TSBB06 TEN2 Instructions

Welcome to TEN2, the final exam in TSBB06!

The written examination in TSBB06 has 2 parts: One part for the course content in HT1 and one for the content from HT2.

There are 10 problems in part I (corresponding to the midterm, KTR1), and these are grouped under the two headings Geometry, and Estimation. There are another 10 problems in part II, grouped under Linear Signal Representations, and Signal Processing Applications.

The exam problems are of two types: **Type A** are 1 point problems, and **Type B** are 2 point problems. Half points can be given.

If you obtained 8p or more on KTR1 you are allowed to use that result on the final exam. This is done by not handing in answers to **any** of the problems in part I of the exam. Your score from KTR1 will then automatically be added to the final exam score.

The maximal score on the final exam is 28p, and grading of the exam is done based on your score as follows:

If you obtain **15p** or more you get **grade 3**.
If you obtain **18p** or more you get **grade 4**.
If you obtain **22p** or more you get **grade 5**.

All tasks should be answered on **separate sheets**. It is fine to answer multiple questions on one sheet.

Write your AID-number and the date on all paper sheets that you hand in. In addition, these sheets should be numbered in consecutive order.

Good luck!
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PART I, Geometry

Problem 1 (A, 1p) A 3D line can be represented by its Plücker coordinates $\mathbf{L}$. In this particular case the line can be factorized as $\mathbf{L} = \mathbf{x}_1\mathbf{x}_2^\top - \mathbf{x}_2\mathbf{x}_1^\top$, where $\mathbf{x}_1$ and $\mathbf{x}_2$ are column vectors. Give an expression for an arbitrary point on this line, and state whether your expression uses Euclidean or homogeneous coordinates.

Problem 2 (A, 1p) A 2D line has dual homogeneous coordinates $(3, 4, 5)$. Draw this line in a Euclidean coordinate system and specify its orientation and distance to the origin.

Problem 3 (A, 1p) Assume you have a rotation matrix $\mathbf{R} \in \mathrm{SO}(3)$. How can you determine the axis of rotation?

Problem 4 (B, 2p) Two 2D lines $\mathbf{l}_1$, $\mathbf{l}_2$ are parallel. Explain why a dual affine 2D transformation preserves this property.

Problem 5 (B, 2p) Instead of representing a rotation by a matrix $\mathbf{R}$, one can use a unit quaternion, $\mathbf{q} = (s, \mathbf{v})$, where $s \in \mathbb{R}$, $\mathbf{v} \in \mathbb{R}^3$, and $\|\mathbf{q}\| = \sqrt{s^2 + \mathbf{v}^T\mathbf{v}} = 1$. Two rotations in unit quaternion form can be combined by

$$\mathbf{q}_1 \circ \mathbf{q}_2 = (s_1 s_2 - \mathbf{v}_1^T\mathbf{v}_2, s_1\mathbf{v}_2 + s_2\mathbf{v}_1 + \mathbf{v}_1 \times \mathbf{v}_2)\,.$$

Explain why it follows from this expression that 3D rotation is a non-commutative operation in general (1p), and also give a condition for when two rotations commute (1p).

PART I, Estimation

Problem 6 (A, 1p) Assume you have a set of noisy, but corresponding 3D points $\{\mathbf{x}_k \leftrightarrow \mathbf{y}_k\}_1^K$ before and after a rigid transformation $\mathbf{y} = \mathbf{R}\mathbf{x} + \mathbf{d}$. You now want to estimate the rotation $\mathbf{R}$, from the point sets $\{\mathbf{x}_k\}_1^K$, $\{\mathbf{y}_k\}_1^K$, using the orthogonal Procrustes algorithm. Before you can do this however, the point sets, should be be modified. Explain how!

Problem 7 (A, 1p) Why, and when, should we apply normalization, e.g., the Hartley normalization, to data before a model is estimated?

Problem 8 (A, 1p) Explain how to rewrite a homogeneous equation system $\mathbf{A}\mathbf{z} = \mathbf{0}$ as an *inhomogeneous* equation system, and state the extra assumption that you need to make.

Problem 9 (B, 2p) You have access to two sets of corresponding points, $\{\mathbf{y}_k\}_1^K$ in one image, and $\{\mathbf{y}'_k\}_1^K$ in another image. The two images are related by a homography, and thus we can expect that: $\mathbf{y}'_k \sim \mathbf{H}\mathbf{y}_k$ will hold approximately.

Formulate an error function ϵ, based on *geometric errors*, for the estimation of $\mathbf{H}$ from the correspondences. The function ϵ should be *symmetric*, in the sense that it measures the geometric errors in both images.

Problem 10 (B, 2p) The relation $\mathbf{y} \sim \mathbf{C}\mathbf{x}$, between a 3D point $\mathbf{x}$ and its projection through a pinhole camera $\mathbf{C}$ onto an image point $\mathbf{y}$, corresponds to a set of homogeneous equations in the elements of $\mathbf{C}$. Explain why we only get two linearly independent equations and not three.

PART II, Linear Signal Representations

Problem 11 (A, 1p) Let $\mathbf{F}$ be a matrix that contains a set of frame vectors in its columns, and let $\tilde{\mathbf{F}}$ be the corresponding matrix for the dual frame. How are the two matrices related, assuming that the scalar product is defined by $\mathbf{G}_0 = \mathbf{I}$?

Problem 12 (A, 1p) Let $\mathbf{F}$ be a matrix that contains a set of frame vectors in its columns. Let $\mathbf{c}$ be a set of coefficients that can reconstruct a vector $\mathbf{v}$ as a linear combination of the frame vectors. Describe the full set of reconstructing coefficients of $\mathbf{v}$ relative to this frame.

Problem 13 (A, 1p) How is a Gram matrix computed from a basis $\mathbf{B} = (\mathbf{b}_1\ \mathbf{b}_2\ , \ldots , \mathbf{b}_N)$?

Problem 14 (B, 2p) Consider the following two functions $f, g : \mathbb{R}^N \times \mathbb{R}^N \to \mathbb{R}$.

$$f(\mathbf{a}, \mathbf{b}) = \mathbf{a}^T \mathbf{L} \mathbf{b} \quad \text{and} \quad g(\mathbf{a}, \mathbf{b}) = \mathbf{a}^T \mathbf{M} \mathbf{b}\,.$$

Here $\mathbf{L}$ and $\mathbf{M}$ are matrices with elements

$$l_{ij} = \cos(i/4) + \sin(j/4) \quad \text{and } m_{ij} = \begin{cases} \exp(-i^2/4) & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$

Which of f, g are valid scalar products, and why?

Problem 15 (B, 2p) The discrete Fourier transform maps a set of N samples $f[k]$ in the time domain to N samples $F[l]$ in the frequency domain, and it is defined according to the equation below.

$$F[l] = \sum_{k=0}^{N-1} f[k]\, e^{-i2\pi kl/N}$$

This expression suggests that the set of all $F[l]$, $l \in [0, N-1]$ are the coordinates of the sequence $f[k]$, $k \in [0, N-1]$, relative to some set of basis functions. What basis functions are used, and what scalar product is used?

PART II, Signal Processing Applications

Problem 16 (A, 1p) A continuous-time signal is reconstructed from a sampled and quantized version of the signal. The noise in the reconstructed signal can be reduced if the original signal is oversampled and certain assumptions are valid. Describe the two assumptions about the sampling noise that we used in the course.

Problem 17 (A, 1p) The epipolar constraint can formulated as $\mathbf{u}^T\mathbf{F}\mathbf{v} = 0$, for the image coordinates, $\mathbf{u} = (u_1, u_2, 1)^T$, $\mathbf{v} = (v_1, v_2, 1)^T$. $\mathbf{F}$ has 9 elements, but fewer degrees of freedom. State how many, and motivate why it is less than 9.

Problem 18 (A, 1p) What problem do you solve when you apply *principal component analysis*? Describe what is known and be specific about what you seek.

Problem 19 (B, 2p) In normalized convolution it is common to use a polynomial basis. Consider the 1D case, with an applicability $a(x) = \exp(-x^2/4)$, and the basis $\{b_1(x), b_2(x), b_3(x)\} = \{1, x, x^2\}$. Also assume full certainty, i.e. $c(x) = 1$. Now you can obtain coordinates in the basis by either: (A) convolving the input $f(x)$ with a set of fixed filters, and correcting the coordinates, or (B) by convolving the input with a set of filters that have been computed beforehand. Give the steps required for determining the coordinates using either of these approaches. You may assume discrete filters with $x \in [-4, 4]$.

Problem 20 (B, 2p) The discrete wavelet transform can be implemented in terms of an iterated filter bank. Describe how this is done, and for which values of the scale and translation parameters, a and b, the transform is computed, given that the filter bank has N levels.