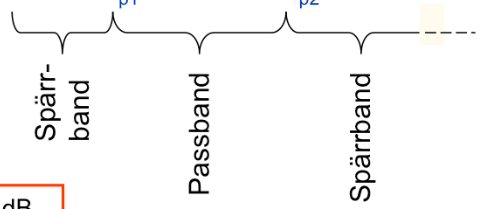
Frekvensselektiv filtrering:

$x(t) = \cos(\omega_0 t)$ & *stabilt LTI-system* ⟹

$y(t) = |H(\omega_0)|\cos(\omega_0 t + \arg H(\omega_0))$

Amplitudnormerat filter: $|H(\omega)|_{max} = 1$





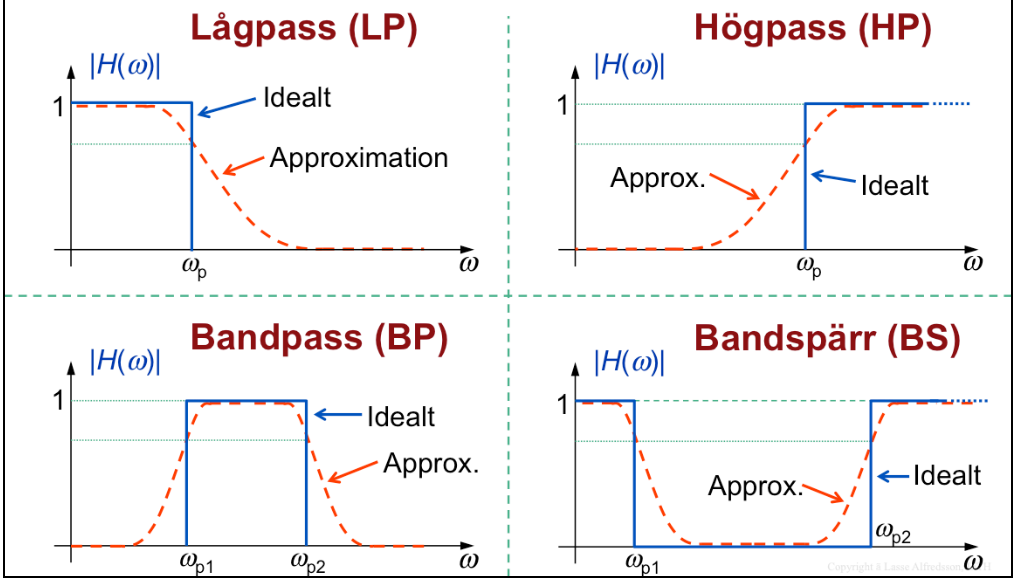
ω_{p1}; undre gränsvinkelfrekvens

ω_{p2}; övre gränsvinkelfrekvens

$\omega_{p2} - \omega_{p1}$; bandbredden

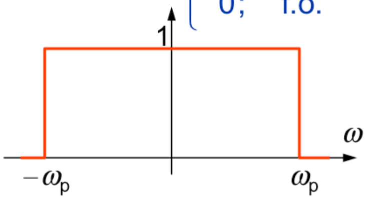
Ofta studeras $|H(\omega)|_{dB} = 20 \cdot {}^{10}\log |H(\omega)|$ dB

Olika frekvensselektiva filtertyper

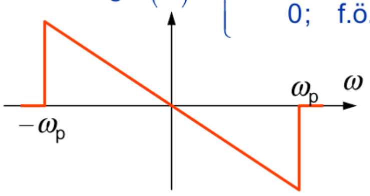


Exempel – Idealt LP-Filter:

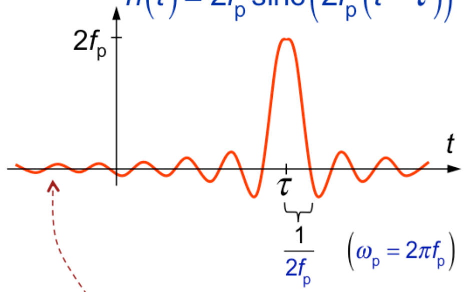
$$|H(\omega)| = \begin{cases} 1; & |\omega| \le \omega_p \\ 0; & \text{f.ö.} \end{cases}$$



$$\arg H(\omega) = \begin{cases} -\omega\tau; & |\omega| \le \omega_p \\ 0; & \text{f.ö.} \end{cases}$$



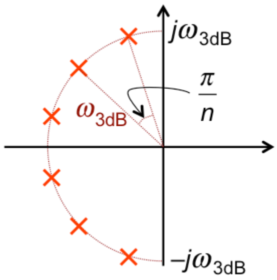
$$h(t) = 2f_p \operatorname{sinc}\left(2f_p\left(t-\tau\right)\right)$$



$h(t) \neq 0$ för $t < 0$,
dvs. icke-kausalt system
$\Longrightarrow$ **ej realiserbart**!

Butterworthfilter – (LP-filter)

Poler hos *H*(*s*) längs en halvcirkel:

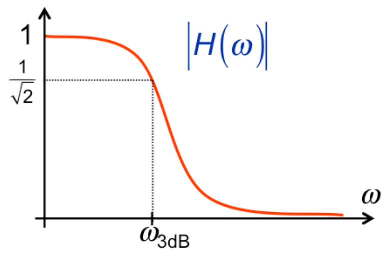


<u>Poler:</u>

$$p_k = \omega_{3dB} \cdot e^{j\left(\frac{\pi}{2} - \frac{1}{2}\cdot\frac{\pi}{n} + k\cdot\frac{\pi}{n}\right)}$$

$$= \omega_{3dB} \cdot e^{j\left(\frac{2k+n-1}{2n}\right)\pi}$$

$k = 1, 2, 3, \ldots, n$



ω_{3dB} = 3 dB-gränsvinkelfrekvensen

$$H(s) = \frac{(\omega_{3dB})^n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + (\omega_{3dB})^n}$$

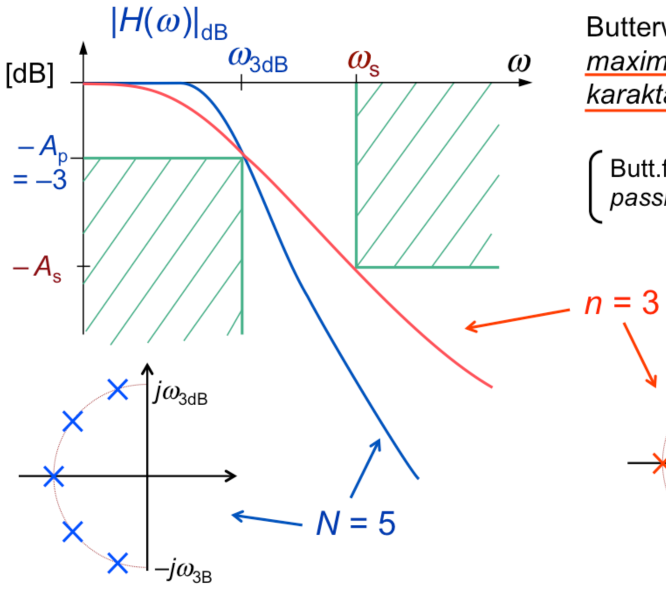
$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_{3dB}}\right)^{2n}}}$$

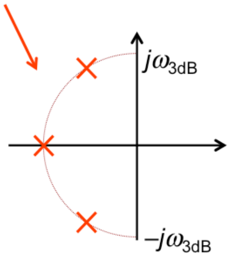
a_i erhålls vanligen från tabell eller genom utveckling av $(s-p_1)(s-p_2)\ldots(s-p_n)$

Butterworthfilter – (LP-filter)

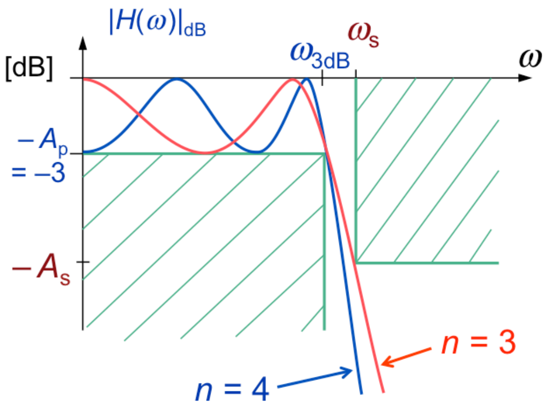
Butterworthfilter har *maximalt flat amplitud-karaktäristik* i passbandet!

(Butt.filter ger *bästa möjliga passbands-approximationen*)

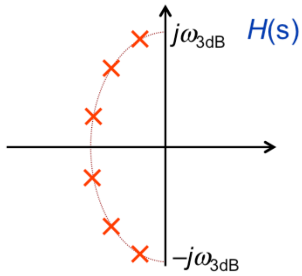




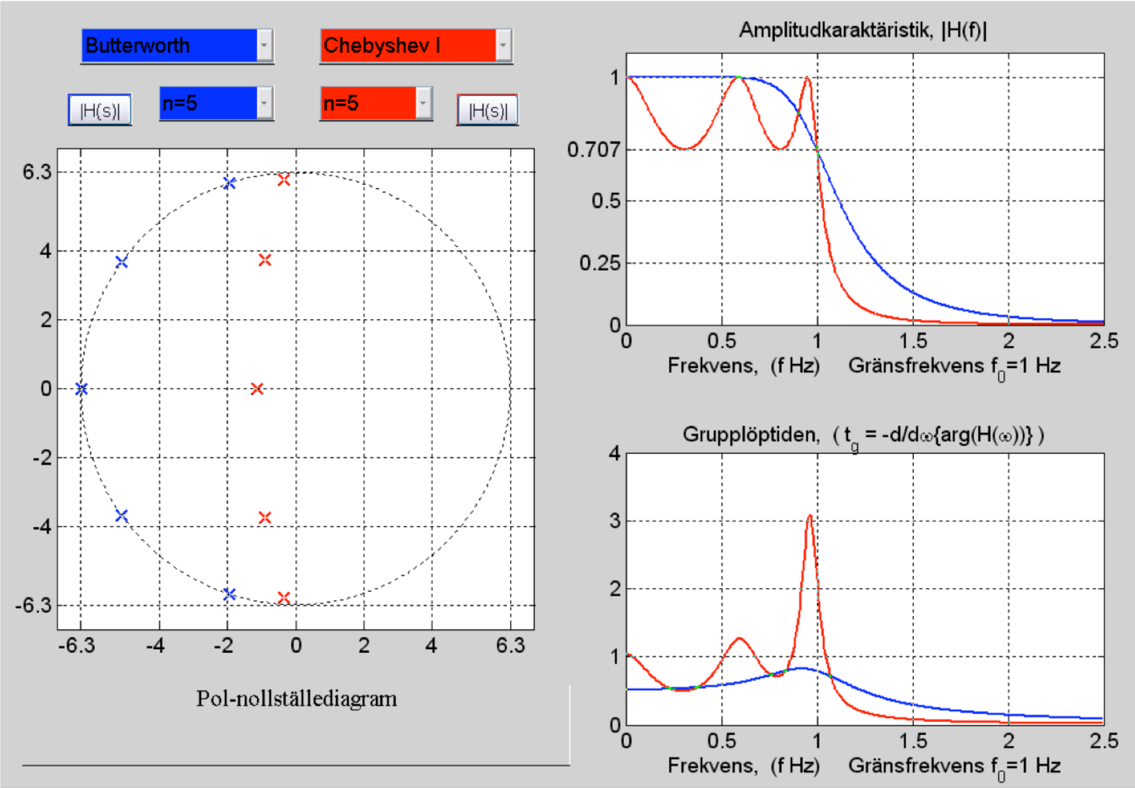
Chebyshev I-filter



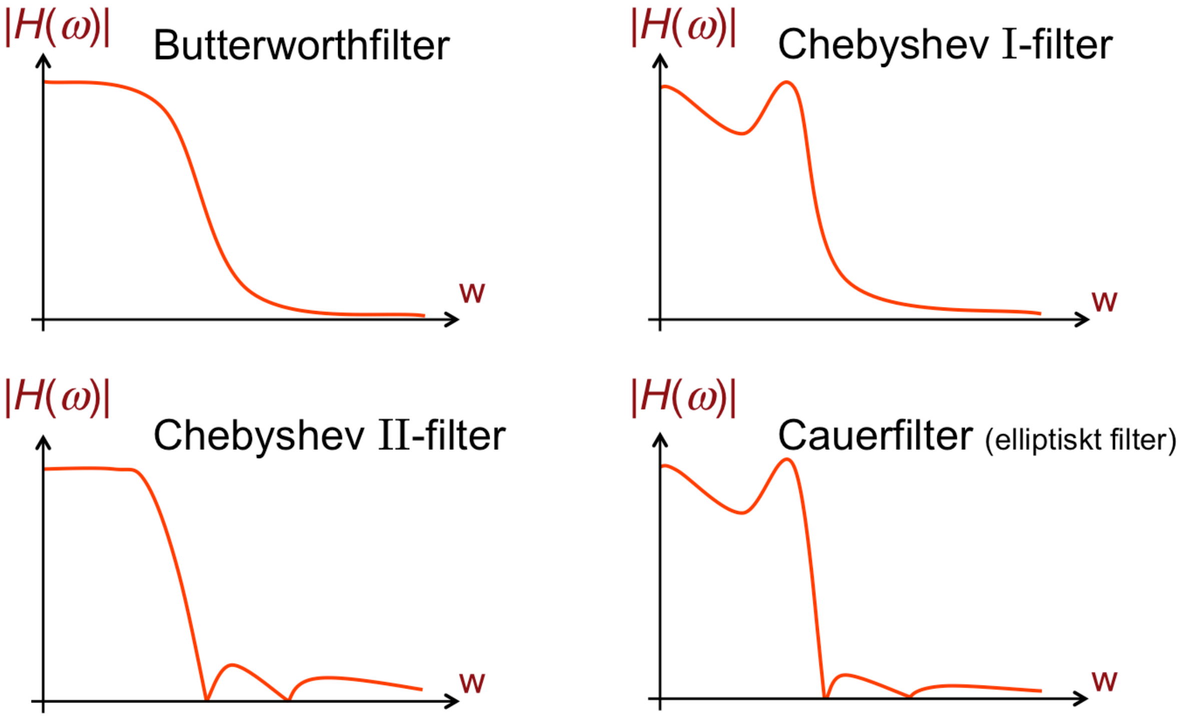
- *Rippel* (A_p dB) i passbandet!
- Optimalt m.a.p. *brantheten i övergångsbandet* $\omega_{3dB} \rightarrow \omega_s$
- Polerna hos $H(s)$ ligger längs en *halv-ellips* i VHP



Matlabdemo, Butterworth- & Chebyshev I-filter:

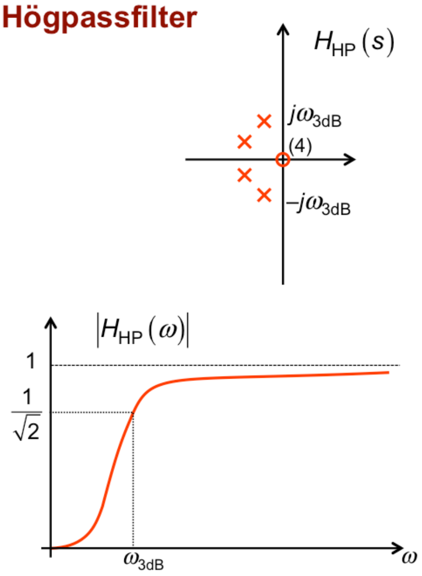


Klassiska ideala LP-approximationer



Typiska HP- & BP-filter

Högpassfilter



Bandpassfilter