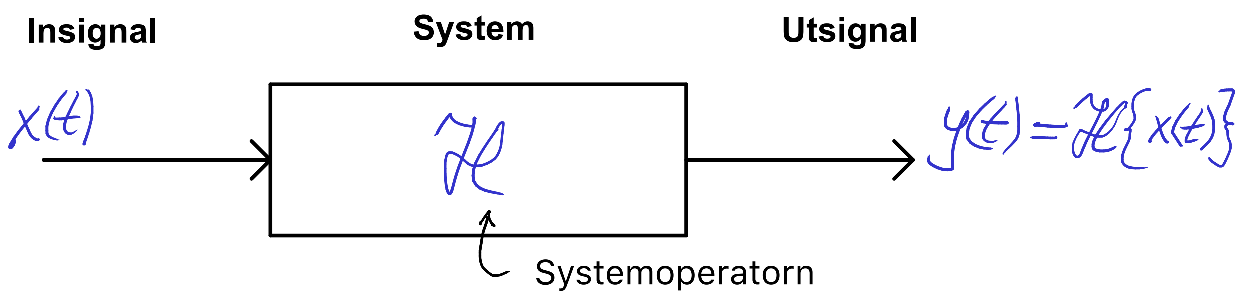
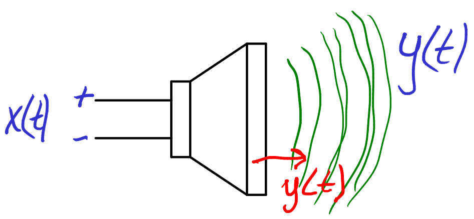
Signaler & System – Föreläsning 1: Inledning, signal- och systemegenskaper

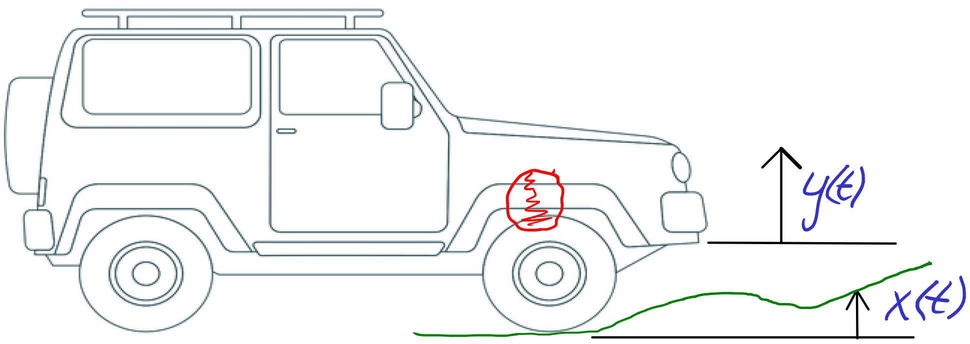


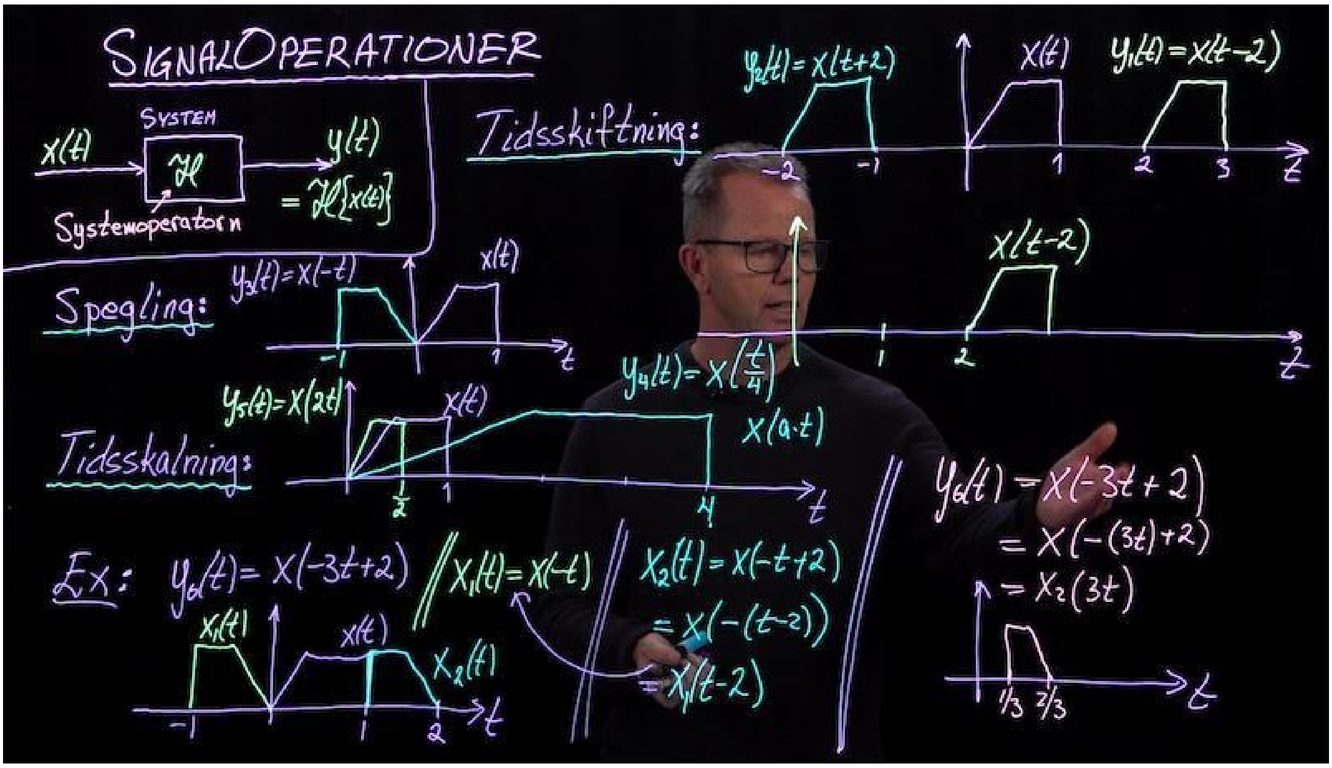
Ett SYSTEM = en matematisk modell av ett fysikaliskt system (alt. en algoritm) som för olika insignaler x genererar olika utsignaler y

En SIGNAL = en informationsbärande matematisk funktion som representerar en (ofta mätbar) fysikalisk storhet.

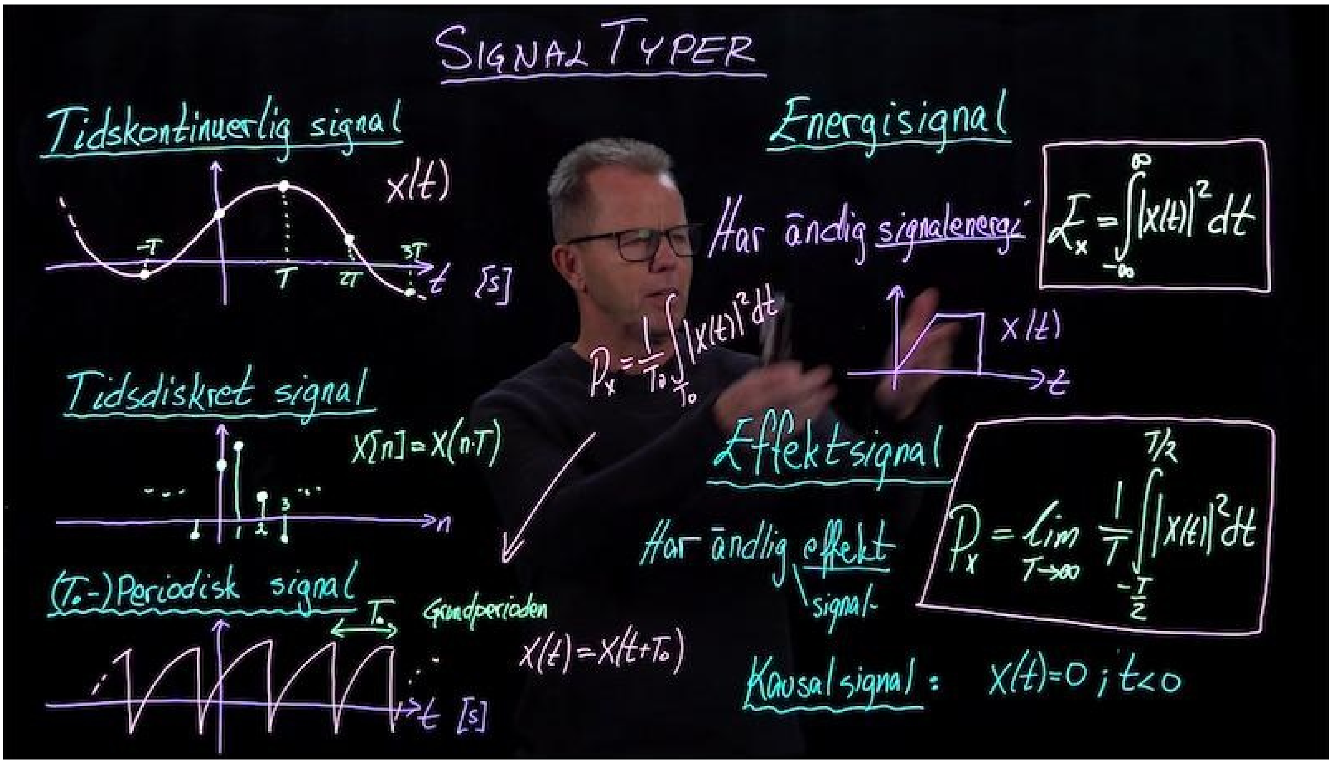
Exempel:



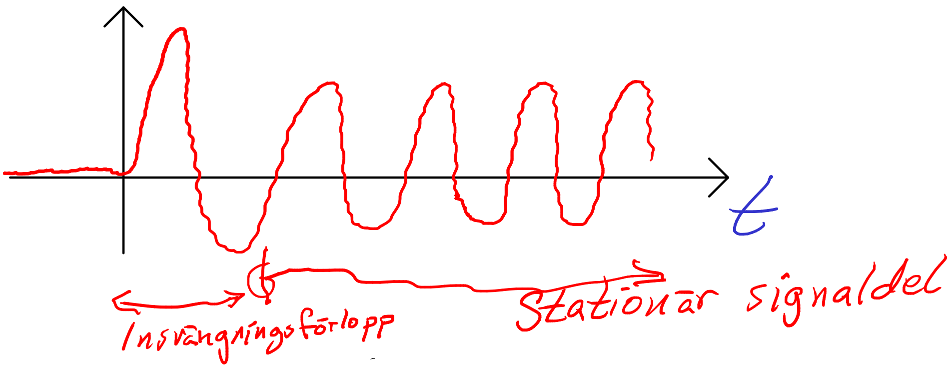




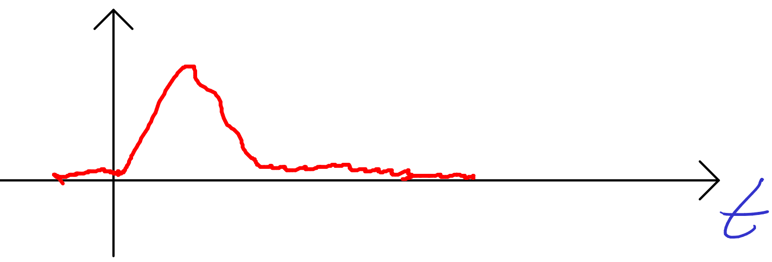
SIGNALOPERATIONER
SYSTEM
x(t)
y(t)
= H{x(t)}
Systemoperatorn
Tidsskiftning:
y2(t) = x(t+2)
x(t)
y1(t) = x(t-2)
x(t-2)
Spegling:
y3(t) = x(-t)
x(t)
y4(t) = x(t/4)
Tidsskalning:
y5(t) = x(2t)
x(t)
x(a·t)
Ex: y6(t) = x(-3t+2)
x1(t) = x(-t)
x2(t) = x(-t+2)
= x(-(t-2))
= x1(t-2)
x1(t)
x(t)
x2(t)
y6(t) = x(-3t+2)
= x(-(3t)+2)
= x2(3t)

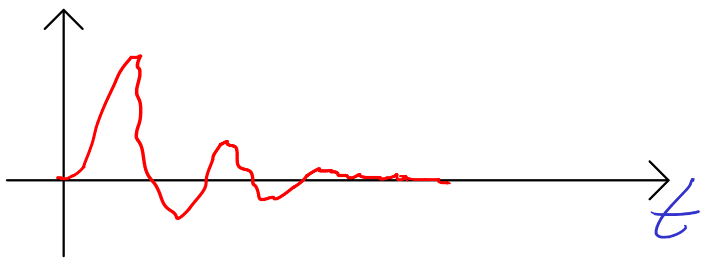


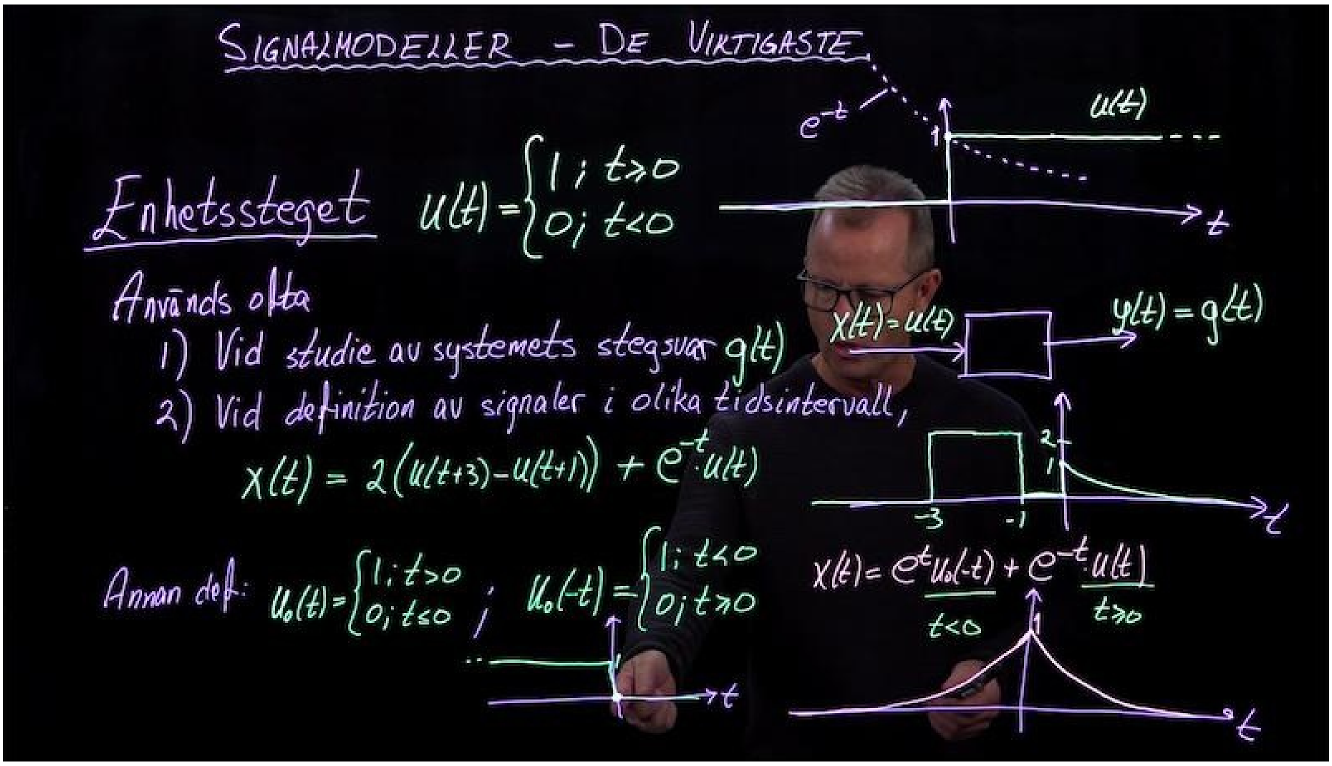
Stationär signal

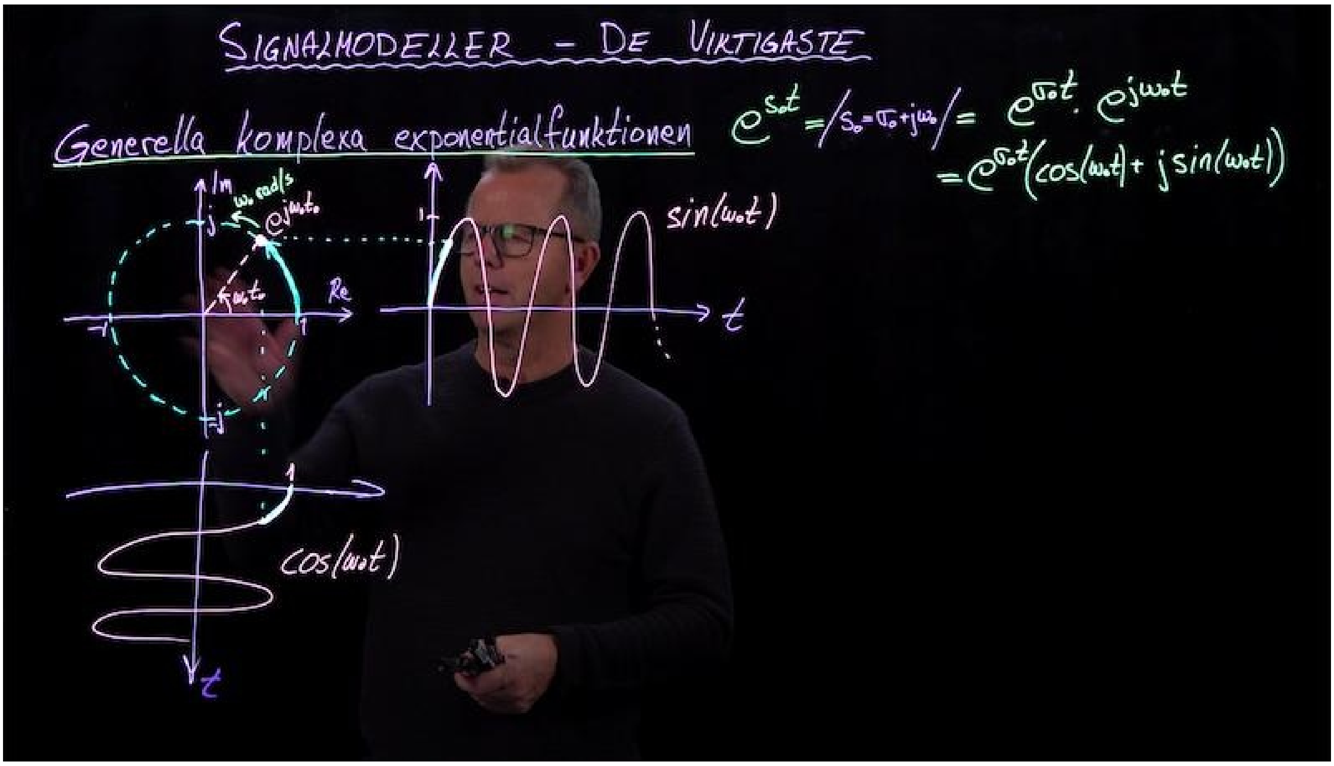


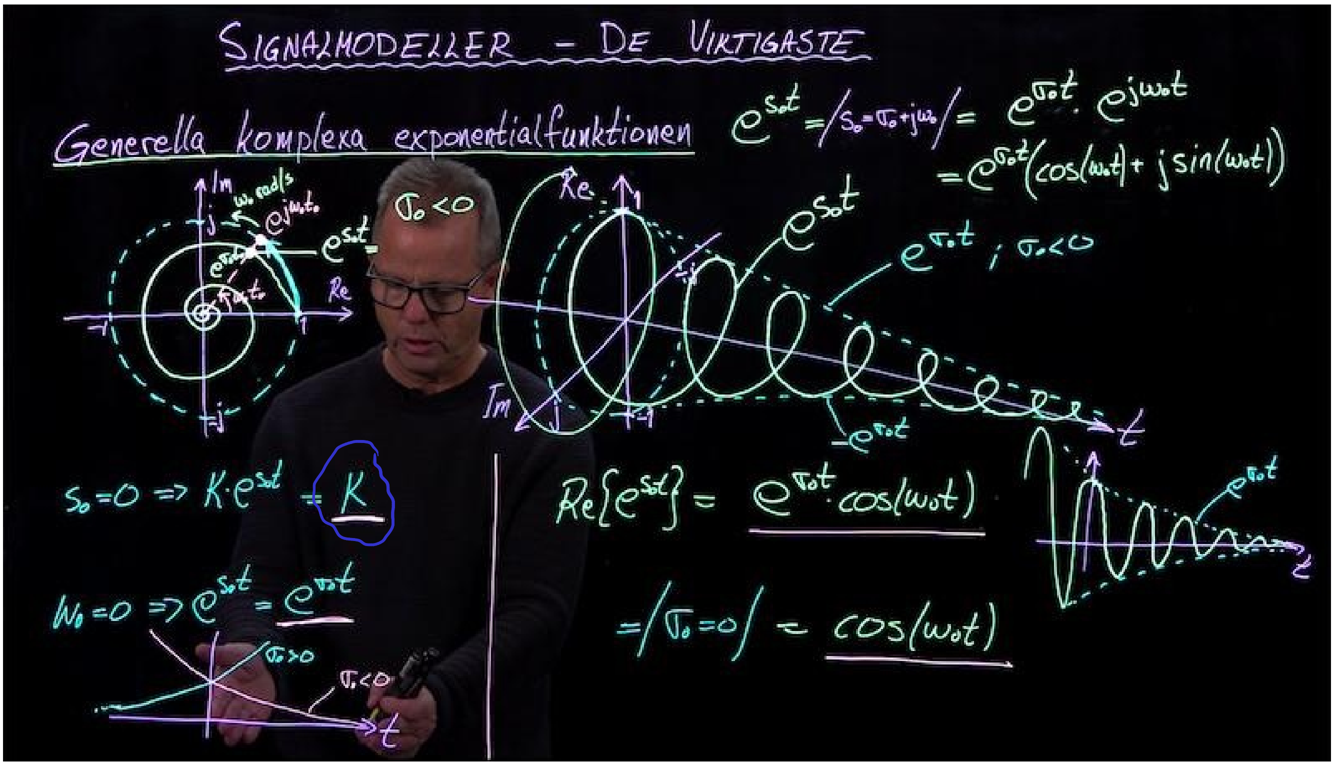
Transient signal



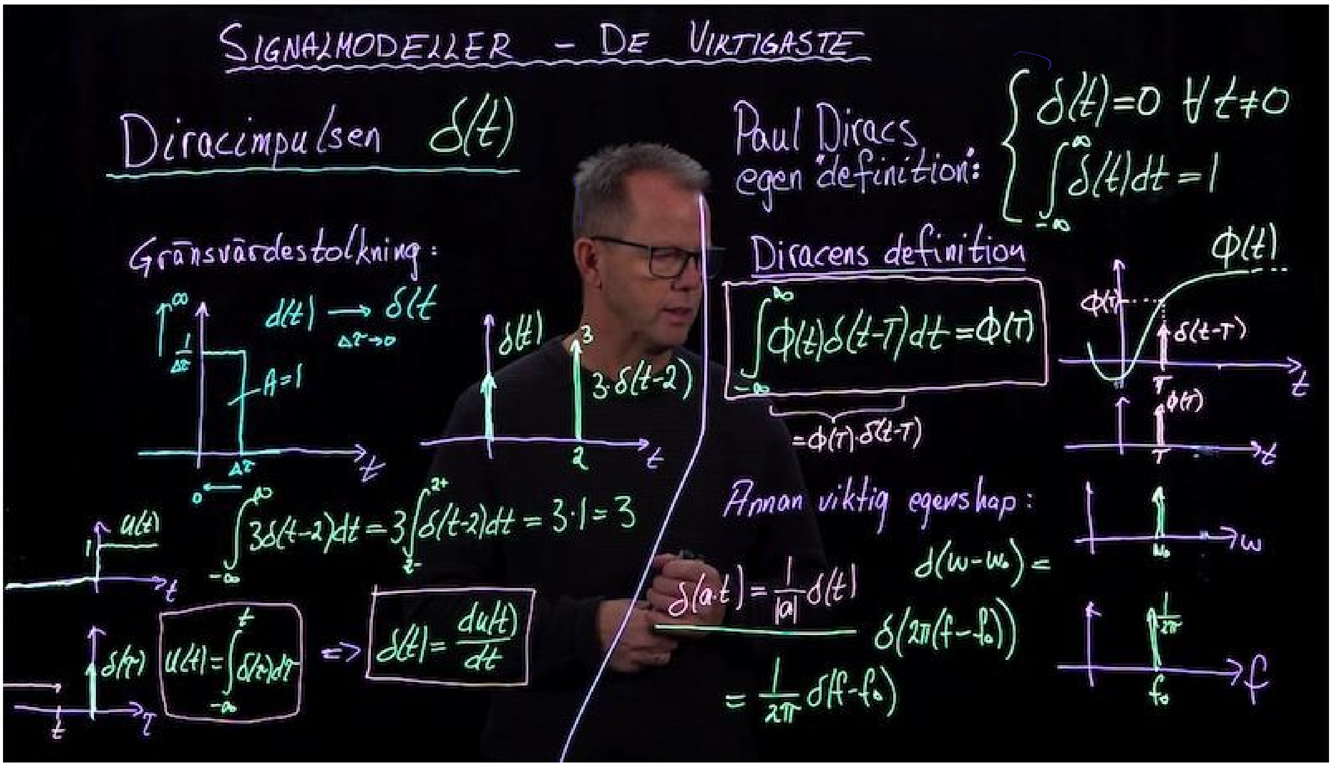



SIGNALMODELLER - DE VIKTIGASTE
Enhetssteget
$u(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$
e^{-t}
$u(t)$
Används ofta
1) Vid studie av systemets stegsvar $g(t)$
2) Vid definition av signaler i olika tidsintervall,
$x(t) = 2(u(t+3) - u(t+1)) + e^{-t} \cdot u(t)$
$x(t) = u(t)$
$y(t) = g(t)$
Annan def: $u_0(t) = \begin{cases} 1; & t > 0 \\ 0; & t \leq 0 \end{cases}$; $u_0(-t) = \begin{cases} 1; & t < 0 \\ 0; & t \geq 0 \end{cases}$
$x(t) = \underbrace{e^{t} u_0(-t)}_{t<0} + \underbrace{e^{-t} \cdot u(t)}_{t \geq 0}$
t


SIGNALMODELLER – DE VIKTIGASTE
Generella komplexa exponentialfunktionen
$e^{s_0 t} = / s_0 = \sigma_0 + j\omega_0 / = e^{\sigma_0 t} \cdot e^{j\omega_0 t}$
$= e^{\sigma_0 t}(\cos(\omega_0 t) + j\sin(\omega_0 t))$
Im
ω_0 rad/s
$e^{j\omega_0 t_0}$
j
$\omega_0 t_0$
Re
-1
1
-j
1
$\sin(\omega_0 t)$
t
$\cos(\omega_0 t)$
t

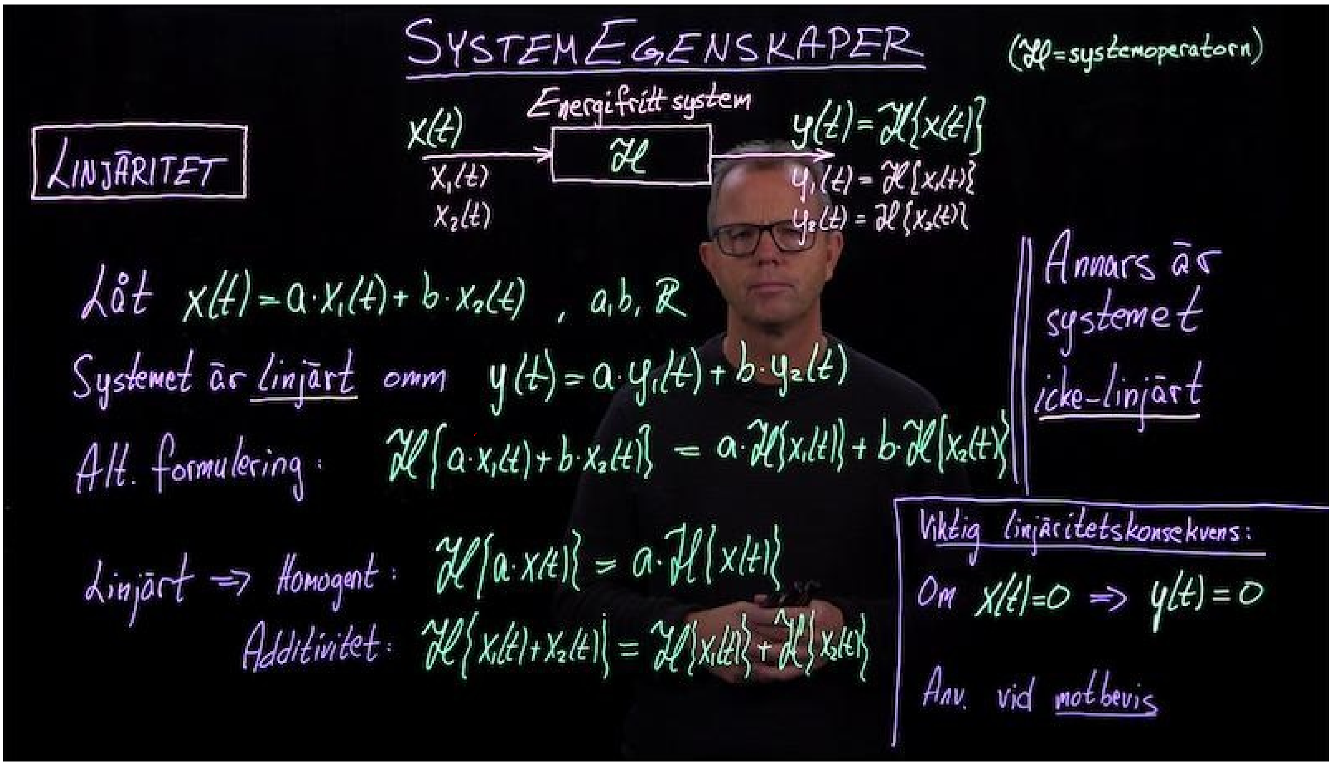


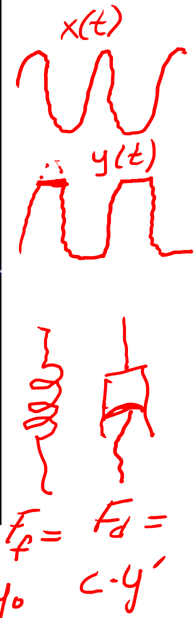
$$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} ds \qquad s = j\omega \qquad x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$



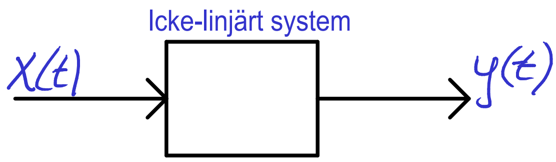
Exempel:

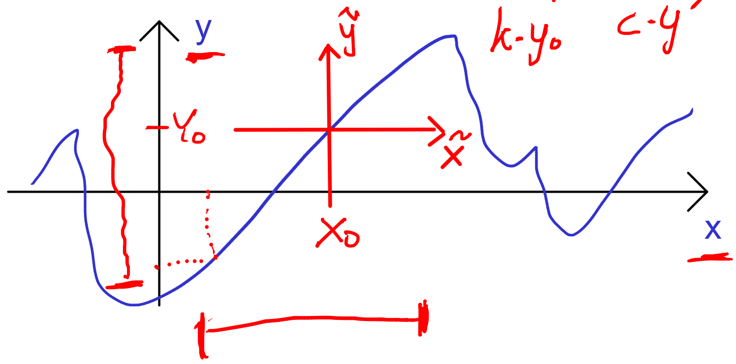
$$\boxed{x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau}$$



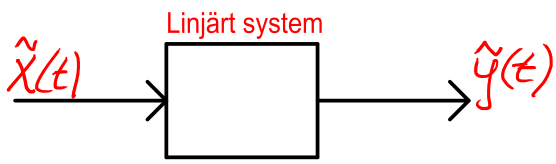


De flesta fysikaliska system är inte linjära.

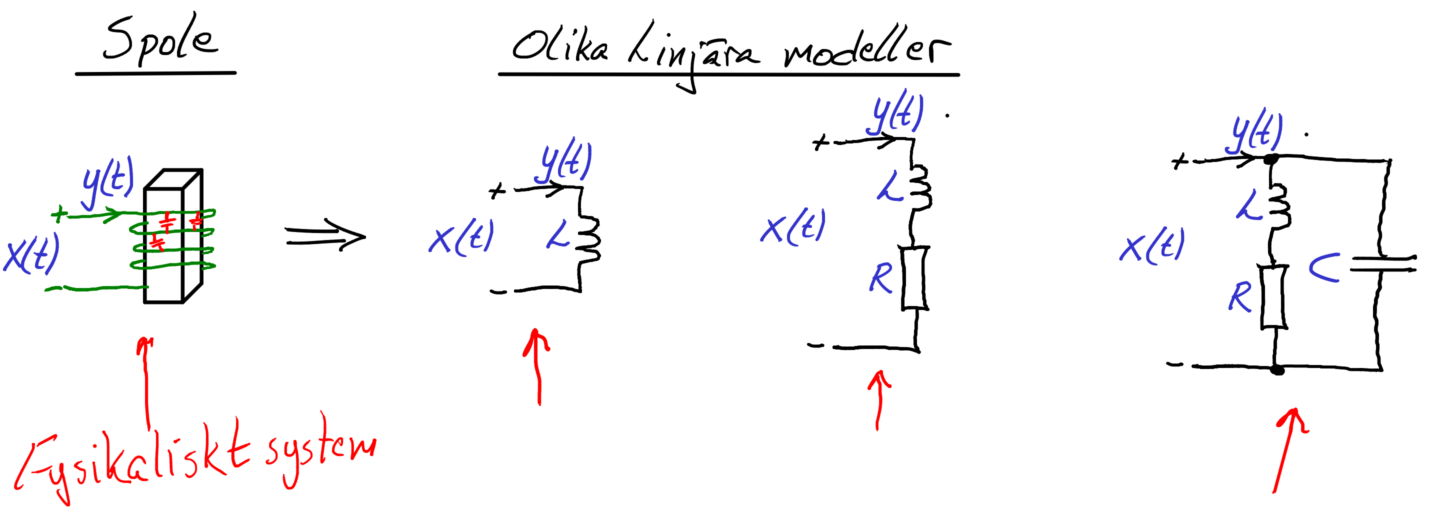




Linjärisera $\Longrightarrow$



Exempel:



Exempel – test av linjäritet

(betrakta energifritt system)

$y(t) = \mathcal{H}\{x(t)\} \Rightarrow y_n(t) = \mathcal{H}\{x_n(t)\}$

Låt $x(t) = a \cdot x_1(t) + b \cdot x_2(t)$ Om linjärt $\Rightarrow y(t) = a \cdot y_1(t) + b \cdot y_2(t)$, $a, b \in \mathbb{R}$

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① $y(t) = t \cdot x(t-4) \Rightarrow y_n(t) = t \cdot x_n(t-4)$

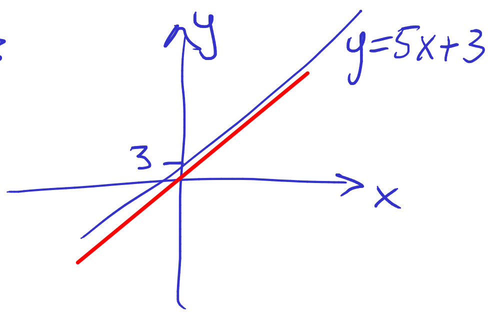
$$y(t) = t(a \cdot x_1(t-4) + b \cdot x_2(t-4)) = a \cdot \underbrace{t \cdot x_1(t-4)}_{} + b \cdot \underbrace{t \cdot x_2(t-4)}_{}$$

$$= a \cdot y_1(t) + b \cdot y_2(t)$$

$\Rightarrow$ Systemet är linjärt

② $y(t) = 5x(t) + 3 \Rightarrow y_n(t) = 5x_n(t) + 3$

Test: låt $x(t) = 0 \Rightarrow y(t) = 3 \neq 0$



$\Rightarrow$ Systemet är icke-linjärt

Testa själv: $y(t) = 5(a \cdot x_1(t) + b \cdot x_2(t)) + \underline{\underline{3}}$

$= \ldots$

③ $$\frac{dy(t)}{dt} + 2y(t) = 3\frac{dx(t)}{dt} \Rightarrow \frac{dy_n(t)}{dt} + 2y_n(t) = 3\frac{dx_n(t)}{dt}$$

$$\left(y_n' + 2y_n = 3x_n'\right) \star$$

$$x = a\cdot x_1 + b\cdot x_2$$

HL: $$3x' = 3(a\cdot x_1 + b\cdot x_2)' = a\cdot 3x_1' + b\cdot 3x_2' \overset{\star}{=}$$

$$= a(y_1' + 2y_1) + b(y_2' + 2y_2)$$

$$= (ay_1 + by_2)' + 2(a\cdot y_1 + b\cdot y_2) = $$ /VL i diff.ekv/

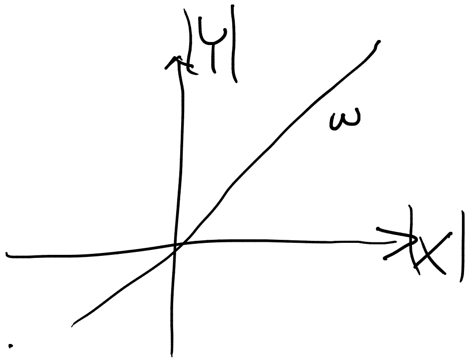
$$= y' + 2\cdot y$$

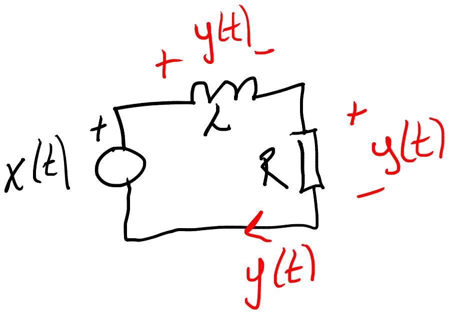
Identifiera $\Rightarrow y(t) = a\cdot y_1(t) + b\cdot y_2(t)$

$\Rightarrow$ Systemet är linjärt

$$y(t) = \frac{dx(t)}{dt}$$

$$Y(\omega) = j\omega\cdot X(\omega)$$





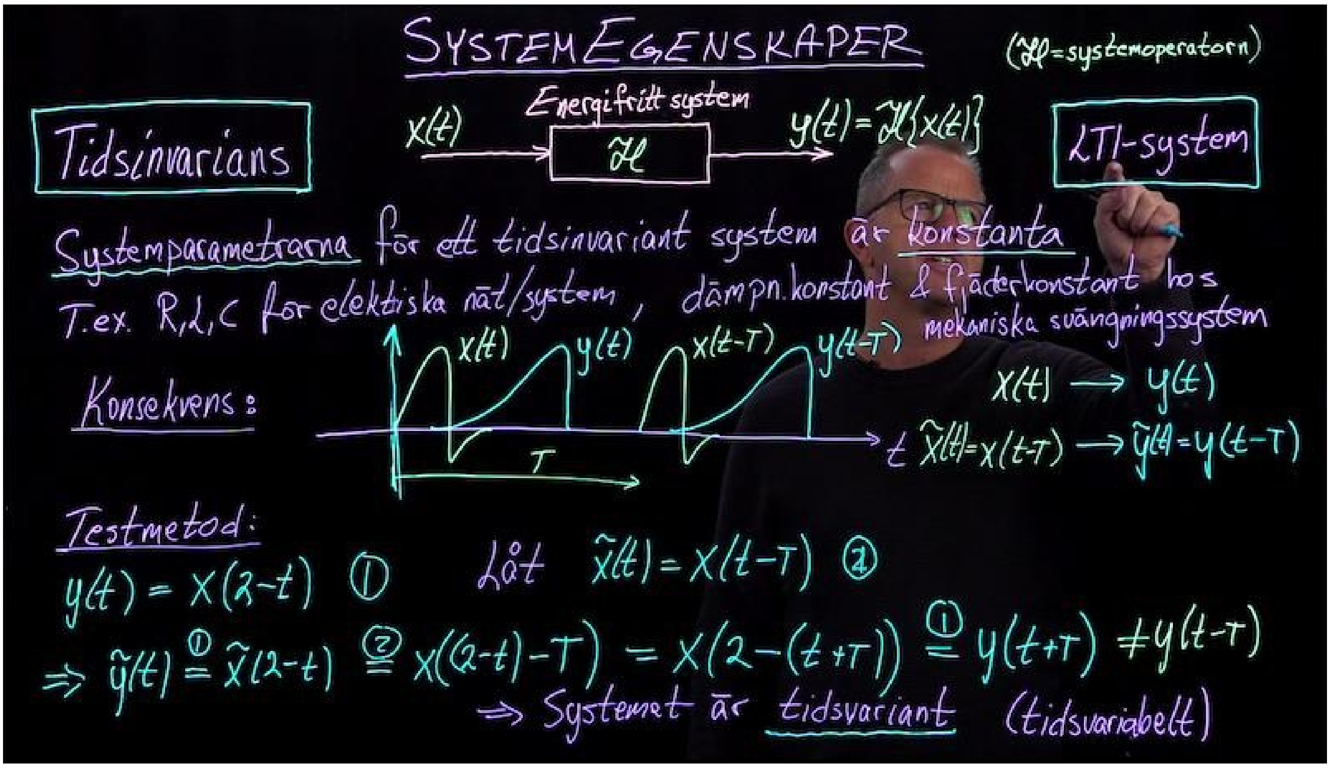
$$\frac{dy(t)}{dt} + a\cdot y(t) = b\frac{dx(t)}{dt} + cx(t)$$

$a = RL$

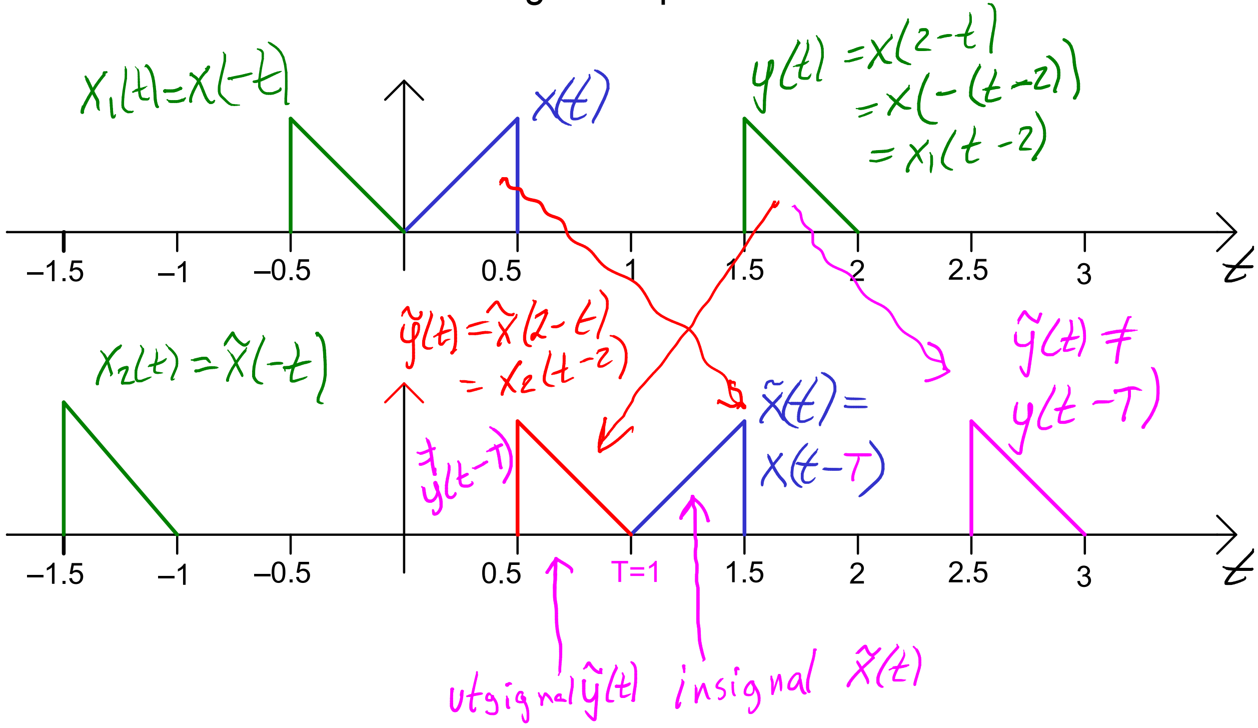
m

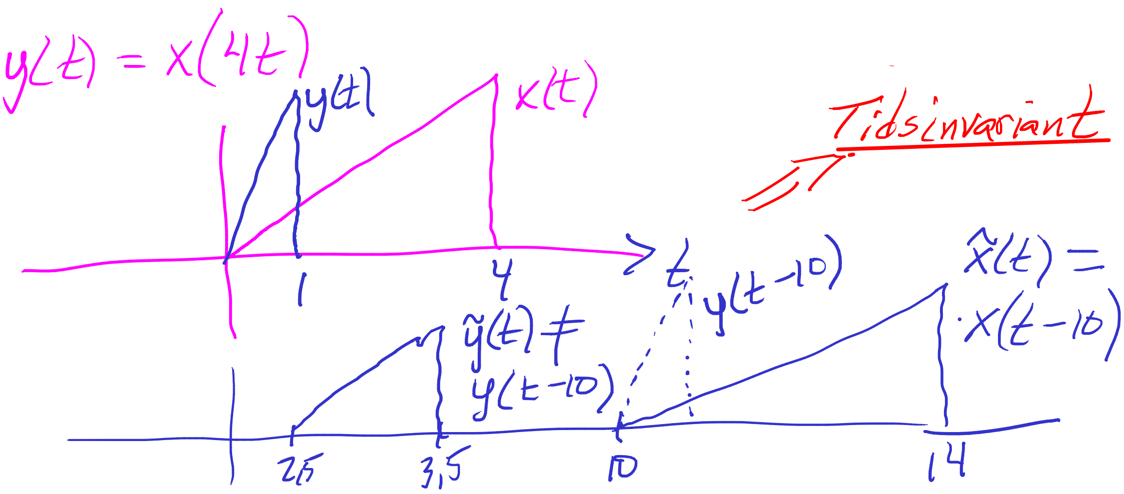
k

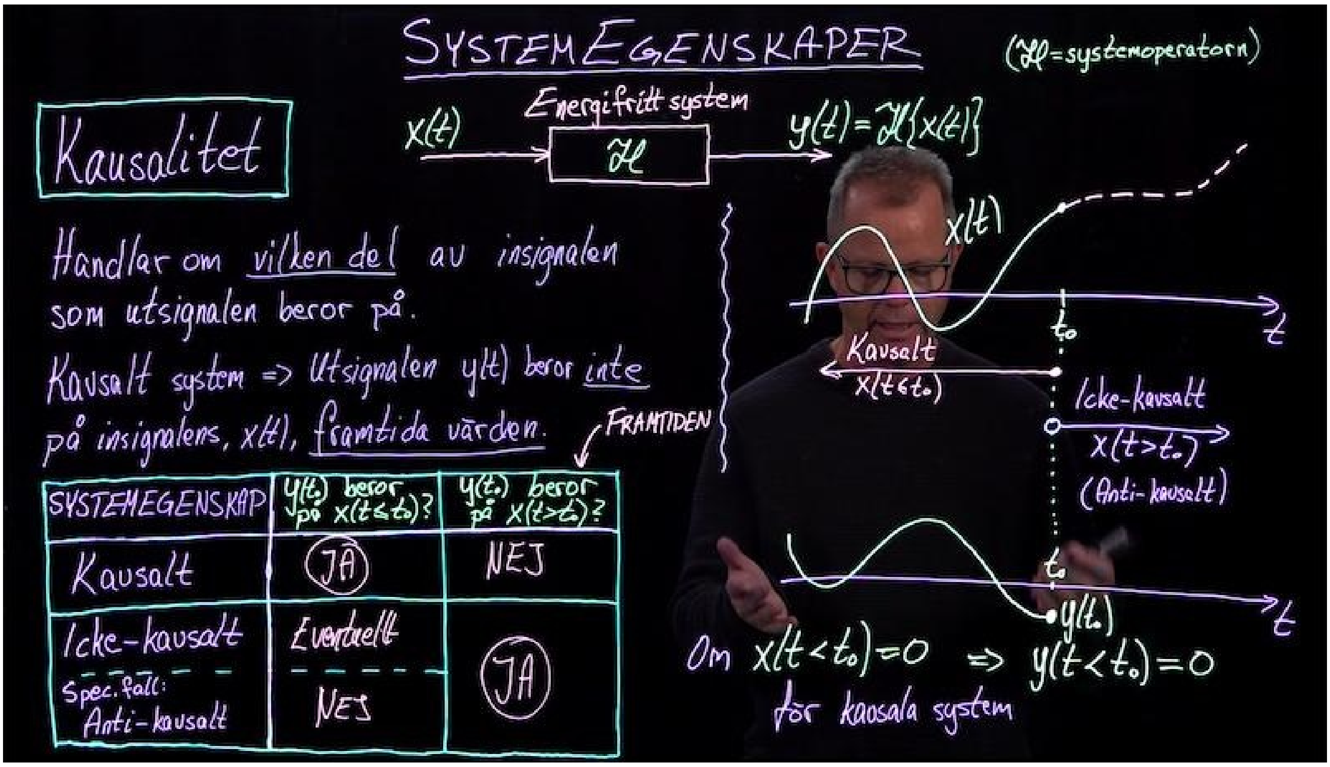
c



Grafisk illustration av föreläsningsexemplet



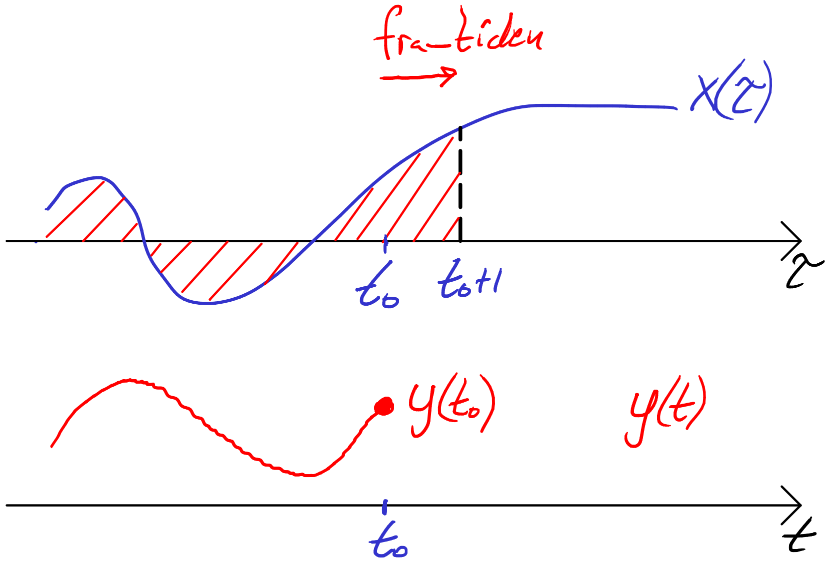




Exempel

① $y(t_0) = \sqrt{x(t_0 - 2)}$ Kausalt

② $y(t) = \int_{-\infty}^{t+1} x(\tau)\,d\tau$ Icke-kausalt

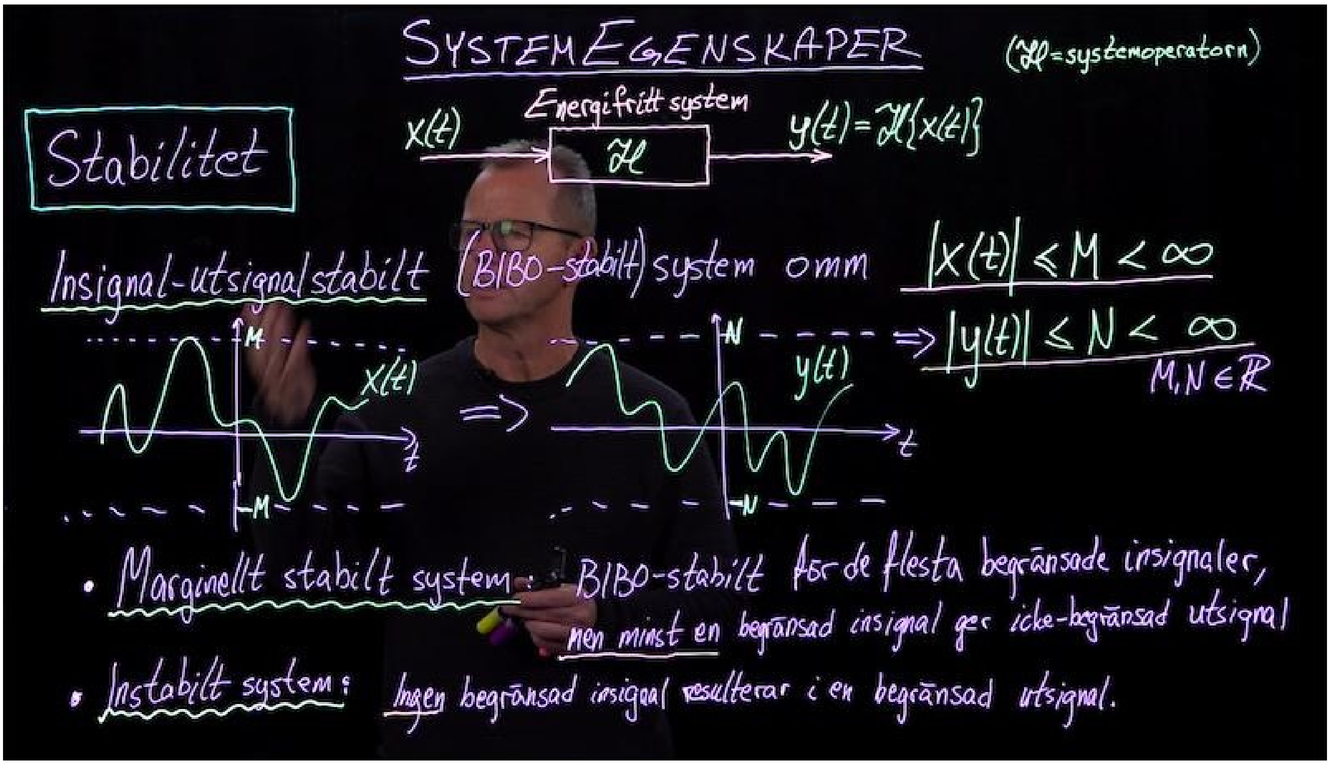


③ $y(t) = x(t+3)$ Icke-kausalt   Anti-kausalt

④ $y(t) = x(-t)$

$y(3) = x(-3)$
$y(-4) = x(4)$

Icke-kausalt



Exempel:

① $y(t) = 5x(t-2) + 3$ — Stabilt

② $y(t) = \dfrac{dx(t-3)}{dt}$ — $|y|$ $\forall$ $|x|$ utom om $x(t)$ inneh. diskontinuitet(er) $\Rightarrow \frac{dx}{dt} = \delta(t)$

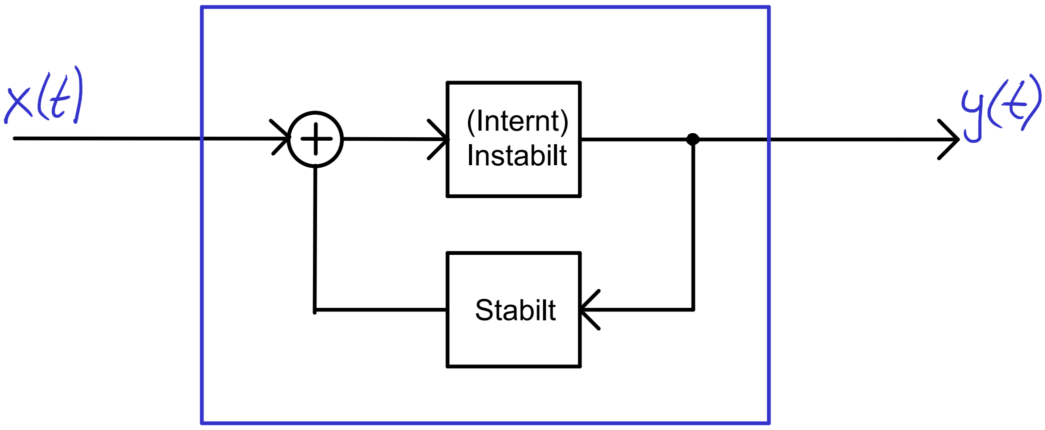
Marginellt stabilt

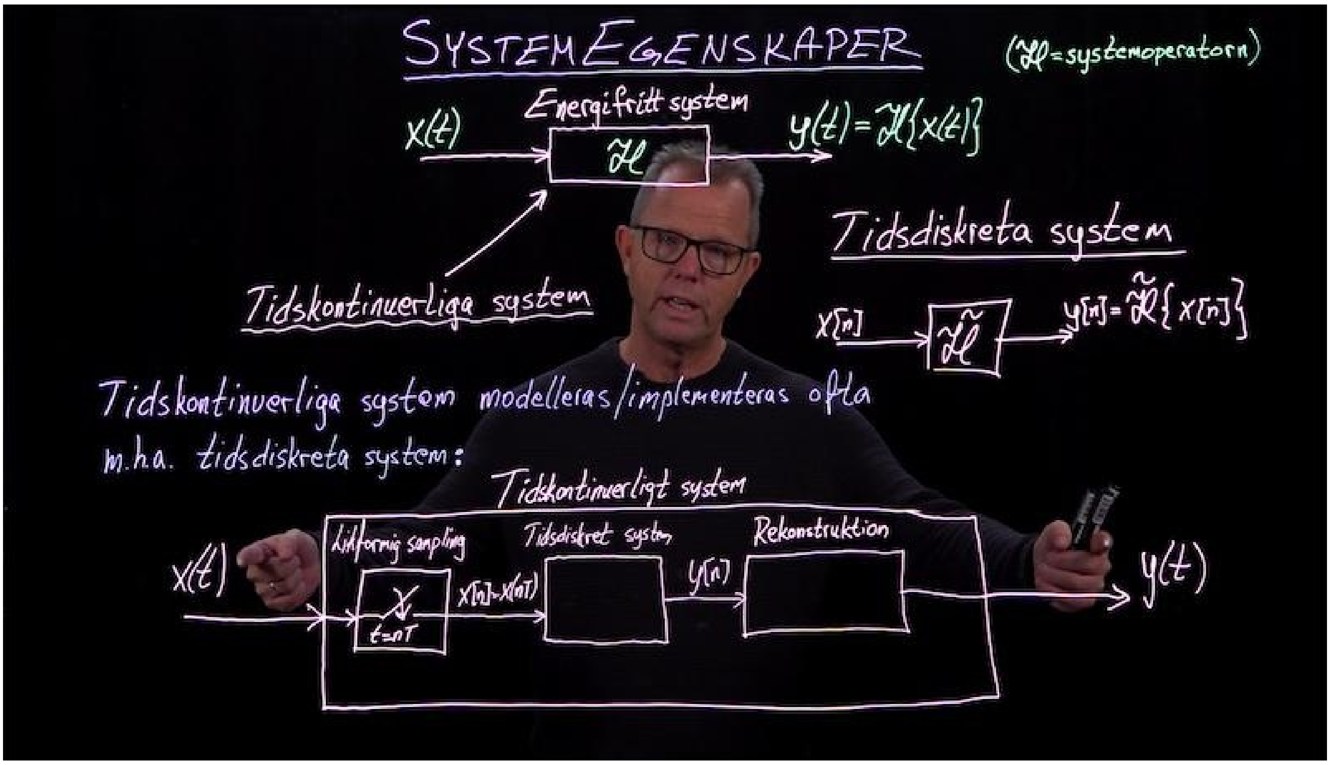
③ $\dfrac{dy(t)}{dt} - y(t) = x(t)$

Stabilt om icke-kausalt
Instabilt om kausalt
} Se senare i kursen!

Stabilisering av instabila system med hjälp av återkoppling (kommer senare i kursen)

Externt stabilt (Insignal-utsignalstabilt)





Tidskontinuerlig signal – Signalen är kontinuerlig längs tidsaxeln

Analog signal – Signalen kan anta alla värden

Tidsdiskret signal – Signalen är diskret längs tidsaxeln

Digital signal – Signalens värden är diskretiserade
(t.e.x. på grund av binär representation)