LINKÖPING UNIVERSITY

# TSBB06 Multi-Dimensional Signal Analysis, Midterm 2020-10-24

**Course module:** KTR1

**Date & Time:** 2020-10-24 14:00–18:00

**Location:** TERE & TER3 (midterm), TER1 (final exam)

**Examiner:** Mårten Wadenbäck
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The examiner will make one visit, approximately one hour after the exam has started, and will be available on the phone the rest of the time.

**Material & aids:** You are allowed to use any reference material you like (books, printouts, your own notes, ...), as long as it does not require an electronic device.

**Scoring:** The maximum score on the midterm is 18 points. To pass the midterm, you need to score at least 10 points. If you pass the midterm, you can choose to use your score from the midterm instead of Part I on the final exam in January.

**Instructions:** Justify your solutions and answers with clear and concise arguments. All solutions and answers should be written on dedicated sheets of paper (i.e. not on the printed exam). Write your AID-number and the exam date on all sheets of paper that you hand in. Start each numbered problem on a new sheet. Right before you hand in, sort your solutions in consecutive order and add page numbers in the upper right corner.

# Part I: Geometry and Estimation

**1.** Let $\mathbf{x} = (-2, 1, 3, 1)$ be homogeneous coordinates of a point in the extended Euclidean space (3D), and let $\mathbf{p}_1 = (1, -2, -2, 3)$ and $\mathbf{p}_2 = (2, 0, -1, -2)$ be dual homogeneous coordinates of two planes.

(a) Which of the planes $\mathbf{p}_1$ and $\mathbf{p}_2$ does the point $\mathbf{x}$ lie the closest to? (2 p)

(b) Compute the dual Plücker coordinates of the *horizon line* of $\mathbf{p}_1$, i.e. the line consisting of all ideal points (points at infinity) that lie on $\mathbf{p}_1$. (1 p)

(c) Show that all planes parallel to $\mathbf{p}_1$ have the same horizon line as $\mathbf{p}_1$. (2 p)

**2.** Consider a 3D *rotation matrix* $\mathbf{R}$ which satisfies

$$\mathbf{R} \sim \begin{pmatrix} 2 & -1 & a \\ 2 & 2 & b \\ -1 & 2 & c \end{pmatrix} \iff \mathbf{R} = \lambda \begin{pmatrix} 2 & -1 & a \\ 2 & 2 & b \\ -1 & 2 & c \end{pmatrix}$$

for some scalars $a, b, c, \lambda \in \mathbb{R}$.

(a) Determine all possible $a, b, c, \lambda \in \mathbb{R}$ which make $\mathbf{R}$ a valid rotation matrix. (2 p)

(b) Choose any of the rotation matrices $\mathbf{R}$ that satisfy the above, and compute an axis–angle representation of the rotation represented by $\mathbf{R}$. (3 p)

**3.** Let $\mathscr{G}$ be the set of rigid transformations in 2D for which the rotation angle is $\frac{k\pi}{2}$ for some $k \in \mathbb{Z}$.

(a) Express a general element $\mathbf{T} \in \mathscr{G}$ in matrix form. (1 p)

(b) Find all $\mathbf{T} \in \mathscr{G}$ that are their own inverses, i.e. all $\mathbf{T}$ for which $\mathbf{T}^2 = \mathbf{I}$. (1 p)

(c) Show that $\mathscr{G}$ is a *group* with respect to composition. (2 p)

**4.** Let $\mathbf{X}_1, \dots, \mathbf{X}_n \in \mathbb{R}^4$ be homogeneous coordinates of points in the extended Euclidean space (3D), and let $\mathbf{x}_1, \dots, \mathbf{x}_n \in \mathbb{R}^3$ be homogeneous coordinates of points in the extended Euclidean plane (2D). Suppose that the points are known, and that we seek a camera matrix $\mathbf{C} \in \mathbb{R}^{3\times4}$ that maps each $\mathbf{X}_k$ to $\mathbf{x}_k$, i.e. $\mathbf{x}_k \sim \mathbf{C}\mathbf{X}_k$. If this cannot be achieved *exactly*, we can still estimate $\mathbf{C}$ by minimising either an algebraic or a geometric cost function (error).

(a) Construct a suitable data matrix $\mathbf{A}$ that can be used to estimate $\mathbf{C}$ from the point correspondences $\mathbf{x}_k \leftrightarrow \mathbf{X}_k$. What is the smallest number of correspondences needed to uniquely (up to scale) determine $\mathbf{C}$? (3 p)

(b) The method based on (a) minimises an algebraic error, which lacks geometric interpretation. Suggest a geometric error $\varepsilon_G(\mathbf{C})$ for this estimation problem. (1 p)