Information page for written examinations at Linköping University

Examination date	2017-10-17
Room (2)	G32(34) G34(11)
Time	14-18
Course code	TSBB06
Exam code	KTR1
Course name **Exam name**	Multidimensional Signal Analysis (Multidimensionell signalanalys) - (Frivillig kontrollskrivning)
Department	ISY
Number of questions in the examination	20
Teacher responsible/contact person during the exam time	Klas Nordberg
Contact number during the exam time	013-281634
Visit to the examination room approximately	16 pm
Name and contact details to the course administrator (name + phone nr + mail)	Carina Lindström 013-284423 carina.e.lindstrom@liu.se
Equipment permitted	Calculator
Other important information	Use cross-ruled paper
Number of exams in the bag	

Guide

The written examination consists of 4 parts, one part for each of the four course aims in the curriculum.

- Part I: Geometry
- Part II: Estimation
- Part III: Linear signal representation
- Part IV: Signal processing applications

Each part consists of 3 exercises where the student should demonstrate ability to explain concepts, phenomena, etc (type A exercises), and 2 additional exercises that test a deeper understanding of various topics in the course, for example, in terms of more detailed explanations or simpler calculations (type B exercises).

Type A exercises give at most 1 point each. Type B exercises give at most 2 points each.

To pass with grade 3: two parts must have at least 3p and two parts must have at least 4p, and there must be 2 B-type exercises passed with full 2p.

To pass with grade 4: two parts must have at least 4p and two parts must have at least 5p, and there must be 4 B-type exercises passed with full 2p.

To pass with grade 5: all parts must have at least 5p, and there must be 6 B-type exercises passed with full 2p.

The answers to the A-exercises should preferably be given in the blank spaces of this examination thesis, below the questions. Use additional sheets if necessary, with no more than one exercise per sheet

Write your anonymous examination ID (AID) at the top of the pages in this examination thesis and any sheet appended to the examination thesis.

Good luck!
Klas Nordberg

AID code:

PART I: GEOMETRY

Exercise 1 (A, 1p) Two point are represented by their homogeneous coordinates: $\mathbf{y}_1 = (3, -1, 2)$ and $\mathbf{y}_2 = (-3, 2, -1)$, respectively. What is the Euclidean distance between the two points?

Exercise 2 (A, 1p) A 2D homography is a mapping in 3D space, from a plane to another plane. Explain how this mapping is made. It may help to draw an illustration. Explain all geometric objects that you introduce.

Exercise 3 (A, 1p) A 3D rotation is specified by a rotation axis $\hat{\mathbf{n}} = (0.6, 0.8, 0)$ and a rotation angle $\alpha = \frac{\pi}{2}$. This rotation is represented by two distinct unit quaternions, $\mathbf{q}_1$ and $\mathbf{q}_2$. Specify $\mathbf{q}_1$ and $\mathbf{q}_2$ in this case.

Exercise 4 (B, 2p) The homogeneous coordinates of three 3D points are given as the vectors $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \in \mathbb{R}^4$. In general, there is a unique plane that intersects all three points. Describe a method that from $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ determines $\mathbf{p}$, the dual homogeneous coordinates of the plane.

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 5 (B, 2p) A 2D point has homogeneous coordinates $\mathbf{y} = (u, v, 0)$; it lies at infinity. This point is subject to a rigid transformation, represented by a 3×3 matrix that is applied to $\mathbf{y}$. Derive and describe the result of this transformation: what point is it, and does it still lie at infinity (1p)? How does the translation part of the transformation affect the point $\mathbf{y}$? Why does this result make sense (1p)?

WRITE YOUR ANSWER ON A SEPARATE SHEET

PART II: ESTIMATION

Exercise 6 (A, 1p) Given a set of 2D points, how is Hartley-normalization done on this data set?

Exercise 7 (A, 1p) In general, algebraic errors lack intuitive geometric interpretations. Despite this, algebraic errors are often used for model estimation. Why?

Exercise 8 (A, 1p) A model is estimated by minimizing an algebraic error, using the homogeneous method. This provides additional information in the form of the SVD profile of the data matrix. Describe how you can use this information to characterize the data from which the model is estimated.

Exercise 9 (B, 2p) The point $\mathbf{y}_k$ lies in one image, and the corresponding point $\mathbf{y}'_k$ lies in another image, $k = 1, \ldots, m$. The two images are related by a homography: $\mathbf{y}'_k \sim \mathbf{H}\,\mathbf{y}_k$. Formulate an error function ϵ, based on *geometric errors*, for the estimation of $\mathbf{H}$ from these points. The function ϵ should be *symmetric*, in the sense that it measures the geometric errors in both images.

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 10 (B, 2p) The same data as in the previous exercise can be used to estimate $\mathbf{H}$ by minimizing an algebraic error (not necessarily a symmetric one). Describe the principal steps that lead to the formulation of the algebraic error, and suggest a method for minimization of the error.

WRITE YOUR ANSWER ON A SEPARATE SHEET

PART III: LINEAR SIGNAL REPRESENTATIONS

Exercise 11 (A, 1p) A vector space has a scalar product $\langle \cdot \mid \cdot \rangle$, and a basis in the form of vectors $\mathbf{b}_l$. How are the corresponding dual basis vectors $\tilde{\mathbf{b}}_k$ related to the basis vectors $\mathbf{b}_l$?

Exercise 12 (A, 1p) $\mathbf{B}$ contains a set of frame vectors in its columns, and correspondingly for the dual frame vectors in $\tilde{\mathbf{B}}$. This means that $\mathbf{v} = \mathbf{B}\,\mathbf{c}$ is solved by $\mathbf{c} = \tilde{\mathbf{B}}^\star \mathbf{v}$ but, in general, there are additional solutions of the type $\mathbf{c} + \mathbf{c}'$. How is $\mathbf{c}$ related to $\mathbf{c}'$ in this case?

Exercise 13 (A, 1p) Two discrete functions are denoted as f and g. Set $h = f * g$. The convolution result at point k, $h[k]$, can be written as a scalar product between f and a function g_0 that depends on g. Express the function g_0 in terms of g.

Exercise 14 (B, 2p) $\mathbf{B}$ is a matrix that holds basis vectors of some subspace in its columns. $\tilde{\mathbf{B}}$ is the matrix for the corresponding dual basis. This implies that $\tilde{\mathbf{B}}^\star \mathbf{x}$ may give the coordinates of vector $\mathbf{x}$, but this is not true in the general case. When is it true, and what does $\tilde{\mathbf{B}}^\star \mathbf{x}$ represent in the general case?

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Exercise 15 (B, 2p) Let $\mathbf{F}$ be the frame operator of some set of frame vectors $\mathbf{b}_k \in V, k = 1, \ldots, N$. $\mathbf{F}$ is a *self-adjoint operator*. Describe how the corresponding dual frame vectors are computed, and show that it is possible to reconstruct any $\mathbf{v} \in V$ from the scalar products between $\mathbf{v}$ and the frame vectors used in a linear combination with the dual frame vectors (or vice versa).

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PART IV: SIGNAL PROCESSING APPLICATIONS

Exercise 16 (A, 1p) A signal s is sampled, which introduces some amount of noise. The signal s is then reconstructed from the samples, resulting in s'. The noise in s' can be reduced if s is over-sampled and certain assumptions about the sampling noise are valid. What assumptions?

Exercise 17 (A, 1p) In normalized convolution, each filter f_i is interpreted as the combination of a basis function b_i and the applicability function a. How is $f_i[k]$ expressed in terms of $b_i[k]$ and $a[k]$?

Exercise 18 (A, 1p) In filter optimization, we can use a spatial mask to set certain filter coefficients to zero and optimize the remaining coefficients. What are the main advantage and disadvantage of this approach compared to optimizing the filter without the spatial mask?

Exercise 19 (B, 2p) Which problem do you solve when you apply *principal component analysis*? Describe what is known and be specific about what you seek (1p). Describe also how this problem is solved (1p).

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 20 (B, 2p) In an orthogonal two-channel filter bank, one of the two reconstructing filters must satisfy a certain condition. What condition? Describe an example of such a filter, with non-zero coefficients, and demonstrate that it satisfy this condition.

WRITE YOUR ANSWER ON A SEPARATE SHEET