TSBB15
Computer Vision

Lecture 5
Global motion estimation
Tracking

LINKÖPING
UNIVERSITY

Motion estimation

BCCE

The inhomogeneous method

The homogeneous method

Least squares solution

Based on $\mathbf{T}_{2D} + \mathbf{s}$

Total least squares solution

Based on $\mathbf{T}_{3D}$

First order differential methods

Motion estimation

- The techniques described next (and in the previous lecture) are suitable for determining an ***the optic flow***, an estimate of **m(x)**, at each point **x** in the image
- This is referred to as ***dense motion estimation***
  - Can still be characterized by a position dependent certainty measure
- An alternative is ***tracking***, where the motions of only a small set of points, or a single point, are determined
  - Later in this lecture…

Motion estimation

- There are other approaches, for example
  - Global smoothness of **v** (Horn & Schunck)
  - Second order differential methods
  - Parametric optical flow
  - Et cetera

Will be covered here

Will not be covered here

The Horn & Schunck method

- At each point we seek the motion vector $\mathbf{v} = (v_1, v_2)$ that satisfies the BCCE:

$$\frac{\partial I}{\partial t} + \frac{\partial I}{\partial u} v_1 + \frac{\partial I}{\partial v} v_2 = 0$$

- Problem: one equation but two unknowns
- Previously, we dealt with this problem by considering a *local* set of equations, assuming **v** constant in a *local* region Ω
- Finding **v** can also be dealt with by means of a *global* approach (with respect to the image)

The Horn & Schunck method

- Let $\mathbf{v}(u, v)$ denote the velocity vector field in an image, as a function of image position (u, v)
- BCCE suggests that we should find $\mathbf{v}(u, v)$ that minimizes

$$\epsilon = \int \left(\mathbf{v}(u, v) \cdot \nabla I + \frac{\partial I}{\partial t} \right)^2 d\mathbf{x}$$

Image gradient at (u, v)

Time derivative at (u, v)

Integration is now made over an entire image!

The Horn & Schunck method

- We can (in principle) always find $\mathbf{v}(u, v)$ that gives $\varepsilon = 0$:

$$\mathbf{v}(u,v) = -\frac{\partial I}{\partial t}\frac{\nabla I}{\|\nabla I\|^2} + \alpha(u,v)\begin{pmatrix} \frac{\partial I}{\partial v} \\ -\frac{\partial I}{\partial u} \end{pmatrix}$$

(why?)

Arbitrary function of (u, v)

The Horn & Schunck method

- Problem I:

  Singularities when $\nabla I = \mathbf{0}$

- Problem II:

  Does not provide a unique solution since $\alpha(u, v)$ can be arbitrary chosen

- Problem III:

  Strong variations in ∇I may not correspond to strong variations in $\mathbf{v}(u, v)$

The Horn & Schunck method

- H&S 1981: Let's make $\mathbf{v}(u, v)$ unique by adding a smoothness term to ε
- This term should assure that $\mathbf{v}(u, v)$ is as smooth at possible, seen as a function of (u, v)
- Smoothness =

  "as little variation in **v** as possible"

The Horn & Schunck method

- H&S used a smoothness term:

$$\|\nabla v_1\|^2 + \|\nabla v_2\|^2$$

- Other types of smoothness terms appear in the literature

The Horn & Schunck method

- New cost function

$$\epsilon = \int \left(\mathbf{v}(u,v) \cdot \nabla I + \frac{\partial I}{\partial t} \right)^2 d\mathbf{x} + \lambda \int \|\nabla v_1\|^2 + \|\nabla v_2\|^2 \, d\mathbf{x}$$

- The integrals are taken *over the entire image*
- λ is a “smoothness weight”
- Our goal: find $\mathbf{v}(u, v)$ that minimizes ε

The Horn & Schunck method

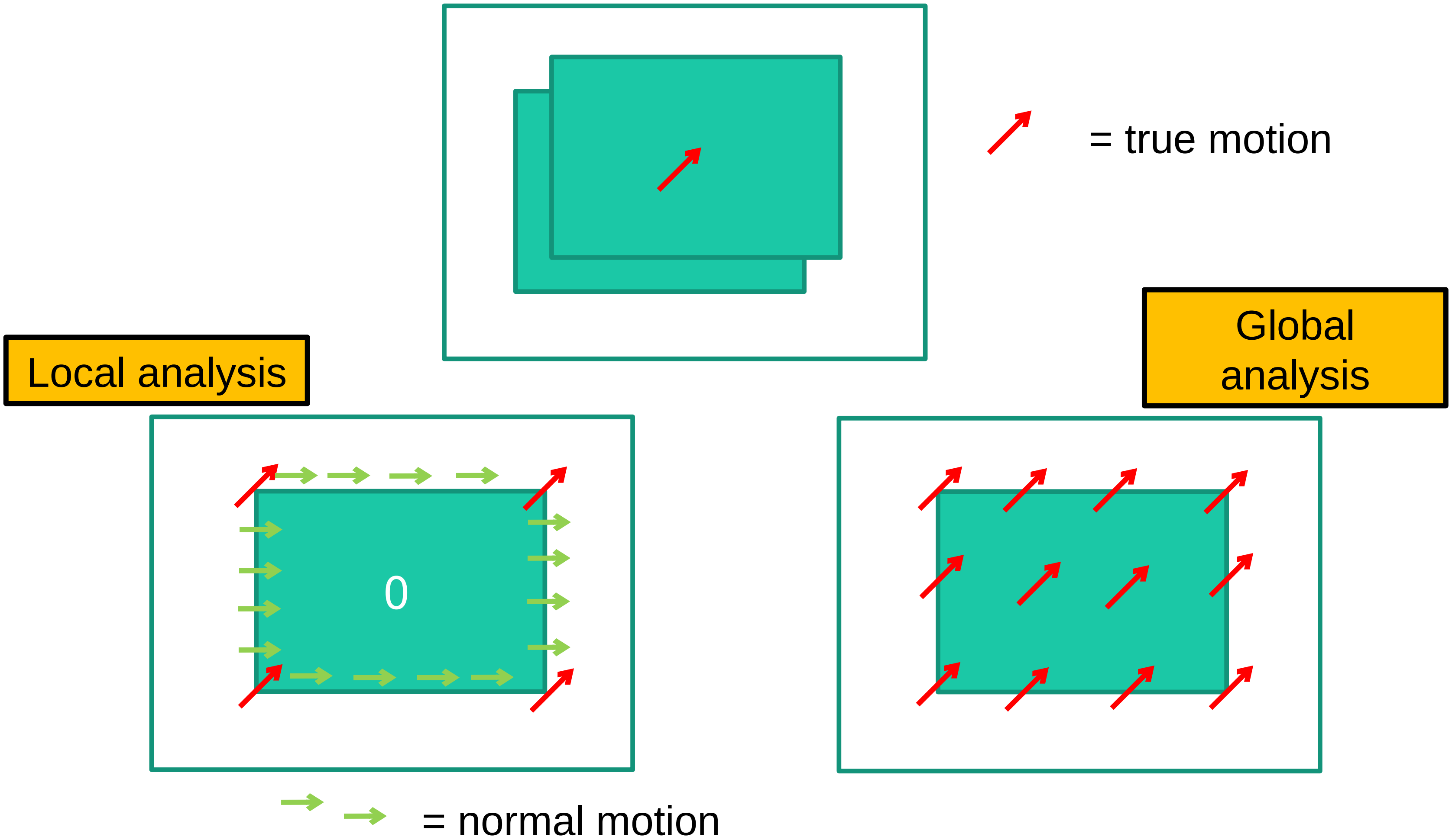
- This was one of the first established methods for motion estimation
- Often referred to as a “global” method
- Can (to some extent) deal with the aperture problem
- In practice: **v** cannot be determined by solving a linear equation, instead iterative methods are required
  - Efficient algorithms exist
  - See e.g. D. Sun, et al, Secrets of Optical Flow Estimation and Their Principles, CVPR 2010.
- Not obvious how to choose λ
  - constant or dependent on **x**?
- The smoothness constraint is not always valid
  - Sharp motion boundaries exist in practice
- More “sophisticated” methods use other types of smoothness terms

The Horn & Schunck method

NOTE!!

- Horn & Schunck's method is not correctly described in the book by R. Szeliski
  - In the printed book and e-book: on page 360, equation (8.70)
  - In the draft version on the web: on page 410, equation (8.70)
- The cost function E_{HS} lacks the regularization term

Local vs global methods



Second order differential methods

- Another approach for obtaining sufficient information to uniquely determine $\mathbf{v}$ at each point is to differentiate BCCE again with respect to u and v
- This method is again based on *local* computations

Second order differential methods

- BCCE:

$$\frac{\partial I}{\partial t} + \frac{\partial I}{\partial u} v_1 + \frac{\partial I}{\partial v} v_2 = 0$$

- Differentiate with respect to u and v:

$$\frac{\partial^2 I}{\partial t \partial u} + \frac{\partial^2 I}{\partial u^2} v_1 + \frac{\partial^2 I}{\partial u \partial v} v_2 = 0$$
$$\frac{\partial^2 I}{\partial t \partial v} + \frac{\partial^2 I}{\partial u \partial v} v_1 + \frac{\partial^2 I}{\partial v^2} v_2 = 0$$

Second order differential methods

- Now we get 2 additional equations in variables $\mathbf{v}(v_1, v_2)$:

$$\mathbf{H}\mathbf{v} = -\frac{\partial}{\partial t}\nabla I$$

- $\mathbf{H}$ is the *Hessian matrix* (second order derivatives) of f w.r.t. u and v
- Solve in a similar way as the LK-equation

Multi order differential methods

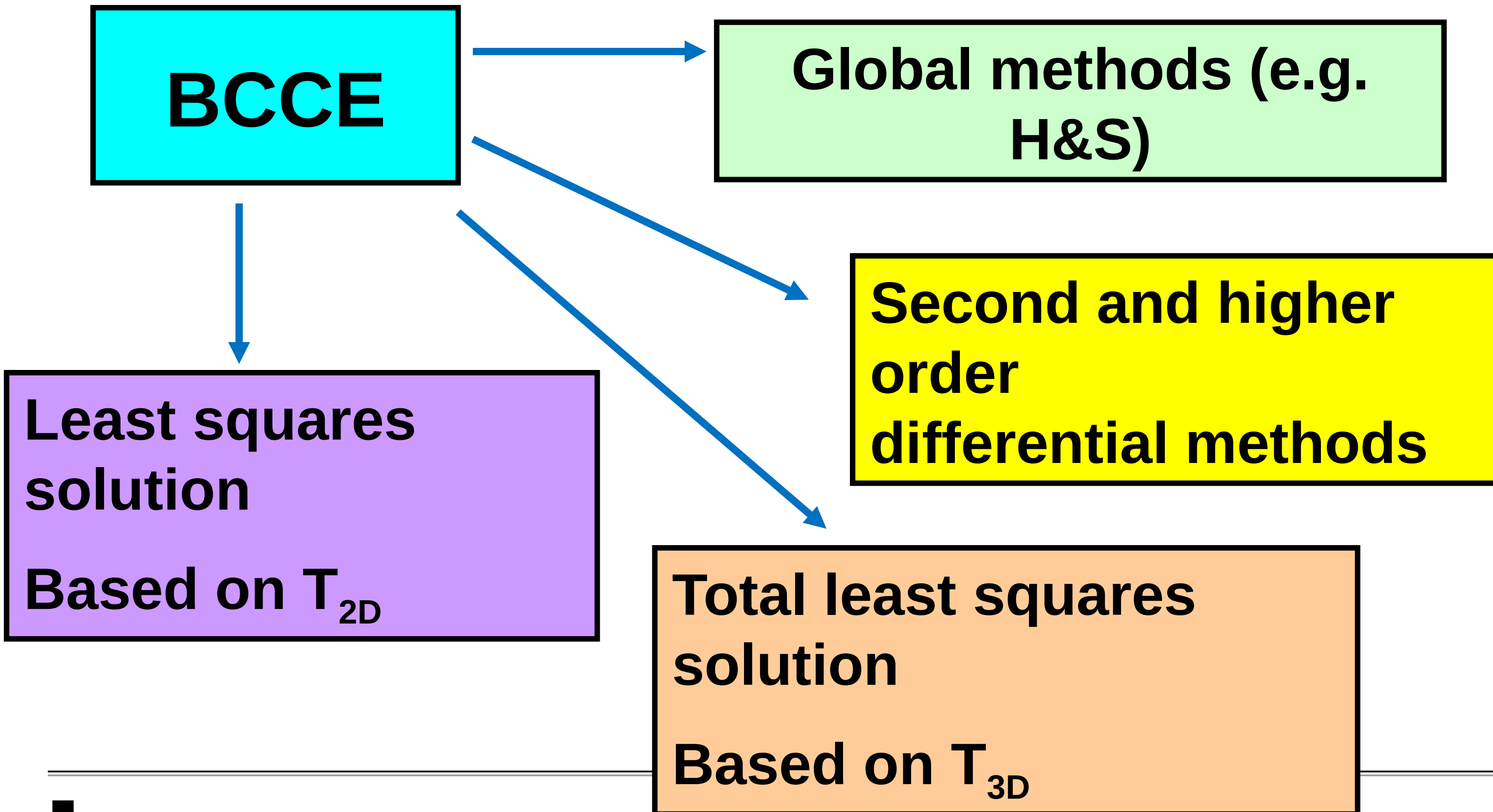
- There is nothing that prevents us from using both first and second order derivatives *simultaneously*!

$$\begin{pmatrix} \nabla^{\mathrm{T}} I \\ \mathbf{H} \end{pmatrix} \mathbf{v} = - \begin{pmatrix} \frac{\partial I}{\partial t} \\ \frac{\partial}{\partial t} \nabla I \end{pmatrix}$$

Multi order differential methods

- We get 3 (or more) equations and have 2 unknowns
- Solutions can still be found using various least squares techniques
(how?)

Motion estimation, summary



Motion estimation, summary

- In the ideal case, all methods (in principle) should give the same solution
- They differ mainly with respect to
  - Sensitivity to
    - noise
    - deviations from model assumptions
  - Computational demand
  - Certainty measures
- For all methods: different sizes of Ω and different ways to estimate gradients give different quality of results

Advanced variations of basic methods

- These basic methods for motion estimation, in particular the local ones, can be significantly improved (at moderate cost) by using one or more ***advanced techniques***, such as
  - Refinement iterations
  - Course-to-fine refinement
  - Spatial filtering of motion estimates
  - Robust error norms
  - Symmetry in I and J
  - Affine transformation

Refinement iterations

- The basic methods described here are based on a set of assumptions, e.g.:
  - Brightness constancy: e.g., for 2-image case:

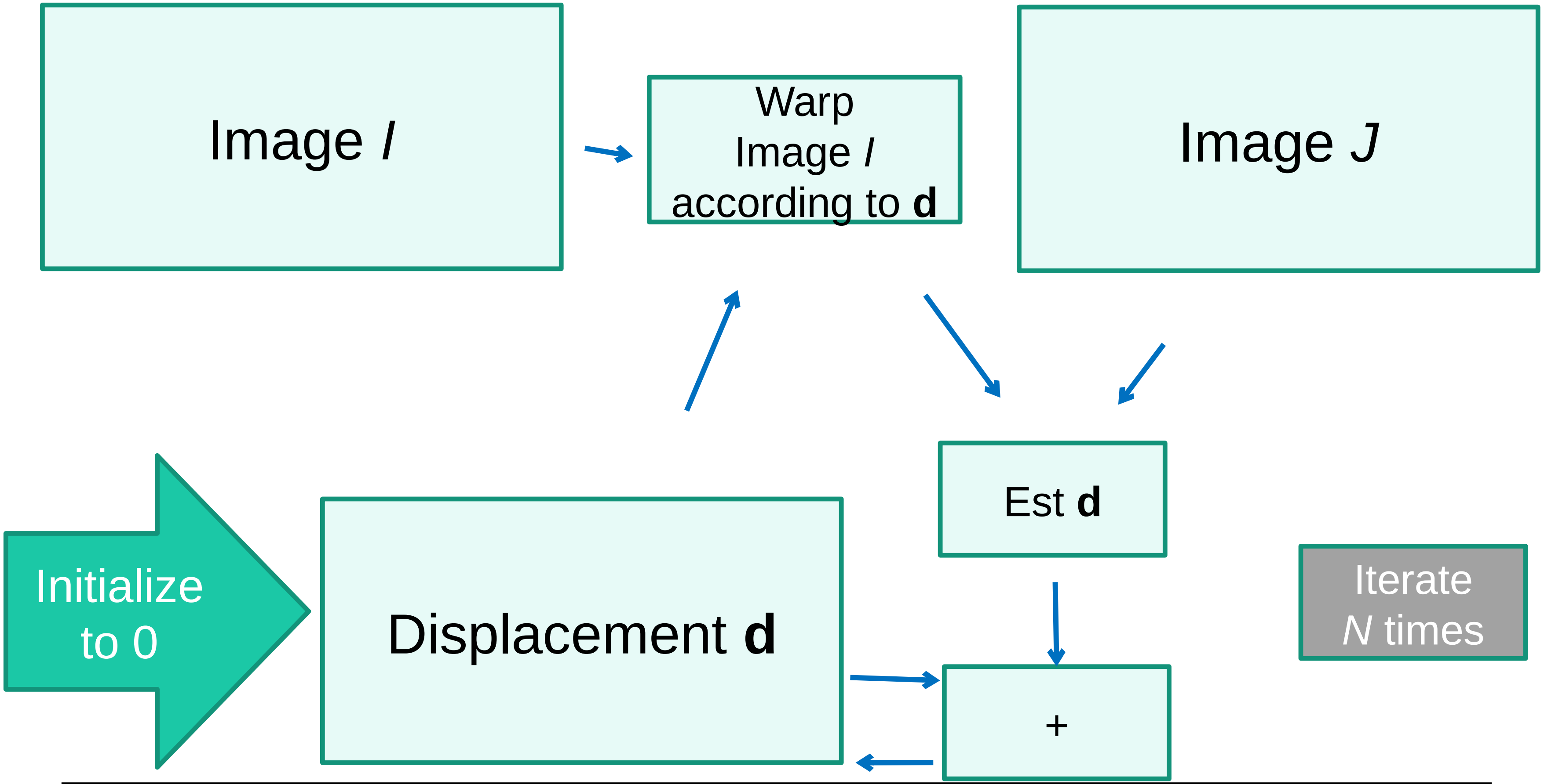
    $$J(\mathbf{x}) = I(\mathbf{x} + \mathbf{d})$$

  - High order terms in Taylor expansions can be neglected
  - Constant **d** (or **v**) within Ω
- In general these assumptions are not all correct: estimate of **d** (or **v**) is inaccurate

Refinement iterations

- The estimate **d** (or **v**) should, however, in most cases be approximately correct
- Warp *I* in accordance to estimated **d** (or **v**)
  - If **v** is correctly estimated, the two images are more or less equal
  - If not, there is some remaining **d** (or **v**) that can be estimated from the new *I* and the old *J*
  - Iterate *N* times and accumulate new estimates of **v** (refine **v**) in each iteration

Refinement iterations



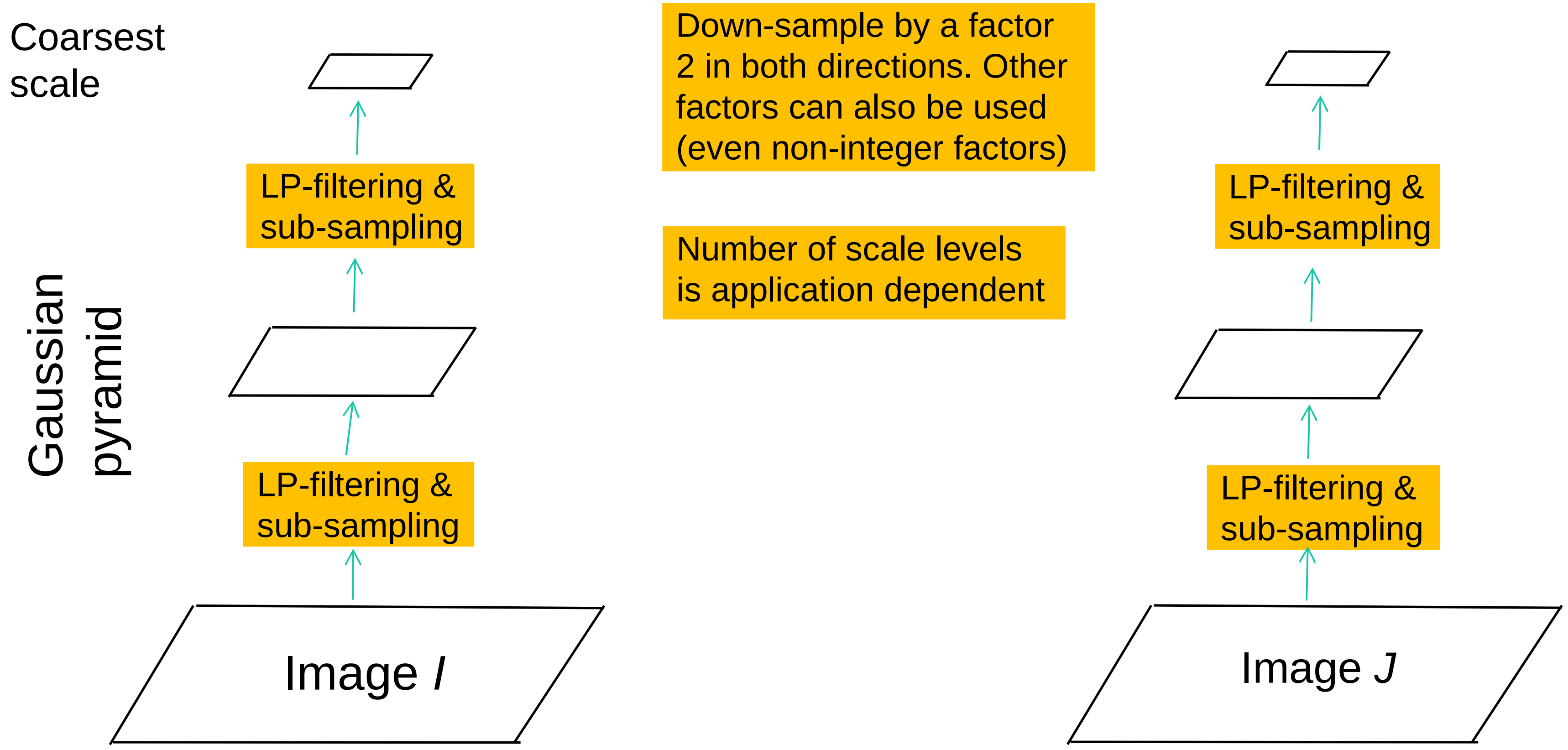
Refinement iterations

- N = number of iterations, depends on the application and on the data (images)
- Does not have to be very large
- For most applications: a “few” iterations are often sufficient

Coarse-to-fine refinement

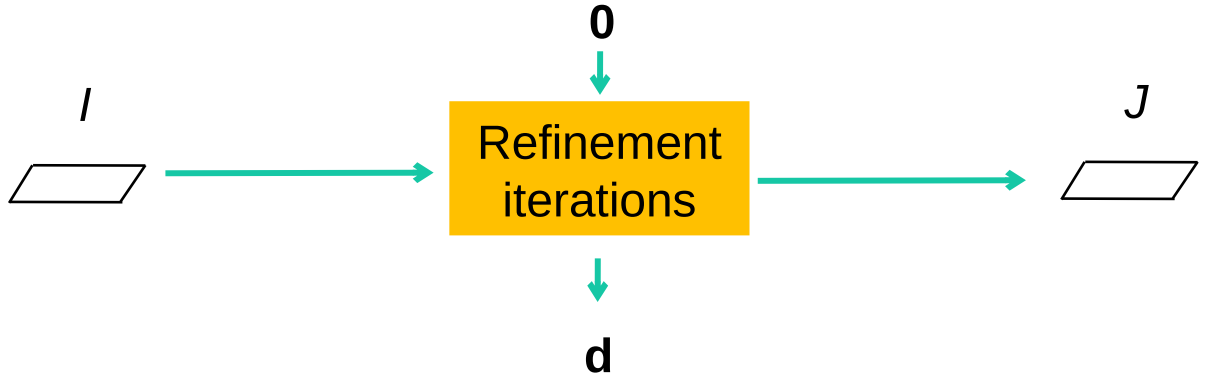
- In local motion analysis, the motion of each point is analyzed within a region Ω
  - Ω has some radius R
- $\mathbf{d}$ cannot be robustly determined if $|\mathbf{d}| > R$
- R cannot be made too large:
  - $\mathbf{d}$ will not be constant in Ω
  - Taylor expansion of $I(\mathbf{x} + \mathbf{y} + \mathbf{d})$ not only linear
- To deal with larger $\mathbf{d}$, use course-to-fine refinement based on scale pyramids
  - See lecture 2

Coarse-to-fine refinement



Coarse-to-fine refinement

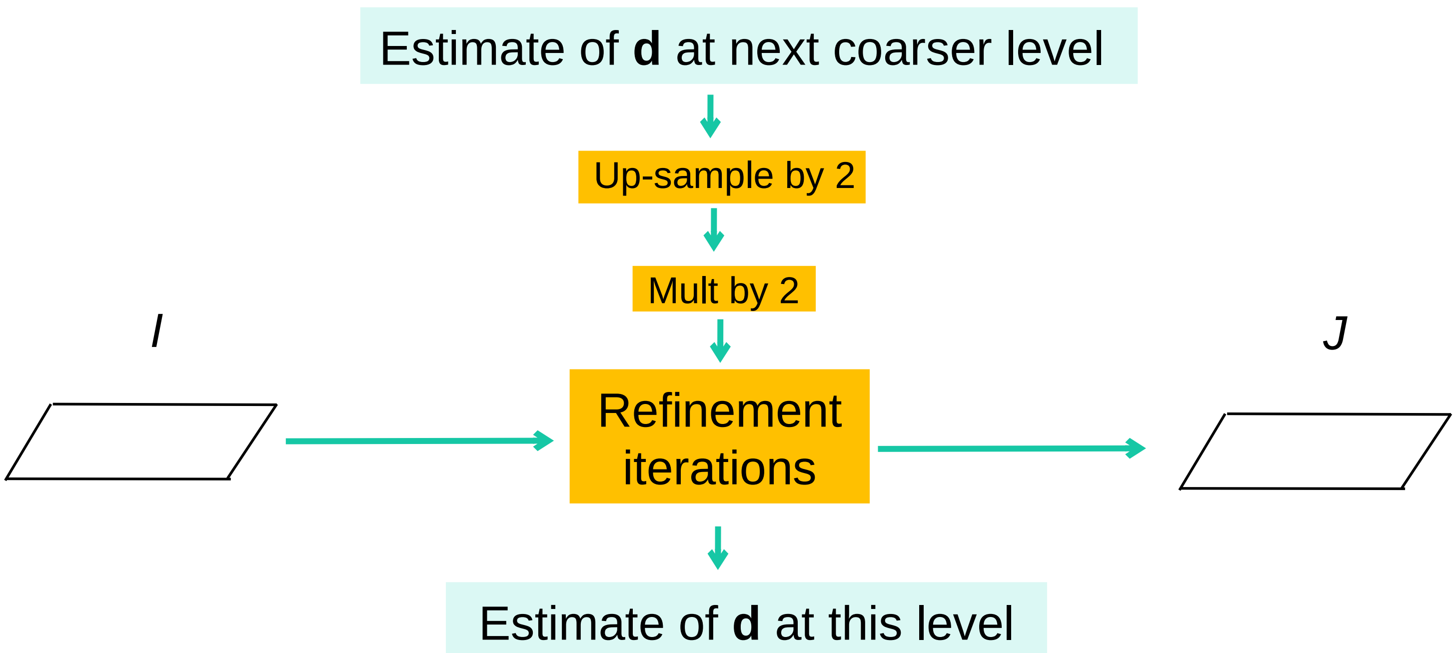
- Start at the coarsest level
- Perform refinement iterations where **d** is initiated to **0** at all points
- Produces an initial estimate of **d** at this level



Coarse-to-fine refinement

- This initial estimate of **d** is then up-sampled to fit the image size at the next finer level
- Also: **d** is multiplied by 2 (or suitable factor) since displacements at the next finer level are 2 times as large as at the previous level
- Use this new **d** as initial estimate in refinement iterations at the finer level

Coarse-to-fine refinement

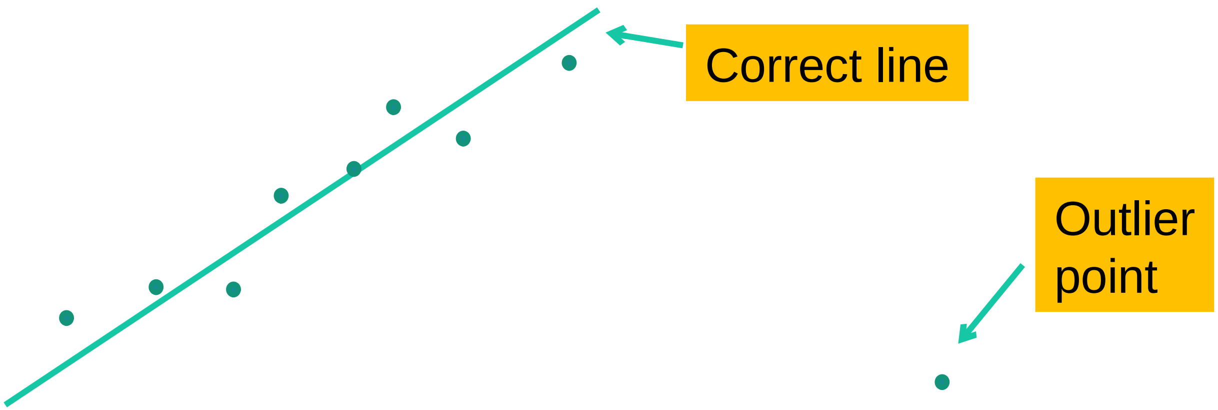


Coarse-to-fine refinement

- Continue this processing from the coarsest level all the way to the finest level
- Estimate of **d** from the finest level is the final estimate from this coarse-to-fine processing
- Can manage magnitudes of **d** which are in the order of R for Ω at the coarsest level
- Note: estimates of **d** at a coarser level does not have to be ***very accurate***, it will be refined at the next finer level!

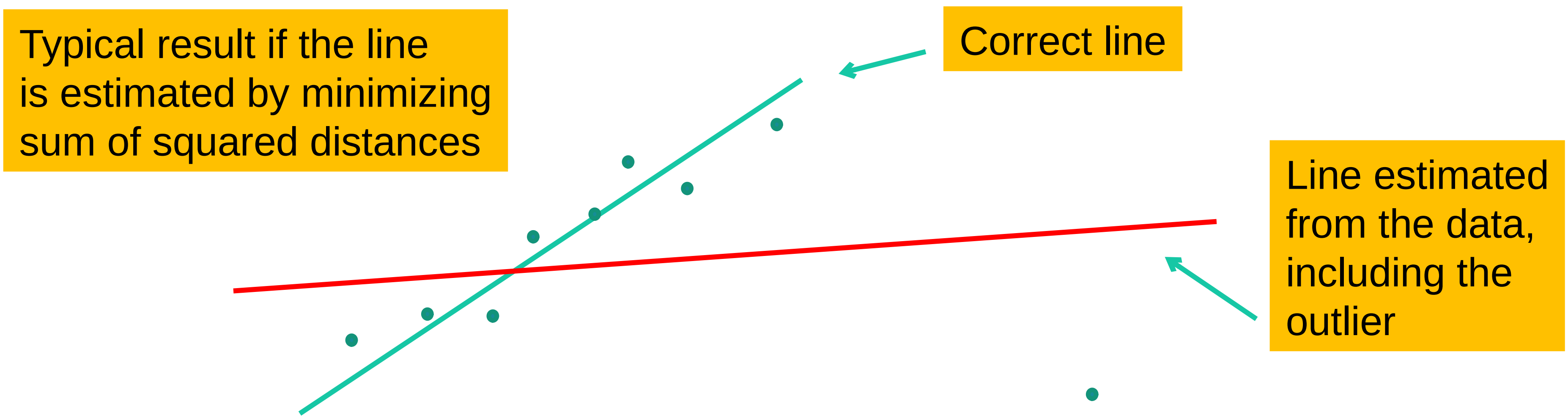
Outliers

- Definition: an ***outlier*** is a point (or data entry) that doesn't fit the model assumed for the data
  - Data that fits the model: ***inliers***
- Example: fitting a line to a set of points



Outliers

- If outliers are allowed to affect estimation of a model in the same way as inliers, the model can become very distorted



Spatial filtering of motion estimates

- Motion estimates at two adjacent pixels should often be very similar
  - The points are projections of 3D points on the same rigid object
  - Not true at ***motion boundaries***!
- Motion estimates can also be degraded by
  - Image noise
  - Invalid assumptions (e.g., because of outliers)

Spatial filtering of motion estimates

- To reduce these effects it makes sense to allow the estimate of **d** to be affected by its neighbors
  - Local averaging, weighted by a spatial window
  - Corresponds to LP-filtering of **d**
- Even better: use normalized convolution
  - Takes certainty of d into account
- Alternatively: use median filtering
  - Avoids large influence from ***outliers***

Robust errors

- Adding squared distances implies:

  Computing a weighted average of the distances, where each weight = the distance

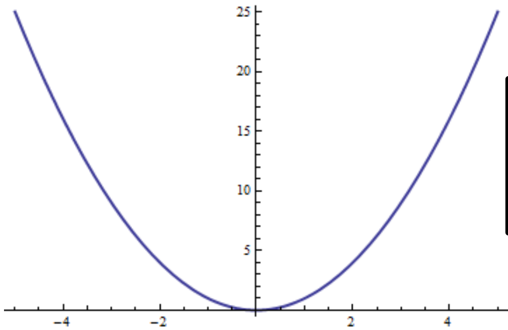
- Implies: outliers are given a high weight
  - Not what we want!!

- This effect can be reduced by using ***robust errors***

Robust errors

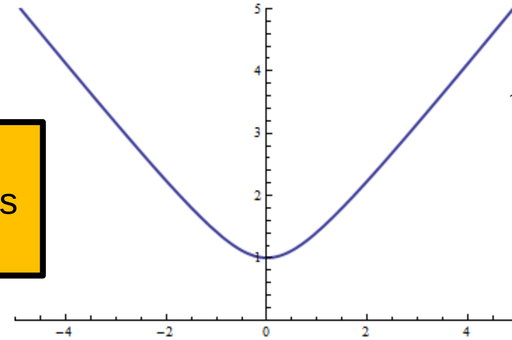
- Replace the square function with alternative function, for example

x^2



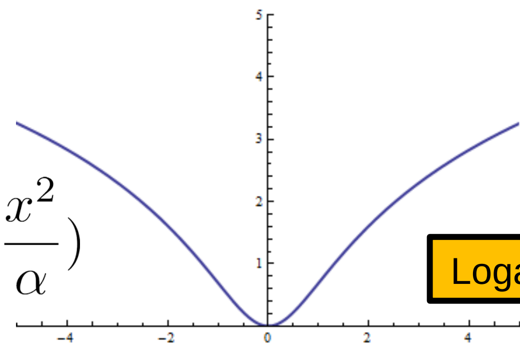
All functions have continuous derivatives!

$\sqrt{x^2 + \alpha^2}$



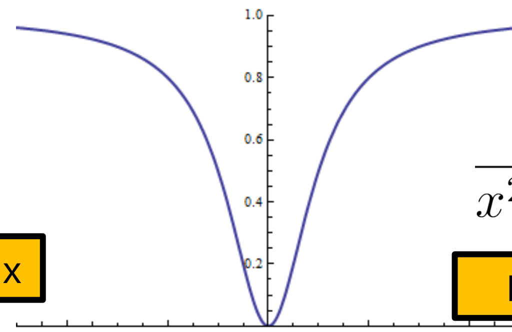
Linear for large x

$\log(1 + \frac{x^2}{\alpha})$



Logarithmic for large x

$\frac{x^2}{x^2 + \alpha^2}$



Never exceeds 1

Symmetric formulation

- The 2-image version of the LK-method does not treat images I and J in the same way
  - Spatial gradients are only computed in I
  - In refinement iterations, only one image is warped
- In the ideal situation, swapping I and J should produce a consistent result
  - Not always true

Symmetric formulation

- Use a symmetric formulation:

$$J(\mathbf{x} - \mathbf{d}/2) = I(\mathbf{x} + \mathbf{d}/2)$$

instead of

$$J(\mathbf{x}) = I(\mathbf{x} + \mathbf{d})$$

Symmetric formulation

- Finding **d** as the minimizer of

$$\epsilon = \int_{\Omega_0} w(\mathbf{y}) \left(I(\mathbf{x}+\mathbf{y}+\mathbf{d}/2) - J(\mathbf{x}+\mathbf{y}-\mathbf{d}/2) \right)^2 \, d\mathbf{y}$$

- Can be solved in a similar way as before:

$$\mathbf{T}\,\mathbf{d} = \mathbf{s}$$

T and **s** contain gradients from both *I* and *J*

(how?)

Affine transformation

- The local motion model for the 2 image case only includes a translation:

$$J(\mathbf{x}) = I(\mathbf{x} + \mathbf{d})$$

- A more complex model could also include an affine transformation:

$$J(\mathbf{x}) = I(\mathbf{A}\,\mathbf{x} + \mathbf{d})$$

Unknown parameters to be estimated, depend on **x**

Affine transformation

- **A** is a 2 × 2 matrix
- In practice, set **A** = **I** + **A'**
  - **A'** is then often a small matrix, easier to estimate
- Set

$$\mathbf{A}' = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}, \quad \mathbf{z} = \begin{pmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \\ d_1 \\ d_2 \end{pmatrix}$$

  and minimize ε over **z** (how?)
- Leads to **T'** **z** = **s'**

T' is 6 × 6
s' is 6-dimensional

Tracking

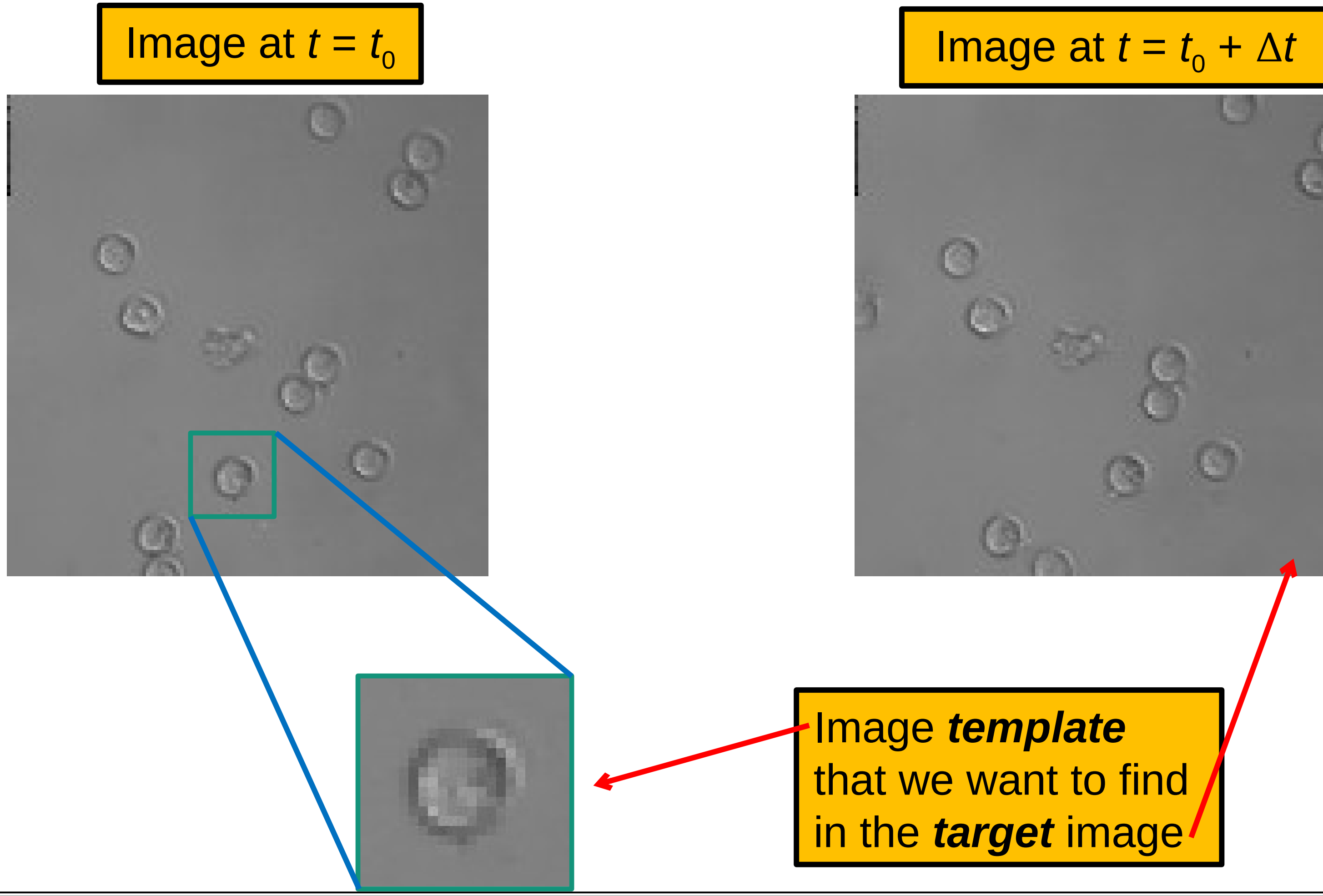
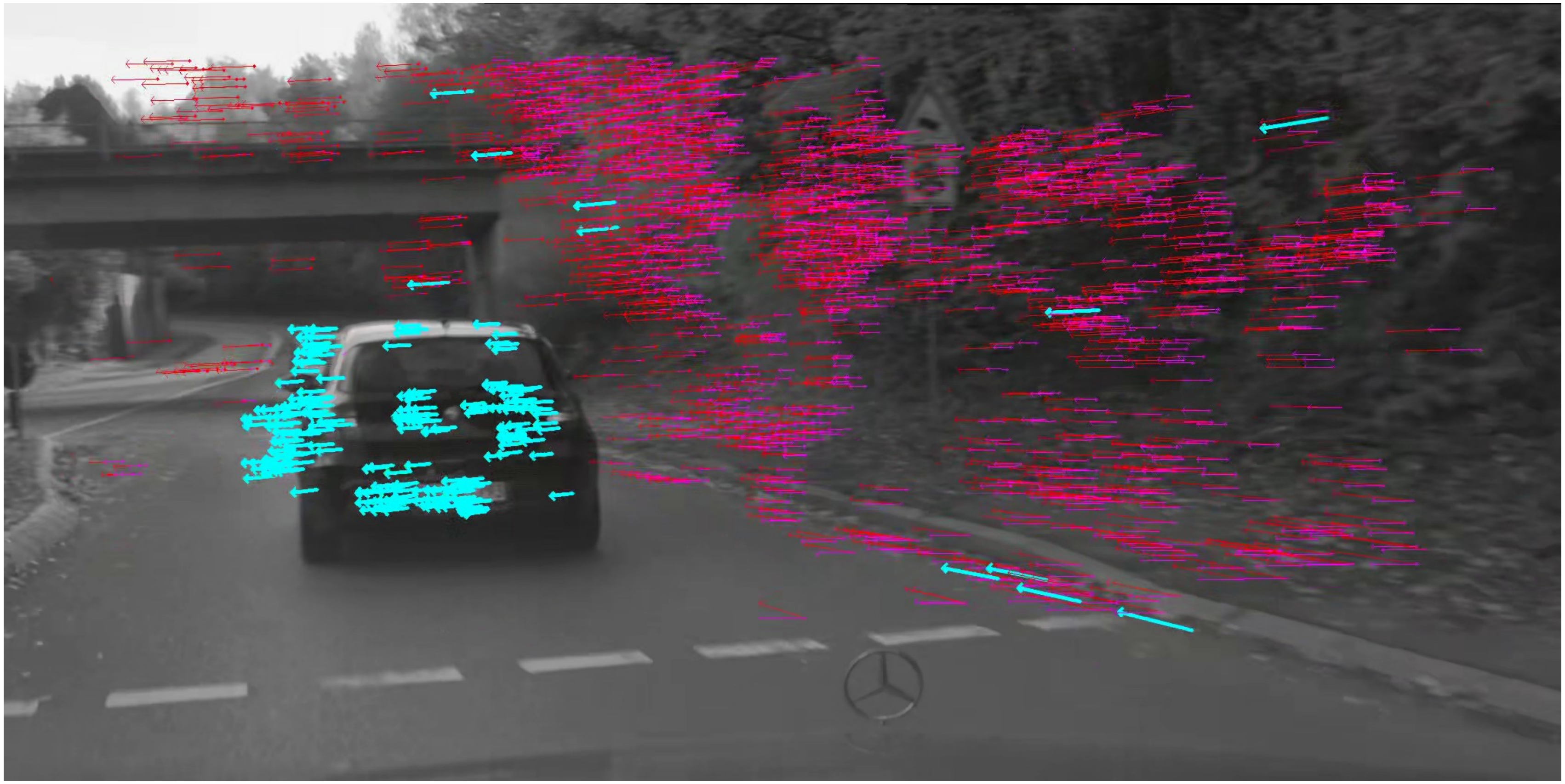


Image at $t = t_0$
Image at $t = t_0 + \Delta t$
Image ***template*** that we want to find in the ***target*** image

Tracking vs. motion estimation

- In ***motion estimation***, the motion field **m**(**x**) is estimated either as a displacement field **d**(**x**) between two images, or as a velocity field **v**(**x**) based on a continuous time model
  – The result is **d**(**x**) (or **v**(**x**)) as a function of x for all image points
- In ***tracking***, we determine **d**(**x**) (or **v**(**x**)) for a single point, or for a region Ω around this point (the template)
  – The result is **d** (or **v**) for this template



Tracking vs. estimation of **m**(**x**)

- Tracking can also be applied to a smaller set of points (templates) determined as interesting to track
  - As a consequence, tracking can be done with low computational cost, alternative it allows more complex methods to be used since they are not applied to every image point
- Typically, tracking of a template is made over several consecutive images in an video sequence
  - As long as the template can be robustly re-identified in each target image

Applications for tracking

Tracking can be used for

- Following specific objects in an image sequence
  - People, vehicles, targets, etc
- For efficiency:
  - assume small **v** between each image
- Producing ***point correspondences*** for specific interest points in two or more images of the same scene
  - *Structure from motion*
  - *Ego-motion estimation*
- Determine 3D motion based on motion in the image
- Segmentation based on distinct objects moving with distinct motions
- Stereo matching (original app for LK-tracking!)
- Video compression

Basic tracking methods

- See tracking as a special case of 2-image motion estimation where image I is the template, and image J is an image from a video sequence (the target image) (or the other way around)
  - Use the LK-approach, or other local methods for motion estimation.
  - Referred to as ***LK-tracking***
  - Use the advanced methods mentioned previously
    - In particular refinement iterations and scale pyramids
  - Can be efficiently implemented in software & hardware
    - GPGPU (Graphics hardware)

Basic tracking methods

- See tracking as the problem of re-identifying a template in a target image
  – Block matching (grid-based method)
- See tracking as the problem of re-identifying a “blob” of pixels that have been determined as “not background”
  – See subsequent lectures

Block matching

A rather straight-forward approach:

- Given
  - A template Ω
  - A target image J
  - A predicted position of Ω in J
  - A range N
- Prediction can be: where Ω was found in the previous image in the sequence
  - Can also include statistical models (Kalman filter)
- Extract a set of regions in J around $\mathbf{x}$, of same size as Ω
  - For example, in the ranges $(x_1 \pm N/2, x_2 \pm N/2)$
  - Typically with integer shifted displacements
  - Number of patches is in the order of N^2

Block matching

- Compare the template with all patches, find best match
  - We need some similarity measure to do this!
  - Generates a matching function $\varepsilon(d_1, d_2)$
  - Find minimum of ε, (or maximum, depending on how ε is defined)
  - Its position in J is $(x_1 + d_1, x_2 + d_2)$, $-N/2 \cdot d_1, d_2 \cdot N/2$
  - The estimated displacement of the template between image 1 and image 2 given by (d_1, d_2)
- Referred to as *block matching* or *template matching*
- Can be implemented efficiently on ***GPGPU hardware***

Block matching

Some issues that need to be resolved

- How do we compare patches (=blocks of pixels)? Examples:
  - Sum of squared differences (SSD)
  - Sum of absolute differences (SAD)
  - Cross-correlation (CC), normalized cross-correlation (NCC)
- How do we choose a reasonable *N*?
  - Must be large enough to cover the displacements that occur for the application
  - Computational complexity grows with N^2
- Best match may not be for a unique displacement
  - Repetitive patterns
- Sub-pixel accuracy
  - $\varepsilon(d_1, d_2)$ can be interpolated to determine inter-pixel optima

Good features to track

- A paper by Tomasi & Kanade analyzes *which* templates are feasible for tracking
- Conclusion: we should consider templates that give $\mathbf{T}_{2D}$ which are definitely non-singular (big surprise?)
- T&K propose that $\min(\lambda_1, \lambda_2) >$ threshold is a useful criteria for template selection

Tomasi-Kanade

- The TK-criteria can be used to find *interest points* in an image, i.e., points that easily can be identified in several images
- In some applications we may be interested in tracking all such interest points
- Compare to the Harris-detector

Practical aspects of tracking

Template update

- 3D objects tend to change appearance over time when moving in a scene
  - Change of aspect and apparent size relative to the camera
- Suggests that the template should be updated from the target image, e.g.,
  - At regular time intervals
  - When the matching measure degrades too much
- Tricky to implement robustly
  - Difficult to avoid that Ω starts to contain the background instead of the relevant object

Practical aspects of tracking

Track-retrack

- 3D Tracking of an object over N images creates a motion trajectory, from image 1 to image N'
  - A "curve" defined by the image coordinates $\mathbf{x}(k)$ of where Ω is found in each image, $k = 1, \ldots, N$
- Generated by starting at $\mathbf{x}(1)$ in image 1 and successively finding the position of Ω in each new, $\mathbf{x}(k)$, image ***forward in time***
- Ideally, if we instead start in image N, at position $\mathbf{x}(N)$, and track Ω ***backward in time***, we should end up at $\mathbf{x}(1)$
- If the forward and backward trajectories differ too much, the tracking can be considered as failed, cannot be trusted for further processing

Practical aspects of tracking

In the literature

- The basic LK-based methods (gradient based) appear in the literature under a variation of names, e.g.,
  - Lucas-Kanade (LK)
  - Kanade-Lucas (KL)
  - Lucas-Kanade-Tomasi (LKT), or permutations
  - Shi-Tomasi (ST)
- Can also be used as a refinement after block matching