the reconstruction in $\mathcal{O}(N^4)$ time. In [Dan92] and [Axe93], Grangeat's 3D conebeam reconstruction algorithm is made amenable to direct Fourier and linogram techniques, resulting in the computation complexity $\mathcal{O}(N^3 \log N)$ for all stages of the reconstruction.

1.3 SPECT

Conventional CT images the attenuation of X-rays in an object slice. Emission CT, on the other hand, images the distribution of a radioactive isotope inside an object slice. One technique is called SPECT (Single Photon Emission Computed Tomography), where one single gamma photon is emitted for each nuclear event. SPECT is well described in the literature, see for example [Kak87].

The data acquisition starts by depositing isotopes in the patient. The isotopes are injected or inhaled in the form of radio-pharmaceuticals. After some time these substances tend to aggregate and deposit themselves in specific organs or tissues. The concentration of the isotope can then be imaged by measuring the gamma photons originating from radioactive decay of the isotope.

As the concentration of the isotope changes with time not only due to radioactive decay, but also because of flow and biochemical kinetics within the body, functional properties of the human body can also be measured. However, it is important that the time constant of these human functions is large enough to measure a sufficient number of gamma photons for computation of one 3D-image.

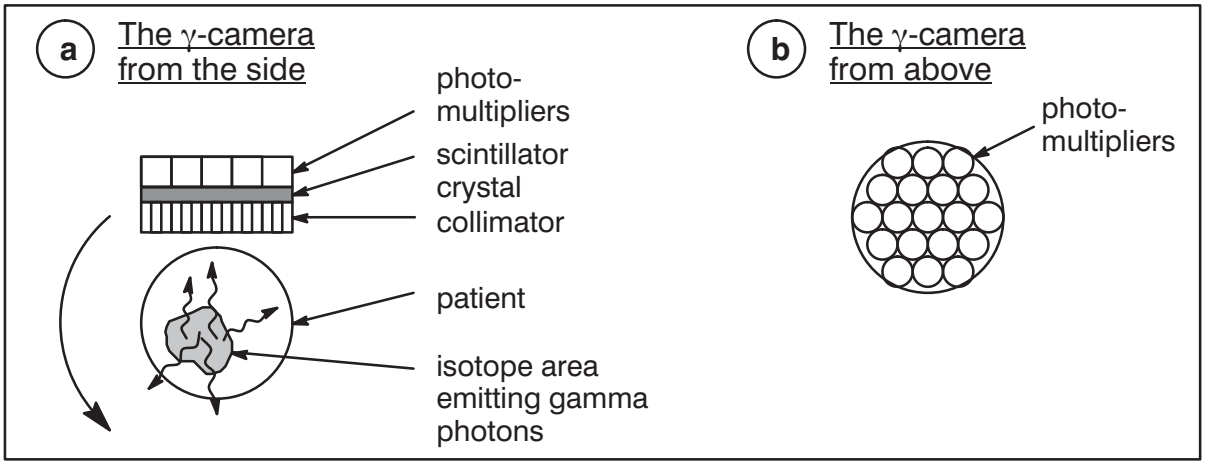

Fig. 1.8 SPECT geometry. **a)** Cutting through the γ-camera and the patient. **b)** The γ-camera from above.

The geometry of SPECT is shown in Fig. 1.8. The radioactive isotopes have migrated and accumulated in a specific area in the patient. Gamma photons are emitted from the isotopes and are then detected by the γ-camera. In the scintillator crystal the gamma photons are converted to light photons, which are detected and magnified by photomultipliers. In front of the γ-camera is a collimator consisting of many parallel tubes of lead. The lead tubes restrict the flight paths of the measured photons so that a parallel projection of the

patient is produced for each position of the camera as it moves in a circle around the patient. In principle, from these 2D projections, the reconstruction can be done in the same way as for X-ray CT where an object cross-section is reconstructed from sets of line integrals, i.e. 1D-projections. Since the γ-camera is two-dimensional, as shown in Fig. 1.8b, it is possible to achieve a whole 3D-volume of image data after reconstruction.

There are several difficulties with SPECT. One is the unwanted attenuation[1] of photons as they travel from the isotope source towards the detector. The attenuation depends both upon the photon energy and the nature of the tissue. In Fig. 1.9, photons originating from an isotope source at g_2 are further away from the detector than g_1-photons, and would therefore be more attenuated. To compensate for the attenuation, the reconstruction formula could be modified. However, this modification is neither straightforward nor simple and the compensation is not perfect.

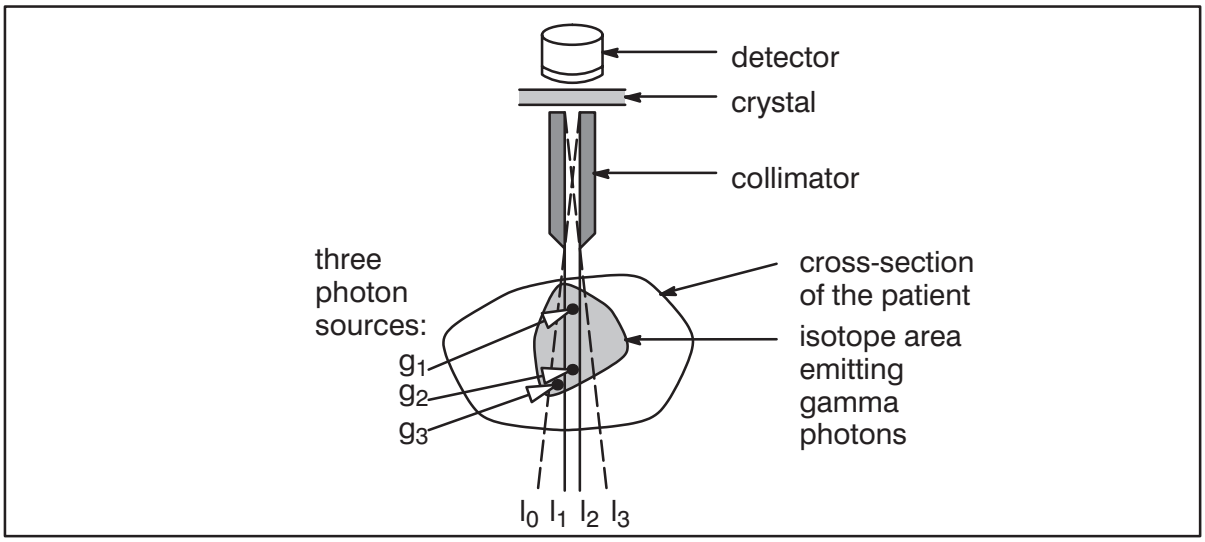

Fig. 1.9 Sources of artifacts in SPECT.

Another source of error is also shown in Fig. 1.9. Photons originating from a source between the two non-parallel lines l_0 and l_3, but outside the area between l_1 and l_2 may also detected by the detector. Seen as a probe, the collimator loses its sharpness with distance. Therefore the resolution becomes lower for points lying far away from the detector.

Typically, the images obtained with SPECT have the size 64 or 128. In Fig. 1.10, a SPECT image of the liver is shown. The rather low image quality in comparison with X-ray CT-images (see for example Fig. 1.19) is due to a lower number of detected photons per pixel. Unfortunately, for safety reasons the dosage of the radioactive isotope cannot be increased. A certain improvement in signal-to-noise ratio (SNR) is obtained by using two or three cameras at the same time.

1. Gamma radiation consists of electromagnetic (EM) waves just as X-rays, however with smaller wave-length (< 0.1 nm). EM-waves with small wave-lengths are often regarded as photons instead of waves. The word attenuation may then be confusing, since the individual photon is not attenuated. Rather, some of the photons are absorbed by the tissue due to a probability function. Therefore, attenuation should be interpreted as probability of absorption.

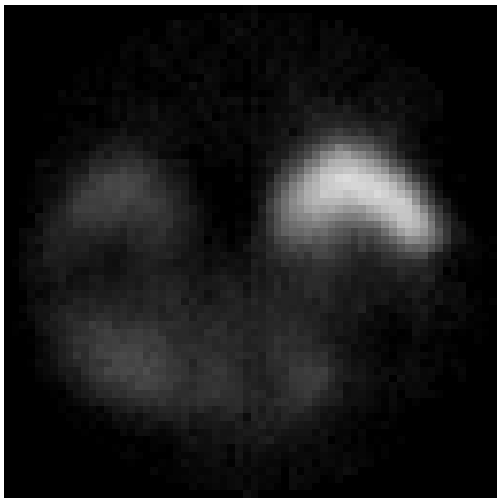

Fig. 1.10 A SPECT image of the liver. This is one of 64 slices constituting a 64^3 SPECT volume produced by a Gamma-11 camera, the Octreoscan tracer at Dept. of Oncology, Sect. of Nuclear Medicine, Uppsala University Hospital. (By courtesy of Jan-Erik Westlin.)

If CT is the filtered version of a special kind of transversal tomography, by the same token **ectomography** [Edh80], [Dal89] can be called the filtered version of circular classical tomography. Ectomography is a new technique which is on the verge of being introduced clinically. It is preferably meant for measurements in the same area as SPECT, i.e. measurement of γ-radiation from an isotope inside the body. Fig. 1.11 shows the geometry of an ectomography scanner. Integrals of slanted lines through the body are recorded by a rotating device. For the reconstruction similar methods as in CT are used. Exact reconstruction is not possible because of missing data. Even so, there are certain situations when this can be tolerated and where ectomography may be more suitable than traditional SPECT.

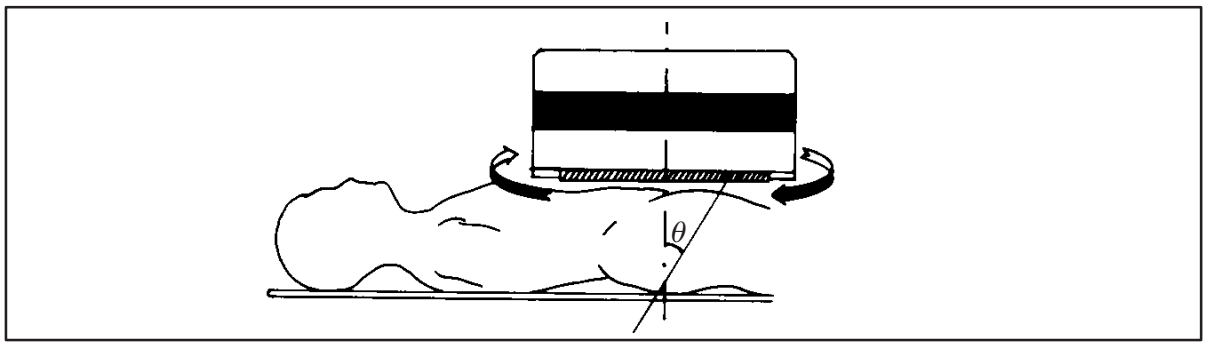

Fig. 1.11 Geometry of ectomography. (From [Dal89].)

1.4 PET

PET or Positron Emission Computed Tomography is another type of emission CT. For surveys of PET, see for example [Kak87], [Boh93]. A PET image shows the concentration of positron emitting isotopes inside a cross-section of the human body. As in SPECT, a radioactive isotope is distributed to the patient, but this isotope emits positrons rather than gamma photons.

The principle of PET is shown in Fig. 1.12. A positron can exist in nature for just a short time interval. It very soon interacts with an electron, which results in annihilation of their masses while two photons are created, each of them having an energy of 511 keV and travelling in exact opposite directions. Note that the mass m of an electron or positron is $9.1091 \cdot 10^{-31}$ kg and due to

$$E = mc^2 = 8.19 \cdot 10^{-14}\,\text{Ws} = 511\,\text{keV}, \tag{1.12}$$

this is is equivalent to the wavelength

$$\lambda = \frac{h}{E} = 8.09 \cdot 10^{-21}\text{m}. \tag{1.13}$$

An emitted positron is detected by *time coincidence* of two photons arriving at two diametrically opposite detector elements. Evidently, the time window must be very narrow (usually 10−25ns [Kak87]) and the number of emission events must be kept rather low to avoid false matches. When a match is confirmed, we know that the positron emitting isotope was on the line (or strip) between the two reacting detectors. A PET device contains a lot of detectors, where many coincidence lines are possible at the same time. In Fig. 1.12, for example, a ring of detectors surrounding the body are indicated. The recorded events constitute line integrals of the isotope concentration within the body. Hence, in principle, the reconstruction can then be done in the same way as for X-ray CT where an object cross-section is reconstructed from sets of line integrals, i.e. projections.

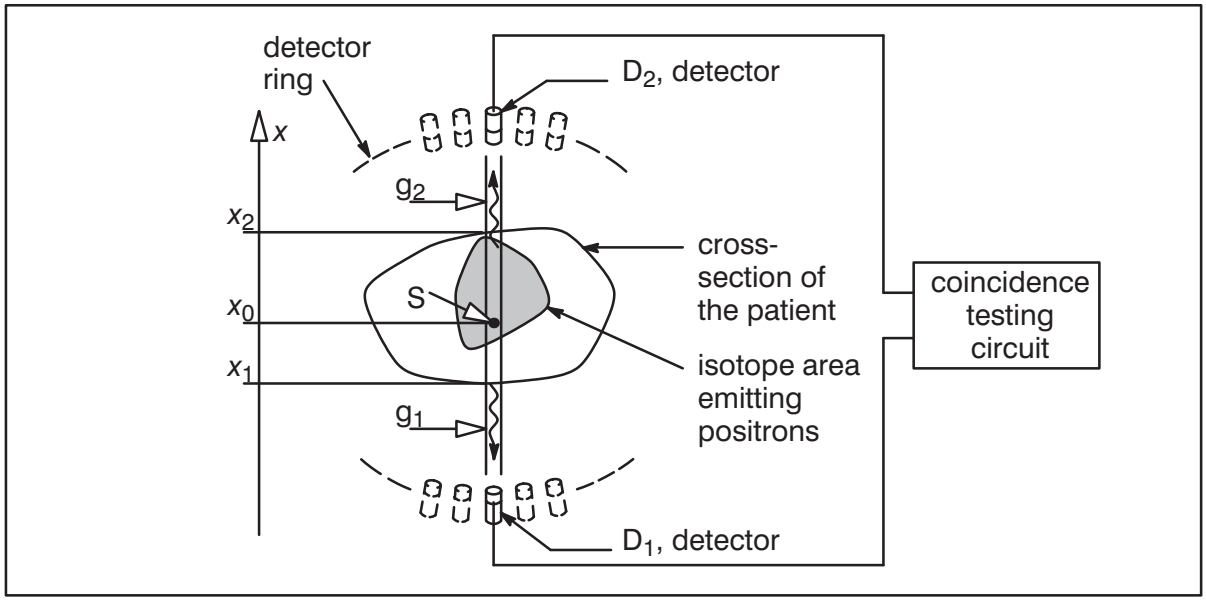

Fig. 1.12 The principle of PET.

There are two clear advantages of PET compared to SPECT. One is that no collimators are needed. Collimation is done by nature itself and the time windowing. Consequently, the photon efficiency is higher. The other advantage is easy attenuation compensation, see Fig. 1.12. From the source S (an annihilated positron-electron pair) two gamma photons g_1 and g_2 are emitted and traveling towards the detectors D_1 and D_2, respectively. The distance that g_1 travel in the body is $x_0 - x_1$ and the distance that g_2 travel in the body is $x_2 - x_0$. The photons g_1 and g_2 are then attenuated with the probability functions

$$\exp\left[-\int_{x_1}^{x_0}\mu(x)\,dx\right] \quad \text{and} \quad \exp\left[-\int_{x_0}^{x_2}\mu(x)\,dx\right], \text{ respectively,}$$

where $\mu(x)$ is the attenuation coefficient along the actual path in the body. The total attenuation μ_{tot} is independent of the source position x_0 along the actual path since

$$\begin{aligned}\mu_{tot} &= \exp\left[-\int_{x_1}^{x_0}\mu(x)\,dx\right]\cdot\exp\left[-\int_{x_0}^{x_2}\mu(x)\,dx\right] = \\ &= \exp\left[-\int_{x_1}^{x_0}\mu(x)\,dx-\int_{x_0}^{x_2}\mu(x)\,dx\right] = \exp\left[-\int_{x_1}^{x_2}\mu(x)\,dx\right].\end{aligned} \tag{1.14}$$

The attenuation factor for every line through the body can then be estimated and put into the reconstruction formula. An elegant and exact way to estimate the attenuation factors is by means of a so called transmission study, which is made as an initial step in the PET examination. During this calibration event 511 keV gamma photons are generated not by annihilated positrons but by a controllable gamma-ray source.

Typically, the resolution of the images obtained with PET are the same as for SPECT, i.e. 64, 128 or maybe 256. A disadvantage with PET is the high cost. An examination with PET requires isotopes which usually have to be produced by a cyclotron. The total system with both PET-camera and cyclotron is considerably more expensive than a SPECT-scanner.

The Linogram Method (LM) and other Direct Fourier Methods (DFM) may replace the Filtered Backprojection Method (FBM) in SPECT and PET as well as in X-ray CT. However, the LM and other DFM have their greatest computation advantage over the FBM for large image sizes, since their complexity is $\mathcal{O}(N^2\log N)$ compared to the $\mathcal{O}(N^3)$-complexity for FBM.

1.5 MRI

The images produced by MRI (Magnetic Resonance Imaging) are similar to those produced by CT in the sense that they represent cross-sections of the human body. MRI is described in for example [Mac83], [Kak87], [Edh93a], and [Weh91]. Here, we will give a simplified description of MRI in terms of classical physics as well as a simplified description of two reconstruction methods.

MRI is based on the fact that any atom with an odd number of nucleons (protons or neutrons) has a magnetic moment, which is due to the *spin* of the nucleus. When the atom is placed in a strong magnetic field, the magnetic moment of the nucleus tends to line up with the field. The magnetic moment can then be perturbed by another rotating magnetic field. This causes the magnetic moment to *precess*. After the perturbation, during the return to