Information page for written examinations at Linköping University

Examination date	2018-04-04
Room (1)	TER3(26)
Time	8-12
Course code	TSBB06
Exam code	TEN2
Course name **Exam name**	Multidimensional Signal Analysis (Multidimensionell signalanalys) Written Examination (Skriftlig tentamen)
Department	ISY
Number of questions in the examination	20
Teacher responsible/contact person during the exam time	Klas Nordberg
Contact number during the exam time	013-281634
Visit to the examination room approximately	10 am
Name and contact details to the course administrator (name + phone nr + mail)	Carina Lindström 013-284423 carina.e.lindstrom@liu.se
Equipment permitted	Calculator
Other important information	Use cross-ruled paper
Number of exams in the bag	

Guide

The written examination consists of 4 parts, one part for each of the four course aims in the curriculum.

- Part I: Geometry
- Part II: Estimation
- Part III: Linear signal representation
- Part IV: Signal processing applications

Each part consists of 3 exercises where the student should demonstrate ability to explain concepts, phenomena, etc (type A exercises), and 2 additional exercises that test a deeper understanding of various topics in the course, for example, in terms of more detailed explanations or simpler calculations (type B exercises).

Type A exercises give at most 1 point each. Type B exercises give at most 2 points each.

To pass with grade 3: two parts must have at least 3p and two parts must have at least 4p, and there must be 2 B-type exercises passed with full 2p.

To pass with grade 4: two parts must have at least 4p and two parts must have at least 5p, and there must be 4 B-type exercises passed with full 2p.

To pass with grade 5: all parts must have at least 5p, and there must be 6 B-type exercises passed with full 2p.

The answers to the A-exercises should preferably be given in the blank spaces of this examination thesis, below the questions. Use additional sheets if necessary, with no more than one exercise per sheet

Write your anonymous examination ID (AID) at the top of the pages in this examination thesis and any sheet appended to the examination thesis.

Good luck!
Klas Nordberg

AID code:

PART I: GEOMETRY

Exercise 1 (A, 1p) What is the distance between the 3D point that has homogeneous coordinates $\mathbf{x} = (1, -3, 1, 2)$ and the plane that has dual homogeneous coordinates $\mathbf{p} = (1, 2, 2, 1)$?

Exercise 2 (A, 1p) Both affine transformations and homographies map lines to lines in the 2D case. What is the difference between affine transformations and homographies regarding the mapping of lines?

Exercise 3 (A, 1p) A 3D point $\bar{\mathbf{x}} = (x_1, x_2, x_3)$ is subject to a rotation that is represented by the unit quaternion q. What is the quaternionic representation of the resulting 3D point?

Exercise 4 (B, 2p) $\mathbf{y} = (a, b, 0)$ and $\mathbf{l} = (0, 0, 1)$ cannot be properly normalized to represent the homogeneous coordinates of a 2D point, or the dual homogeneous coordinates of a 2D line, respectively. Motivate why it makes sense to have them as representing a point and line at infinity, respectively.

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 5 (B, 2p) Define what we mean by the Plücker coordinates of a 3D line that passes trough two points (1p). Show that this representation of the line does not depend on the choice of points, as long as they are distinct and lie on the same line (1p).

WRITE YOUR ANSWER ON A SEPARATE SHEET

AID code:

PART II: ESTIMATION

Exercise 6 (A, 1p) The homogeneous equation $\mathbf{A}\,\mathbf{x} = \mathbf{0}$ has approximately the same solution regardless of whether we use the homogeneous or the inhomogeneous method for computing it. What is the advantage of using the homogeneous method?

Exercise 7 (A, 1p) How can you formulate an algebraic error of a 2D homography transformation $\mathbf{H}$ relative to a point $\mathbf{y}$ before and $\mathbf{y}'$ after the transformation? Here, $\mathbf{y}$ and $\mathbf{y}'$ represent homogeneous coordinates.

Exercise 8 (A, 1p) What is the problem that Hartley-normalization tries to solve?

Exercise 9 (B, 2p) Two sets of 3D points are related by a rigid transformation. Assuming that we know correspondences between points in the two sets how can we estimate the rigid transformation?

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 10 (B, 2p) The relation $\mathbf{y} \sim \mathbf{C}\,\mathbf{x}$, between a 3D point $\mathbf{x}$ and its projection through a pinhole camera $\mathbf{C}$ onto an image point $\mathbf{y}$, corresponds to a set of homogeneous equations in the element of $\mathbf{C}$. Explain why we only get two linearly independent equations and not three.

WRITE YOUR ANSWER ON A SEPARATE SHEET

PART III: LINEAR SIGNAL REPRESENTATIONS

Exercise 11 (A, 1p) An n-dimensional vector space V has a basis $\mathbf{b}_k, k = 1, \ldots, n$. Given a vector $\mathbf{v} \in V$, how do you compute the coordinates of $\mathbf{v}$ relative to $\mathbf{b}$? Explain the concepts that you introduce in your solution.

Exercise 12 (A, 1p) $\mathbf{B}$ contains a set of frame vectors in its columns, and correspondingly for the dual frame vectors in $\tilde{\mathbf{B}}$. This means that $\mathbf{v} = \mathbf{B}\,\mathbf{c}$ is solved by $\mathbf{c} = \tilde{\mathbf{B}}^{\star}\mathbf{v}$ but, in general, there are additional solutions of the type $\mathbf{c} + \mathbf{c}'$. How is $\mathbf{c}$ related to $\mathbf{c}'$ in this case?

Exercise 13 (A, 1p) Set $h = f * g$. The convolution result at point k, $h[k]$, can be written as a scalar product between f and a function that depends on g. Express the latter function in terms of g.

Exercise 14 (B, 2p) $\mathbf{B}$ is a matrix that holds basis vectors of some subspace in its columns. $\tilde{\mathbf{B}}$ is the matrix for the corresponding dual basis. This suggests that $\tilde{\mathbf{B}}^{\star}\mathbf{x}$ gives the coordinates of vector $\mathbf{x}$, but this is not true in the general case. When is it true and what does $\tilde{\mathbf{B}}^{\star}\mathbf{x}$ represent in the general case?

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 15 (B, 2p) The discrete Fourier transform maps a set of N samples $f[k]$ in the time domain to N samples $F[m]$ in the frequency domain, and it is defined according to the equation below. This expression suggests that the set of all $F[m]$ can be seen as the coordinates of the sequence $f[k]$ relative to some set of basis functions. Describe these basis functions, and describe the scalar product is used for the functions $f[k]$.

$$F[m] = \sum_{k=0}^{N-1} f[k] e^{-i2\pi km/N}$$

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PART IV: SIGNAL PROCESSING APPLICATIONS

Exercise 16 (A, 1p) The noise in the reconstructed signal can be reduced if the original signal is over-sampled and certain assumptions about the sampling noise are valid. What assumptions?

Exercise 17 (A, 1p) In normalized convolution, each filter f_i is interpreted as the combination of a basis function b_i and the applicability function a. How is $f_i[k]$ expressed in terms of $b_i[k]$ and $a[k]$?

Exercise 18 (A, 1p) In wavelet theory a function $\phi(t)$ is a scaling function if it satisfies two conditions. What are these two conditions?

Exercise 19 (B, 2p) In filter design, the number of coefficients in the filter is an important choice, subject to a trade-off. Why is it in general desirable to optimize a filter with many non-zero coefficients? Why is it in general desirable to have has few non-zero coefficients in the optimization, e.g., using the spatial mask?

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 20 (B, 2p) A large set of signals $\mathbf{v} \in \mathbb{R}^n$ are stored with only $m < n$ coefficients per $\mathbf{v}$. Describe how to do this so that ϵ, the mean square of the norm of the reconstruction error, is minimal. What coefficients are stored for each $\mathbf{v}$ and how can $\mathbf{v}$ be reconstructed from these coefficients? How large is ϵ in this case?

WRITE YOUR ANSWER ON A SEPARATE SHEET