TSBB06 TEN2 Instructions

Welcome to TEN2, the final exam in TSBB06!

The written examination in TSBB06 has 2 parts: One part for the course content in HT1 and one for the content from HT2.

There are 10 problems in part I (corresponding to the midterm, KTR1), and these are grouped under the two headings Geometry, and Estimation. There are another 10 problems in part II, grouped under Linear Signal Representations, and Signal Processing Applications.

The exam problems are of two types: **Type A** are 1 point problems, and **Type B** are 2 point problems. Half points can be awarded.

If you obtained 8p or more on KTR1 you are allowed to use that result on the final exam. This is done by not handing in answers to **any** of the problems in part I of the exam. Your score from KTR1 will then automatically be added to the final exam score.

The maximal score on the final exam is 28p, and grading of the exam is done based on your score as follows:

If you obtain **15p** or more you get **grade 3**.
If you obtain **18p** or more you get **grade 4**.
If you obtain **22p** or more you get **grade 5**.

All tasks should be answered on **separate sheets** (i.e. not on the printed exam). It is fine to answer multiple questions on one sheet.

Write your AID-number and the date on all paper sheets that you hand in. In addition, these sheets should be numbered in consecutive order.

Good luck!
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PART I, Geometry

Problem 1 (A, 1p) Consider two points in the extended Euclidean plane, with homogeneous coordinates given by $\mathbf{x}_1 = (0, 1, 1)$ and $\mathbf{x}_2 = (1, -2, 1)$, and let $\mathbf{l}$ be the line through $\mathbf{x}_1$ and $\mathbf{x}_2$.

Determine the *ideal point* (the point at infinity) on $\mathbf{l}$.

Problem 2 (A, 1p) Let $\mathbf{L}$ be Plücker coordinates for a line in the extended Euclidean space, and let $\mathbf{p}$ be dual homogeneous coordinates of a plane.

What is the point of of intersection? How can we tell whether the plane contains the whole line?

Problem 3 (A, 1p) Let $\mathbf{H} \in \mathbb{R}^{3\times 3}$ be a non-singular matrix representing a planar homography, and let $\mathbf{l}$ be dual homogeneous coordinates of a line ℓ.

If we apply $\mathbf{H}$ to all points in the extended Euclidean plane, what are the new dual homogeneous coordinates of ℓ?

Problem 4 (B, 2p) Recall that a *group* is a set S equipped with an operation $\star$ satisfying four conditions, namely *closure* (i.e. $a, b \in S \implies a \star b \in S$), *associativity*, *existence of neutral element* (*identity*), and *existence of inverse for each element* (the inverse must of course lie in S).

Show that affine transformations form a group with respect to composition.

Problem 5 (B, 2p) Let $\mathbf{p}_1, \ldots, \mathbf{p}_n \in \mathbb{R}^4$ be dual homogeneous coordinates for $n \geq 4$ planes in the extended Euclidean space. The intersection of the planes is in general empty, but can in special cases be non-empty.

(a) What are the possible objects representing the intersection? (1p)

(b) Describe a practical (numerical) method for deciding what kind of object the intersection is. (1p)

PART I, Estimation

Problem 6 (A, 1p) What is the purpose of *Hartley normalisation*, and what characterises a set of points $\{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ that is Hartley normalised?

Problem 7 (A, 1p) Suppose $\mathbf{x}_1, \dots, \mathbf{x}_n \in \mathbb{R}^3$ are homogeneous coordinates of points in the plane. Let $f(\mathbf{l})$ be the error function obtained as the sum of all squared perpendicular distances from $\mathbf{x}_1, \dots, \mathbf{x}_n$ to the line with dual homogeneous coordinates $\mathbf{l}$.

Write down an explicit formula for $f(\mathbf{l})$.

Problem 8 (A, 1p) Many geometric estimation problems (e.g. homography estimation) are solved by minimising either an *algebraic error* or a *geometric error*.

Describe advantages and disadvantages for each of these error measures.

Problem 9 (B, 2p) Suppose two sets of 3D points are related through a rigid transformation. Assuming that we know the point correspondences between the two sets, describe a method for computing the rigid transformation.

Problem 10 (B, 2p) One way of finding a planar homography which (approximately) satisfies $\hat{\mathbf{x}}_k \sim \mathbf{H}\mathbf{x}_k$ for $k = 1, \dots, n$ is to use the *Direct Linear Transformation* (DLT).

(a) What does the data matrix look like for the DLT? (1p)
(b) What is the largest rank the data matrix can have (in terms of n)? (1p)

PART II, Linear Signal Representations

Problem 11 (A, 1p) Let $\langle \mathbf{u} \mid \mathbf{v} \rangle = \mathbf{v}^\top \mathbf{G}_0 \mathbf{u}$ define the scalar product in $\mathbb{R}^n$, and let $\mathbf{b}_1, \ldots, \mathbf{b}_n$ be a basis for $\mathbb{R}^n$.

Define the *dual basis* to the basis $\mathbf{b}_1, \ldots, \mathbf{b}_n$.

Problem 12 (A, 1p) Let $\langle \mathbf{u} \mid \mathbf{v} \rangle = \mathbf{v}^\top \mathbf{G}_0 \mathbf{u}$ define the scalar product in $\mathbb{R}^n$, and let $\mathbf{b}_1, \ldots, \mathbf{b}_r$ be a basis for some r-dimensional subspace U of $\mathbb{R}^n$.

For a given vector $\mathbf{v} \in \mathbb{R}^n$, what is the closest vector $\mathbf{u} \in U$?

Problem 13 (A, 1p) Suppose $\mathbf{B}$ is a matrix that contains a set of frame vectors as its columns, and correspondingly for the dual frame vectors in $\tilde{\mathbf{B}}$.

Describe the set of reconstructing coefficients for $\mathbf{v}$, i.e. the set of coefficients $\mathbf{c}$ for which $\mathbf{v} = \mathbf{B}\mathbf{c}$.

Problem 14 (B, 2p) Let $\mathbf{B}$ be a matrix whose columns span $\mathbb{R}^n$ (with the usual scalar product, i.e. $\mathbf{G}_0 = \mathbf{I}$).

(a) What is the *frame condition* (in terms of either $\mathbf{B}$ or its columns)? (1p)

(b) How can the *frame operator* $\mathbf{F}$ be expressed in terms of $\mathbf{B}$? (1p)

Problem 15 (B, 2p) *Normalised convolution* can be used to compute a local expansion of a signal in terms of some basis. Assume that we use a monomial basis (as we did in the lab).

(a) What interpretation can be made of the corresponding coordinates? (1p)

(b) How does the Gramian at signal position k, i.e. $\mathbf{G}_0[k]$, depend on the *applicability function* and the *signal certainty*? (1p)

PART II, Signal Processing Applications

Problem 16 (A, 1p) In wavelet theory, a function $\phi(t)$ is a scaling function if it satisfies two conditions. Which are these two conditions?

Problem 17 (A, 1p) A continuous-time signal is reconstructed from a sampled and quantized version of the signal. The noise in the reconstructed signal can be reduced if the original signal is oversampled and certain assumptions are valid. Describe the two assumptions about the sampling noise that we used in the course.

Problem 18 (A, 1p) The continuous wavelet transform $W_f(a, b)$ of a signal f is defined in terms of a *mother wavelet* $\psi(t)$. What criterion does $\psi(t)$ need to satisfy in order to make the transform invertible?

Problem 19 (B, 2p) In *Principal Component Analysis* (PCA) we determine a basis for a certain (often low-dimensional) subspace of the whole signal space.

(a) This is done in a way such that a certain error is minimised. How is this error defined? (1p)

(b) How do we find the basis in practice? (1p)

Problem 20 (B, 2p) Suppose you are given two camera projection matrices $\mathbf{C}$ and $\hat{\mathbf{C}}$ (with distinct camera centres).

(a) What are the *epipoles*, and how can you compute them? (1p)

(b) Let $\mathbf{x}$ and $\hat{\mathbf{x}}$ be hoomogeneous coordinates points in the two views, corresponding to the same 3D point $\mathbf{X}$. How can we (linearly) solve for $\mathbf{X}$ if we know $\mathbf{x}$ and $\hat{\mathbf{x}}$? (1p)