TSBB06 KTR1 Instructions

Welcome to KTR1, the voluntary midterm exam for TSBB06!

The written examination in TSBB06 has 2 parts: One part for the course content in HT1 and one for the content from HT2. This document is KTR1, the voluntary midterm exam, which corresponds to part 1 of the main exam.

There are 10 problems in KTR1 (and in part 1 of the final exam), and these are grouped under the two headings Geometry, and Estimation.

The exam problems are of two types: **Type A** are 1 point problems, and **Type B** are 2 point problems. Half points can be awarded.

The maximal score on KTR1 is 14p, and if you obtain 8p or more on KTR1 you are allowed to use that result on the final exam. (This is done by not handing in answers to **any** of the problems in part 1 of the exam. Your score from KTR1 will then automatically be added to the final exam score.)

The maximal score on the final exam is 28p, and grading of the exam is done based on your score as follows:

If you obtain **15p** or more you get **grade 3**.
If you obtain **18p** or more you get **grade 4**.
If you obtain **22p** or more you get **grade 5**.

All tasks should be answered on **separate sheets**. It is fine to answer multiple questions on one sheet.

Write your AID-number and the date on all paper sheets that you attach to the examination. In addition, these sheets should be numbered in consecutive order.

Good luck!
Mårten Wadenbäck

PART I, Geometry

Problem 1 (A, 1p) Consider two points in the extended Euclidean plane, with homogeneous coordinates given by $\mathbf{x}_1 = (3, 1, 1)$ and $\mathbf{x}_2 = (3, -2, 0)$.

Find dual homogeneous coordinates for the line $\mathbf{l}$ through $\mathbf{x}_1$ and $\mathbf{x}_2$.

Problem 2 (A, 1p) A 2D homography is represented by the matrix

$$\mathbf{H} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

The three points in the set $\{(2, 2),\ (0, 3),\ (2, -2)\}$ are transformed using this homography into the points in $\{(-1, 3),\ (4, 2),\ (x, y)\}$, but possibly listed in a different order in the two sets.

Determine the coordinates for the unknown point (x, y) in the second set.

Problem 3 (A, 1p) Different types of transformations preserve different geometric notions/concepts. For example, rigid transformations preserve *angles*, since two lines intersecting at an angle α before the transformation also will intersect at *the same* angle α after the transformation.

Name two additional notions that are preserved by rigid transformations but not (in general) by affine transformations.

Problem 4 (B, 2p) The 3D rotation matrix $\mathbf{R}$ corresponding to the axis–angle representation $(\hat{\mathbf{n}}, \alpha)$, where $\|\hat{\mathbf{n}}\| = 1$, can be computed (for example) using Rodrigues' formula:

$$\mathbf{R} = \mathbf{I} + \sin\alpha\,[\,\hat{\mathbf{n}}\,]_\times + (1 - \cos\alpha)[\,\hat{\mathbf{n}}\,]_\times^2.$$

Given a rotation matrix, how can we determine (one of) its axis–angle representation(s)?

Problem 5 (B, 2p) Homogeneous representations for four distinct points in the extended Euclidean space (3D) are given as the vectors $\mathbf{x}_1, \mathbf{x}_2, \mathbf{y}_1, \mathbf{y}_2 \in \mathbb{R}^4$.

Assuming the points are co-planar, describe a method for computing the intersection of the line through $\mathbf{x}_1$ and $\mathbf{x}_2$ and the line through $\mathbf{y}_1$ and $\mathbf{y}_2$. Your method should work even if some or all points involved (i.e., even the intersection point) lie on the plane at infinity.

PART I, Estimation

Problem 6 (A, 1p) A model is fitted to a very large amount of experimental data, and it turns out that the model fits the data *exactly* (i.e. zero error).

Explain why this most likely is a bad model.

Problem 7 (A, 1p) Explain the orthogonal Procrustes problem (OPP). What is known, and what is to be determined? How is this done?

Problem 8 (A, 1p) Suppose two images are related by a homography $\mathbf{H}$ that we wish to estimate from point correspondences. If we use the homogeneous method to minimise an algebraic error for this problem, we should pay attention to the singular values of the data matrix.

Explain what characterises a 'good' set of singular values, and why this is so.

Problem 9 (B, 2p) A symmetric matrix $\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{pmatrix}$ can be used to represent a *conic section*. The conic section then consists of all $\mathbf{x} \sim (x, y, 1)$ satisfying the equation $\mathbf{x}^\top \mathbf{C} \mathbf{x} = 0$, or, written out explicitly,

$$c_{11}x^2 + 2c_{12}xy + 2c_{13}x + c_{22}y^2 + 2c_{23}y + c_{33} = 0.$$

Suppose that we want to estimate $\mathbf{C}$ from a number of known points $\mathbf{x}_1, \ldots, \mathbf{x}_m$.

a) The equation of the conic section is linear in the entries c_{ij}. Construct a suitable data matrix, and explain how it can be used to estimate $\mathbf{C}$ from the known points $\mathbf{x}_1, \ldots, \mathbf{x}_m$. (1p)

b) What is the smallest number of points needed to determine $\mathbf{C}$? (1p)

Problem 10 (B, 2p) Estimation of geometric objects or transformations can often be formulated as minimising $\|\mathbf{A}\mathbf{z}\|$, where $\mathbf{z}$ is a parameter vector representing the unknown entity that we want to estimate, and $\mathbf{A}$ is a known data matrix. The minimisation problem is solved with an additional constraint on $\mathbf{z}$ to avoid the solution $\mathbf{z} = \mathbf{0}$. Describe two common approaches to defining such a constraint, and explain how $\mathbf{z}$ is computed in the two cases.