

# GEOMETRY FOR COMPUTER VISION

LECTURE 6B:  
SAMPLE CONSENSUS  
STRATEGIES

# LECTURE 6B: SAMPLE CONSENSUS STRATEGIES

- ✱ LO-RANSAC

- ✱ Preemptive RANSAC

- ✱ DEGENSAC

- ✱ Today's paper: PROSAC

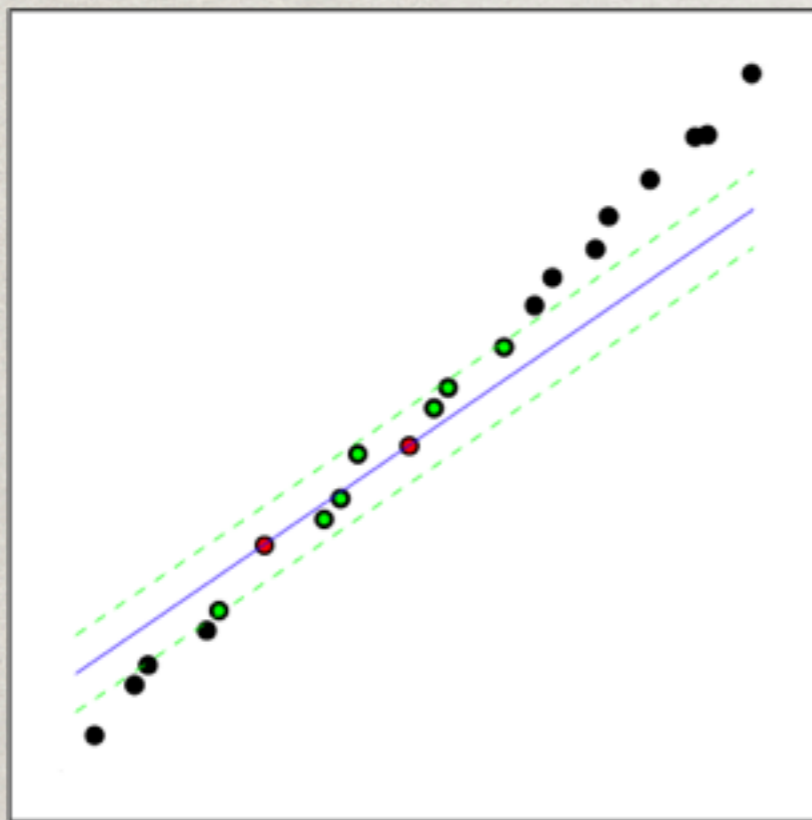
- ✱ Not covered here: All the other variants  
MLESAC, NAPSAC etc.

# RANSAC ISSUES

- ✻ In lecture 3 we introduced RANSAC (Fischler&Bolles 81).
- ✻ It finds a **model with maximal support** in the presence of **outliers**
- ✻ Approach: randomly **generate hypotheses** and **score** them.
- ✻ Most novelties since 1981 covered in thesis by: **Ondrej Chum, *Two-View Geometry Estimation by Random Sample and Consensus*, July 2005**

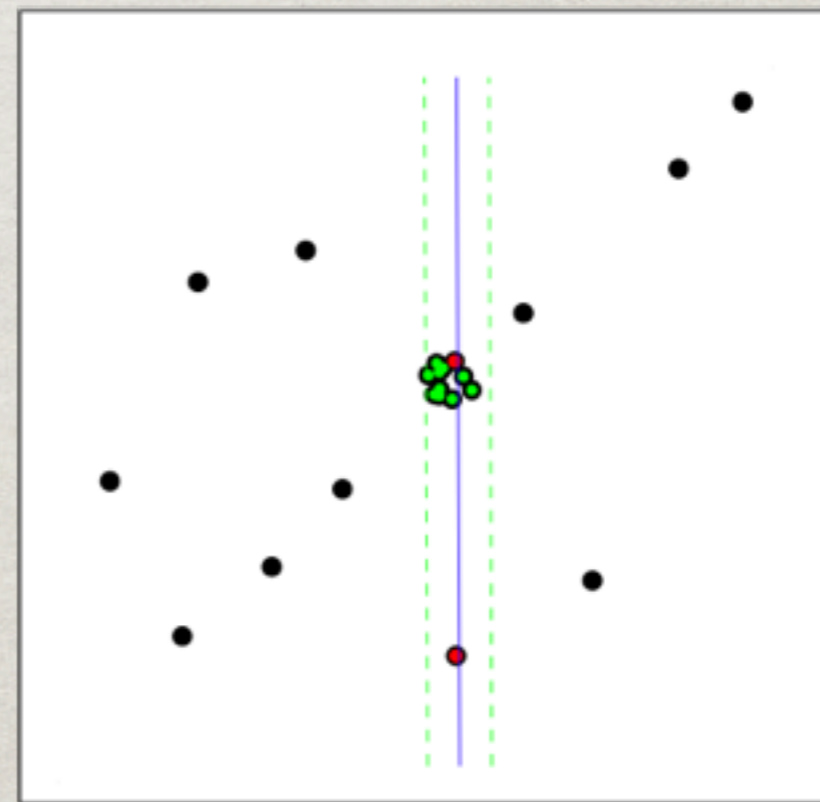
# RANSAC ISSUES

✱ Two problems with the original approach:



(a)

Inlier noise



(b)

Near degeneracies

# RANSAC ISSUES

- ✱ Near degeneracies can be dealt with by sampling non-randomly, e.g.
- ✱ DEGENSAC, for F estimation in plane dominant scenes. *Chum et al., Two-view Geometry estimation unaffected by a Dominant Plane, CVPR05*
- ✱ Distance constraint for points used in E estimation. *Hedborg et al., Fast and Accurate Structure and Motion Estimation, ISVC09*  
Reduces #iterations by 50% in forward motion.

# LO-RANSAC

- ✱ Inlier noise means that the heuristic for number of samples to draw:

$$N = \log(1 - p) / \log(1 - w^s)$$

is overly optimistic.

- ✱ A small modification makes the heuristic work again: **Chum et al., *Locally Optimized RANSAC*, DAGM03**

# LO-RANSAC

☼ Small modification

## RANSAC

loop:

1. Select random sample
2. Estimate model
3. Score model
4. If new high-score  
store model and score

# LO-RANSAC

✻ Small modification

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## LO-RANSAC

loop:

1. Select random sample
2. Estimate model
3. Score model
4. If new high-score  
**run local optimisation**  
then store model and score



# LO-RANSAC

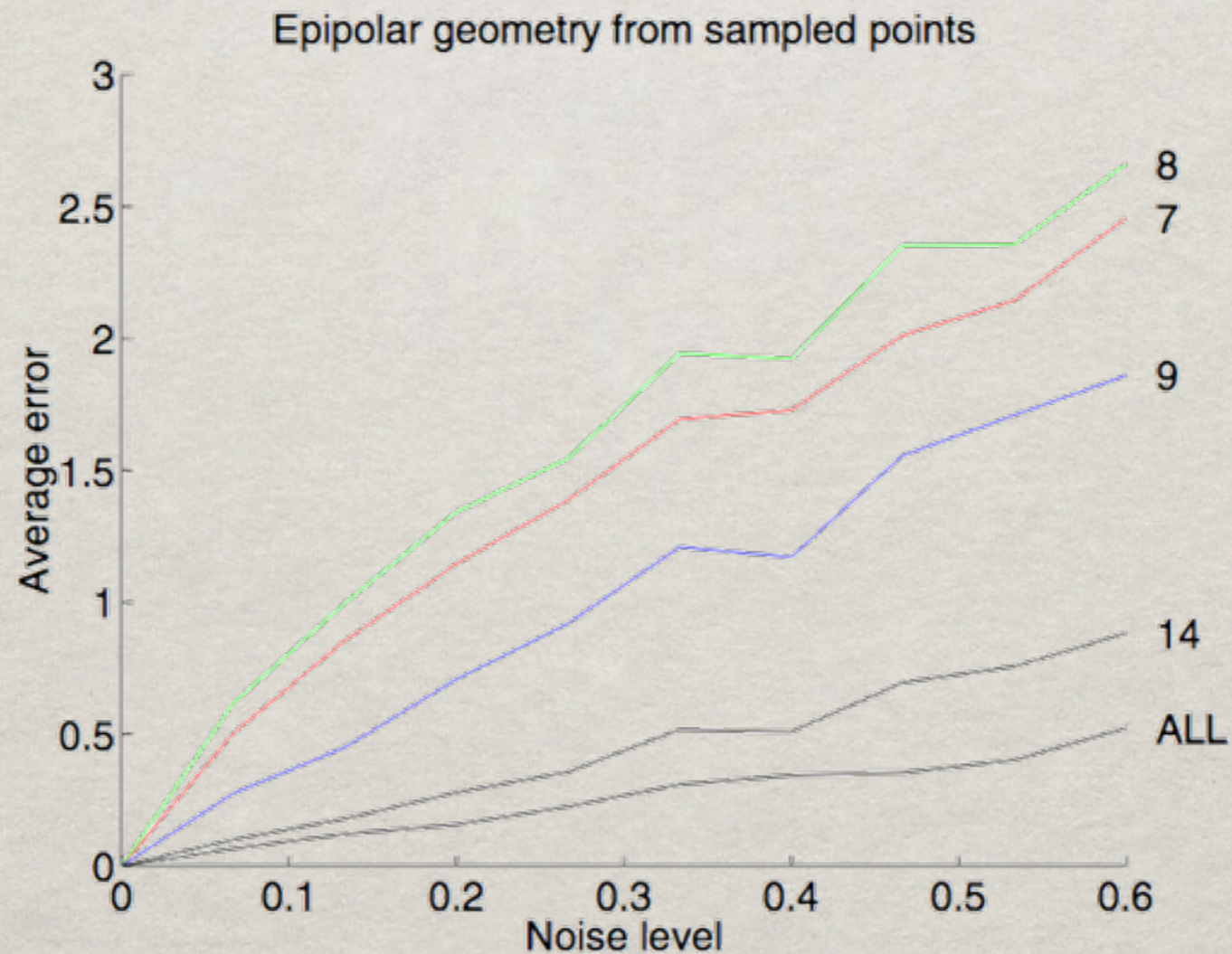
☼ Chum tries four variants of local optimisation:

1. Linear estimation from all inliers
2. Iterative linear estimation with decreasing inlier threshold.
3. Inner RANSAC
4. Inner RANSAC with #2.

☼ #2 and #4 worked best, and came close to the heuristically expected #samples.

# LO-RANSAC

- ✱ The inner RANSAC step uses non-minimal sample sets. Errors for linear  $F$  estimation:



# PREEMPTIVE RANSAC

✱ David Nister, *Preemptive RANSAC for live structure and motion estimation*, ICCV03

✱ Total time for RANSAC is given by:

$$t = k(t_M + E[m_S]t_V)$$

✱  $k$ - #iterations  $t_M$ -model estimation time,  
 $t_V$ -verification time.  $m_S$  - #models/iteration

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✱ If many correspondences,  $t_V$  will dominate.

# PREEMPTIVE RANSAC

- ✱ Idea: Do a probabilistic verification instead.

$$t = k(t_M + E[m_S]t_V)$$

- ✱ In a real-time system,  $t$  is fixed, so if we reduce  $t_V$  we may increase  $k$ .
- ✱ Preemptive RANSAC does this by evaluating all hypotheses in parallel.

# PREEMPTIVE RANSAC

## ☼ Preemptive RANSAC:

1. Generate  $f(1)$  hypotheses in parallel.
2. For  $n=1$  to  $N$
3. Evaluate  $f(n)$  hypotheses on a random correspondence
4. Keep the  $f(n+1)$  best hypotheses according to accumulated score.

☼  $f(1)=M$  and  $f(n+1) \leq f(n)$

# PREEMPTIVE RANSAC

- ✱  $f(n)$  - the preemption function

$$f(n) = \lfloor M2^{-\lfloor \frac{n}{B} \rfloor} \rfloor$$

- ✱  $B$  - block size ( $f$  only changes every  $B$  steps)

- ✱  $M$  - number of models

- ✱ Accumulated scoring  $L(m) = \sum_{n=1}^N \rho(n, m)$

- ✱ Log-likelihood of sample  $n$  given model  $m$

$$\rho(n, m)$$

# DEGENSAC

- ✻ Chum, et al., *Two-view Geometry Estimation Unaffected by a Dominant Plane*, CVPR'05
- ✻ Planar **dominant** scenes are also problematic





# DEGENSAC

- ✱ Actually, the  $F$  estimation problem is even worse than it might appear, as 5 points in a plane +2 *arbitrary* correspondences gives an  $F$  compatible with the plane.

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- ✱ Actually, the  $\mathbf{F}$  estimation problem is even worse than it might appear, as 5 points in a plane +2 *arbitrary* correspondences gives an  $\mathbf{F}$  compatible with the plane.
- ✱ In le5 we saw that if all seven points are in a plane, then

$$\mathbf{x}_k^T \mathbf{F} \mathbf{y}_k = 0, \quad \mathbf{x}_k = \mathbf{H} \mathbf{y}_k, \quad k = 1 \dots 7$$

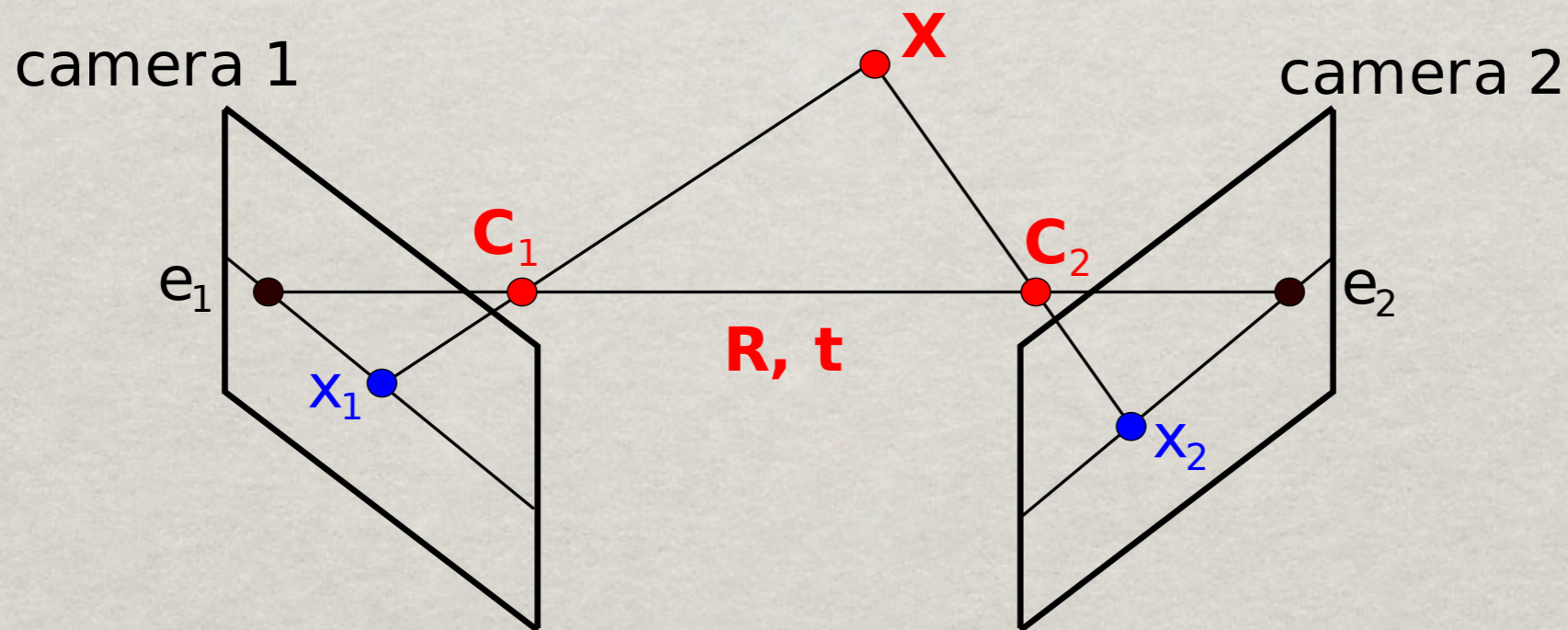
and  $\mathbf{F} = [\mathbf{e}]_{\times} \mathbf{H}$  for any epipole  $\mathbf{e}$  (**why epipole?**)

# DEGENSAC

✱ If *six* points are in a plane

$$\mathbf{x}_k^T \mathbf{F} \mathbf{y}_k = 0, \quad k = 1 \dots 7 \quad \mathbf{x}_k = \mathbf{H} \mathbf{y}_k, \quad k = 1 \dots 6$$

$$\mathbf{F} = [\mathbf{e}]_{\times} \mathbf{H} \quad \text{for } \mathbf{e} \in \mathbb{R}^3, \quad \mathbf{e}^T (\mathbf{H} \mathbf{x}_7 \times \mathbf{y}_7) = 0$$



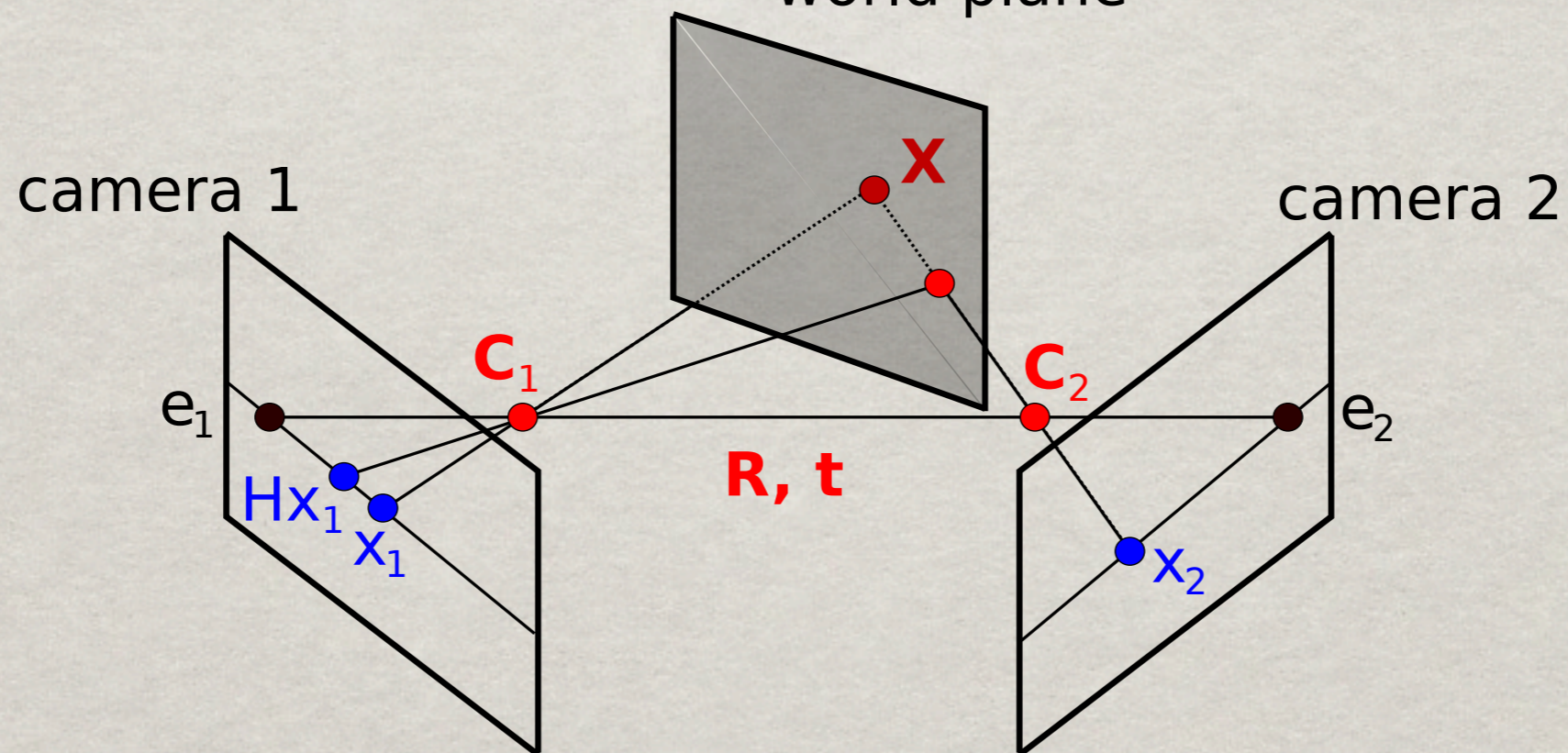
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world plane



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✱ For *five* points in the plane

$$\mathbf{x}_6 \times (\mathbf{H} \mathbf{y}_6) \quad \text{and} \quad \mathbf{x}_7 \times (\mathbf{H} \mathbf{y}_7)$$

define two lines that intersect in  $\mathbf{e}$ .  $\mathbf{F}$  will have all points consistent with  $\mathbf{H}$  as inliers.

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✱ Also used in plane+parallax algorithm

# DEGENSAC

✱ From  $\mathbf{F}$  and  $\{\mathbf{x}_k \leftrightarrow \mathbf{y}_k\}_{k=1}^3$  we can compute a homography

$$\mathbf{H} = \mathbf{A} - \mathbf{e}_1(\mathbf{M}^{-1}\mathbf{b})^T$$

where  $\mathbf{A} = [\mathbf{e}_1]_{\times}\mathbf{F}$        $\mathbf{M} = [\mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{x}_3]^T$

and  $b_k = (\mathbf{x}_k \times \mathbf{A}\mathbf{y}_k)^T (\mathbf{x}_k \times \mathbf{e}_1) \|\mathbf{x}_k \times \mathbf{e}_1\|^{-2}$

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- ✱ This  $\mathbf{H}$  is now checked for two additional inliers. If found,  $\mathbf{F}$  is said to be  $\mathbf{H}$ -degenerate



# DEGENSAC

- ✱ There are  $\binom{7}{5} = 21$  ways to pick five points from 7.
- ✱ But, if we pick the 3 points that define  $H$  as  $\{1, 2, 3\}, \{4, 5, 6\}, \{1, 2, 7\}, \{4, 5, 7\}, \{3, 6, 7\}$
- ✱ We will have covered all 21 permutations.
- ✱ Thus at most **five**  $H$  need to be computed and tested to find out if  $F$  is  $H$ -degenerate.

# DEGENSAC

## ✱ DEGENSAC algorithm

1. Select 7 random correspondences and estimate  $F$
2. IF best support this far
3. IF  $H$ -degeneracy
4. Do inner RANSAC and estimate  $F$   
from  $H$  and 2 correspondences  
(Plane+Parallax algorithm)  
that are inconsistent with  $H$
5. IF new  $F$  has even bigger support, store  $F$
6. ELSE store  $H$

# DISCUSSION

- ✻ Discussion of the paper:  
Ondrej Chum and Jiri Matas, *Matching with PROSAC -- Progressive Sample Consensus*,  
CVPR'05

# FOR NEXT WEEK...

- ✻ Hartley & Zisserman, Appendix A4.3
- ✻ K. Shoemake, *Animating Rotation with Quaternion Curves*, SIGGRAPH85