LINKÖPING UNIVERSITY

# TSBB06 Multi-Dimensional Signal Analysis, Solutions 2020-10-24

**1.** (a) The distance between $\mathbf{x}$ and $\mathbf{p}_1$ is given by

$$d_1 = \left|(\mathrm{norm}_\mathrm{P}\,\mathbf{x})^\top(\mathrm{norm}_\mathrm{D}\,\mathbf{p}_1)\right| = \left|(-2,1,3,1)\cdot\frac{-1}{3}(1,-2,-2,3)\right| = \frac{7}{3},$$

and the distance between $\mathbf{x}$ and $\mathbf{p}_2$ is given by

$$d_2 = \left|(\mathrm{norm}_\mathrm{P}\,\mathbf{x})^\top(\mathrm{norm}_\mathrm{D}\,\mathbf{p}_2)\right| = \left|(-2,1,3,1)\cdot\frac{1}{\sqrt{5}}(2,0,-1,-2)\right| = \frac{9}{\sqrt{5}}.$$

Since $d_1 < d_2$, $\mathbf{x}$ lies closer to $\mathbf{p}_1$ than to $\mathbf{p}_2$.

**Answer:** The point $\mathbf{x}$ lies closer to $\mathbf{p}_1$ than to $\mathbf{p}_2$.

(b) The horizon line of a plane is the intersection of the plane and the ideal plane $\mathbf{p}_\infty = (0,0,0,1)$. The dual Plücker coordinates of a line can be formed using any two (distinct) planes that contain the line, so in this case we have

$$\tilde{\mathbf{L}} = \mathbf{p}_1\mathbf{p}_\infty^\top - \mathbf{p}_\infty\mathbf{p}_1^\top = \begin{pmatrix}1\\-2\\-2\\3\end{pmatrix}\begin{pmatrix}0 & 0 & 0 & 1\end{pmatrix} - \begin{pmatrix}0\\0\\0\\1\end{pmatrix}\begin{pmatrix}1 & -2 & -2 & 3\end{pmatrix} = \begin{pmatrix}0 & 0 & 0 & 1\\0 & 0 & 0 & -2\\0 & 0 & 0 & -2\\-1 & 2 & 2 & 0\end{pmatrix}.$$

**Answer:** The dual Plücker coordinates are $\tilde{\mathbf{L}} = \begin{pmatrix}0 & 0 & 0 & 1\\0 & 0 & 0 & -2\\0 & 0 & 0 & -2\\-1 & 2 & 2 & 0\end{pmatrix}$.

(c) All planes parallel to $\mathbf{p}_1$ are of the form $\mathbf{p}(s) = (1,-2,-2,s)$ for some $s \in \mathbb{R}$. By computing the horizon line of $\mathbf{p}(s)$ in the same way as we did for $\mathbf{p}_1$ in (b), we see that

$$\mathbf{p}(s)\mathbf{p}_\infty^\top - \mathbf{p}_\infty\mathbf{p}(s)^\top = \ldots \sim \begin{pmatrix}0 & 0 & 0 & 1\\0 & 0 & 0 & -2\\0 & 0 & 0 & -2\\-1 & 2 & 2 & 0\end{pmatrix}.$$

In other words, we obtain the same horizon line irrespective of the value of $s$.

**2.** (a) A rotation matrix in 3D is an orthogonal matrix with determinant equal to one. To satisfy the length requirement for the first two columns, it is clear that the only possibilities for $\lambda$ are $\lambda = \pm\frac{1}{3}$.

To satisfy the orthogonality requirement, it is clear that

$$(a, b, c) \sim (2, 2, -1) \times (-1, 2, 2) = (6, -3, 6),$$

and the length requirement now gives $(a, b, c) = \pm(2, -1, 2)$.
Finally, the determinant requirement. For $(a, b, c) = (2, -1, 2)$ we obtain

$$1 = \det \mathbf{R} = \lambda^3 \begin{vmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{vmatrix} = 27\lambda^3 \iff \lambda = \frac{1}{3}.$$

In the same way, for $(a, b, c) = (-2, 1, -2)$ we obtain instead

$$1 = \det \mathbf{R} = \lambda^3 \begin{vmatrix} 2 & -1 & -2 \\ 2 & 2 & 1 \\ -1 & 2 & -2 \end{vmatrix} = -27\lambda^3 \iff \lambda = -\frac{1}{3}.$$

**Answer:** The two possibilities are $(a, b, c, \lambda) = \pm\left(2, -1, 2, \frac{1}{3}\right)$.

(b) If we let $(\hat{\mathbf{n}}, \alpha)$ denote an axis–angle representation, we have seen in the lectures that

$$\sin\alpha\,[\hat{\mathbf{n}}]_\times = \frac{\mathbf{R} - \mathbf{R}^\top}{2} \qquad \text{and} \qquad \cos\alpha = \frac{\operatorname{tr}\mathbf{R} - 1}{2}.$$

Let us use the rotation matrix $\mathbf{R}$ obtained using $(a, b, c, \lambda) = \left(2, -1, 2, \frac{1}{3}\right)$, i.e.

$$\mathbf{R} = \frac{1}{3}\begin{pmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{pmatrix}.$$

We compute

$$\sin\alpha\,[\hat{\mathbf{n}}]_\times = \frac{\mathbf{R} - \mathbf{R}^\top}{2} = \frac{1}{6}\left(\begin{pmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{pmatrix} - \begin{pmatrix} 2 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix},$$

which gives $\hat{\mathbf{n}} = \frac{1}{\sqrt{3}}(1, 1, 1)$ and $\sin\alpha = \frac{\sqrt{3}}{2}$. To determine $\alpha$ we also compute

$$\cos\alpha = \frac{\operatorname{tr}\mathbf{R} - 1}{2} = \frac{\frac{1}{3}(2 + 2 + 2) - 1}{2} = \frac{1}{2}.$$

The point $(\cos\alpha, \sin\alpha) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ now determines $\alpha = \frac{\pi}{3}$ (plus an arbitrary integer multiple of $2\pi$).

**Answer:** The rotation matrix obtained using $(a, b, c, \lambda) = \left(2, -1, 2, \frac{1}{3}\right)$ performs a rotation the angle $\alpha = \frac{\pi}{3}$ around the axis $\hat{\mathbf{n}} = \frac{1}{\sqrt{3}}(1, 1, 1)$.

(The other possible rotation matrix, where $(a, b, c, \lambda) = \left(-2, 1, -2, -\frac{1}{3}\right)$, gives $\hat{\mathbf{n}} = \frac{1}{\sqrt{11}}(-1, 1, -3)$, and $(\cos\alpha, \sin\alpha) = \left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right)$. This determines the angle $\alpha$ in the second quadrant, but it is not given by a nice expression in this case.)

**3.** (a) **Answer:** A general element $\mathbf{T} \in \mathscr{G}$ is of the form

$$\mathbf{T} = \begin{pmatrix} \mathbf{R}(\alpha_k) & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha_k & -\sin\alpha_k & t_x \\ \sin\alpha_k & \cos\alpha_k & t_y \\ 0 & 0 & 1 \end{pmatrix},$$

where $\alpha_k = \frac{k\pi}{2}$ for some $k \in \mathbb{Z}$.

(b) Considering the geometric action of the transformation, it is clear that the rotation part must be either a rotation an angle $\alpha = 0$ or $\alpha = \pi$. Furthermore, if $\alpha = 0$ the translational part must be $\mathbf{t} = \mathbf{0}$, since $\mathbf{T}^2$ in this case will have translational part $2\mathbf{t}$. On the other hand, if $\alpha = \pi$, the translation will cancel itself out regardless of what $\mathbf{t}$ is.

This can of course also be verified algebraically, since

$$\mathbf{I} = \mathbf{T}^2 = \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{pmatrix}\begin{pmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{R}^2 & \mathbf{R}\mathbf{t} + \mathbf{t} \\ \mathbf{0}^\top & 1 \end{pmatrix}.$$

It follows that $\mathbf{R}^2 = \mathbf{I} \iff \mathbf{R} = \pm\mathbf{I}$ (since $\mathbf{R}$ is a rotation). If $\mathbf{R} = \mathbf{I}$, we see that the translational part becomes $\mathbf{R}\mathbf{t} + \mathbf{t} = 2\mathbf{t}$, so $\mathbf{t}$ must be zero. If $\mathbf{R} = -\mathbf{I}$, we see that the translational part becomes $\mathbf{R}\mathbf{t} + \mathbf{t} = \mathbf{0}$, irrespective of $\mathbf{t}$.

**Answer:** The only $\mathbf{T} \in \mathscr{G}$ which satisfy $\mathbf{T}^2 = \mathbf{I}$ are $\mathbf{T} = \mathbf{I}$ or

$$\mathbf{T} = \begin{pmatrix} -\mathbf{I} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{pmatrix}$$

for arbitrary translations $\mathbf{t}$.

(c) To show that $\mathscr{G}$ is a group with respect to composition, we need to show *closure*, *associativity*, *existence of identity*, and *existence of inverse*.

**Closure:** Let

$$\mathbf{T}_1 = \begin{pmatrix} \mathbf{R}(\alpha_k) & \mathbf{t}_1 \\ \mathbf{0}^\top & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{T}_2 = \begin{pmatrix} \mathbf{R}(\alpha_\ell) & \mathbf{t}_2 \\ \mathbf{0}^\top & 1 \end{pmatrix}$$

be two transformations in $\mathscr{G}$. We now verify that

$$\mathbf{T}_1\mathbf{T}_2 = \begin{pmatrix} \mathbf{R}(\alpha_k) & \mathbf{t}_1 \\ \mathbf{0}^\top & 1 \end{pmatrix}\begin{pmatrix} \mathbf{R}(\alpha_\ell) & \mathbf{t}_2 \\ \mathbf{0}^\top & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{R}(\alpha_k)\mathbf{R}(\alpha_\ell) & \mathbf{R}(\alpha_k)\mathbf{t}_2 + \mathbf{t}_1 \\ \mathbf{0}^\top & 1 \end{pmatrix} =$$
$$= \begin{pmatrix} \mathbf{R}(\alpha_k + \alpha_\ell) & \mathbf{t}_3 \\ \mathbf{0}^\top & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{R}(\alpha_{k+\ell}) & \mathbf{t}_3 \\ \mathbf{0}^\top & 1 \end{pmatrix} \in \mathscr{G},$$

since $\alpha_{k+\ell} = \frac{(k+\ell)\pi}{2}$ is an allowed angle.

**Associativity:** This follows from the associativity of matrix multiplication.

**Existence of identity:** The identity transformation $\mathbf{T}_{\text{id}} = \mathbf{I}$ is in $\mathscr{G}$ (it corresponds to $\alpha_0 = \frac{0\pi}{2} = 0$ and $\mathbf{t} = \mathbf{0}$).

**Existence of inverse:** For a general rigid transformation $\mathbf{T}_{\text{general}}$, we have

$$\mathbf{T}_{\text{general}} = \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{pmatrix} \iff \mathbf{T}_{\text{general}}^{-1} = \begin{pmatrix} \mathbf{R}^\top & -\mathbf{R}^\top\mathbf{t} \\ \mathbf{0}^\top & 1 \end{pmatrix}.$$

The same expression for the inverse will work for $\mathbf{T} \in \mathscr{G}$ too, since if $\alpha_k$ is an allowed angle for $\mathbf{R}$, the angle for $\mathbf{R}^\top$ will be $-\alpha_k = \alpha_{-k}$, which is also allowed.

**4.** (a) We rewrite the relation $\mathbf{x}_k \sim \mathbf{C}\mathbf{X}_k$ using the cross product, and obtain

$$\mathbf{x}_k \sim \mathbf{C}\mathbf{X}_k \iff \mathbf{x}_k \times \mathbf{C}\mathbf{X}_k = \mathbf{0} \iff [\mathbf{x}_k]_\times \mathbf{C}\mathbf{X}_k = \mathbf{0} \iff \left(\mathbf{X}_k^\top \otimes [\mathbf{x}_k]_\times\right) \operatorname{vec}\mathbf{C} = \mathbf{0}.$$

As usual, the cross product matrix makes the three rows linearly dependent, so that $\operatorname{rank}\left(\mathbf{X}_k^\top \otimes [\mathbf{x}_k]_\times\right) = 2$, and we can let $\mathbf{A}_k \in \mathbb{R}^{2\times 12}$ be the first two rows of this matrix. We can then form the data matrix

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_n \end{pmatrix} \in \mathbb{R}^{2n\times 12}.$$

This rank of this matrix is at most $r = \operatorname{rank}\mathbf{A} = \min(2n, 12)$. If $n < 6$ we are therefore guaranteed to have an infinite number of solutions (the null space will at least two-dimensional).

**Answer:** The smallest number of point correspondences needed to determine $\mathbf{C}$ uniquely (up to scale) is $n = 6$.

Note: For $n = 6$ the null space will in general be zero-dimensional (i.e. consist only of $\mathbf{0}$), so here we would most likely need the best one-dimensional *approximate null space*.

(b) **Answer:** We could sum the squares of the point-to-point distances between $\mathbf{x}_k$ and the projected point $\mathbf{C}\mathbf{X}_k$, as

$$\varepsilon_G(\mathbf{C}) = \sum_{k=1}^{n} \left\| \operatorname{norm}_P \mathbf{x}_k - \operatorname{norm}_P(\mathbf{C}\mathbf{X}_k) \right\|^2.$$

**5.** (a) The function $f_1$ is a scalar product since the matrix

$$\mathbf{G}_0 = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

is symmetric and positive definite.

The positive definiteness of $\mathbf{G}_0$ can easily be verified directly from the definition, as for $\mathbf{u} = (u_1, u_2, u_3) \neq \mathbf{0}$, we have

$$\begin{aligned} \mathbf{u}^\top \mathbf{G}_0 \mathbf{u} &= \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} \begin{pmatrix} 2u_1 + u_2 \\ u_1 + 2u_2 + u_3 \\ u_2 + 2u_3 \end{pmatrix} = \\ &= 2u_1^2 + u_1u_2 + u_1u_2 + 2u_2^2 + u_2u_3 + u_2u_3 + 2u_3^2 = \\ &= 2u_1^2 + 2u_1u_2 + 2u_2^2 + 2u_2u_3 + 2u_3^2 = \\ &= (u_1 + u_2)^2 + (u_2 + u_3)^2 + u_1^2 + u_3^2 > 0. \end{aligned}$$

(It is of course also possible to use Sylvester's criterion instead.)

Since $f_2(\mathbf{u}, \mathbf{u}) = f_3(\mathbf{u}, \mathbf{u}) = 0$ for *all* $\mathbf{u} \in \mathbb{R}^3$, both $f_2$ and $f_3$ fail the requirement that $\langle \mathbf{u} \mid \mathbf{u} \rangle > 0$ when $\mathbf{u} \neq \mathbf{0}$.

(b) The Gram matrix is defined as

$$\mathbf{G} = \begin{pmatrix} \langle \mathbf{b}_1 \mid \mathbf{b}_1 \rangle & \cdots & \langle \mathbf{b}_n \mid \mathbf{b}_1 \rangle \\ \vdots & \ddots & \vdots \\ \langle \mathbf{b}_1 \mid \mathbf{b}_n \rangle & \cdots & \langle \mathbf{b}_n \mid \mathbf{b}_n \rangle \end{pmatrix} = \begin{pmatrix} \mathbf{b}_1^\top \mathbf{G}_0 \mathbf{b}_1 & \cdots & \mathbf{b}_1^\top \mathbf{G}_0 \mathbf{b}_n \\ \vdots & \ddots & \vdots \\ \mathbf{b}_n^\top \mathbf{G}_0 \mathbf{b}_1 & \cdots & \mathbf{b}_n^\top \mathbf{G}_0 \mathbf{b}_n \end{pmatrix}.$$

If we let $\mathbf{B} = \begin{pmatrix} \mathbf{b}_1 & \dots & \mathbf{b}_n \end{pmatrix}$, we can write the above as $\mathbf{G} = \mathbf{B}^\top \mathbf{G}_0 \mathbf{B}$.
In this particular problem, since we use the standard basis $\mathbf{B} = \mathbf{I}$, the Gram matrix $\mathbf{G}$ will be the same as $\mathbf{G}_0$. Since $\mathbf{G}_0 \neq \mathbf{I}$, the basis is not orthonormal with respect to the chosen scalar product.

**Answer:** The Gram matrix is $\mathbf{G} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$, and we see that the basis is *not* orthonormal with respect to this scalar product.

(c) From the definition of the dual basis, we should have $\langle \tilde{\mathbf{b}}_i \mid \mathbf{b}_j \rangle = \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$
This gives us three linear equations for $\tilde{\mathbf{b}}_1 = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$, and these result in

$$\begin{cases} \langle \tilde{\mathbf{b}}_1 \mid \mathbf{b}_1 \rangle = 1 \\ \langle \tilde{\mathbf{b}}_1 \mid \mathbf{b}_2 \rangle = 0 \\ \langle \tilde{\mathbf{b}}_1 \mid \mathbf{b}_3 \rangle = 0 \end{cases} \iff \begin{cases} 2\tilde{b}_1 + \tilde{b}_2 \phantom{+ \tilde{b}_3} = 1 \\ \tilde{b}_1 + 2\tilde{b}_2 + \tilde{b}_3 = 0 \\ \phantom{\tilde{b}_1 +} \tilde{b}_2 + 2\tilde{b}_3 = 0 \end{cases} \iff \begin{cases} \tilde{b}_1 = \frac{3}{4} \\ \tilde{b}_2 = -\frac{1}{2} \\ \tilde{b}_3 = \frac{1}{4} \end{cases} \iff \tilde{\mathbf{b}}_1 = \frac{1}{4}\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}.$$

**Answer:** The first dual basis vector is $\tilde{\mathbf{b}}_1 = \frac{1}{4}(3, -2, 1)$.

**6.** (a) **Answer:** Since $\mathbf{G}_0 = \mathbf{I}$, the frame operator is given by

$$\mathbf{F} = \mathbf{B}\mathbf{B}^\top = \begin{pmatrix} 1 & -1 & 2 & 1 \\ 1 & 1 & 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}.$$

(b) By definition, the lower frame bound $L$ and upper frame bound $U$ are defined as the largest $L > 0$ and smallest $U \geq L$ such that for every $\mathbf{v}$

$$0 < L\,||\mathbf{v}||^2 \leq ||\mathbf{B}^\top \mathbf{G}_0 \mathbf{v}||_F^2 \leq U\,||\mathbf{v}||^2 \iff 0 < L\,||\mathbf{v}||^2 \leq \mathbf{v}^\top \mathbf{G}_0 \mathbf{F} \mathbf{v} \leq U\,||\mathbf{v}||^2.$$

Since in this case $\mathbf{F}\mathbf{v} = 7\mathbf{v}$, the frame condition is

$$0 < L\,||\mathbf{v}||^2 \leq 7\underbrace{\mathbf{v}^\top \mathbf{G}_0 \mathbf{v}}_{\langle \mathbf{v} | \mathbf{v} \rangle = ||\mathbf{v}||^2} \leq U\,||\mathbf{v}||^2,$$

so we see that $L = U = 7$.

**Answer:** The lower and upper frame bounds are $L = U = 7$.

(c) **Answer:** Since the upper and lower frame bounds are equal, this means that the frame is tight.

**7.** (a) Replacing all occurrences of $H_0$, $G_0$, and $G_1$ in (FB2) with their definitions in terms of $H_1$ gives

$$\begin{aligned}
&H_1(u)H_0(u+\pi) + G_1(u)G_1(u+\pi) = \\
&= H_1(u)H_1^*(u+\pi) + e^{-iu}H_1^*(u+\pi)e^{i(u+\pi)}H_1(u+\pi+\pi) = \\
&= H_1(u)H_1^*(u+\pi) + e^{i\pi}H_1^*(u+\pi)H_1(u+2\pi) = \\
&= H_1(u)H_1^*(u+\pi) - H_1^*(u+\pi)H_1(u) = 0,
\end{aligned}$$

since the discrete-time Fourier transform is periodic with a period $2\pi$.

(b) Replacing all occurrences of $H_0$, $G_0$, and $G_1$ in (FB2) with their definitions in terms of $H_1$ gives

$$\begin{aligned}
&H_1(u)H_0(u) + G_1(u)G_0(u) = 2 \iff \\
\iff\ &H_1(u)H_1^*(u) + e^{-iu}H_1^*(u+\pi)e^{iu}H_1(u+\pi) = 2 \iff \\
\iff\ &\left|H_1(u)\right|^2 + \left|H_1(u+\pi)\right|^2 = 2.
\end{aligned}$$

**Answer:** The condition (FB1) can be written as $\left|H_1(u)\right|^2 + \left|H_1(u+\pi)\right|^2 = 2$.

**8.** (a) The epipolar constraint is $\mathbf{x}_k^\top \mathbf{F}\mathbf{x}'_k = 0$, and direct verification yields

$$\mathbf{x}_1^\top \mathbf{F}\mathbf{x}'_1 = -6, \qquad \mathbf{x}_2^\top \mathbf{F}\mathbf{x}'_2 = -4, \qquad \mathbf{x}_3^\top \mathbf{F}\mathbf{x}'_3 = -4,$$

which means that none of the three pairs of points satisfy the epipolar constraint.

**Answer:** None of the pairs of points satisfy the epipolar constraint.

(b) If $\mathbf{F}$ maps points in the second view to epipolar lines in the first view, $\mathbf{F}^\top$ maps in the opposite direction, and we obtain $\mathbf{l}' = \mathbf{F}^\top\mathbf{x}_1 = (-3, 0, 3) \sim (-1, 0, 1)$.

**Answer:** The sought epipolar line is $\mathbf{l}' \sim (-1, 0, 1)$.

(c) The epipoles satisfy $\mathbf{F}^\top\mathbf{e} = \mathbf{0}$ and $\mathbf{F}\mathbf{e}' = \mathbf{0}$, i.e. they are found as the left and right null spaces of $\mathbf{F}$. With the particular form in this problem, $\mathbf{F} = [(1, 2, 1)]_\times$, it is clear that the epipoles are $\mathbf{e} = \mathbf{e}' = (1, 2, 1)$.

**Answer:** The epipoles are $\mathbf{e} = \mathbf{e}' = (1, 2, 1)$.