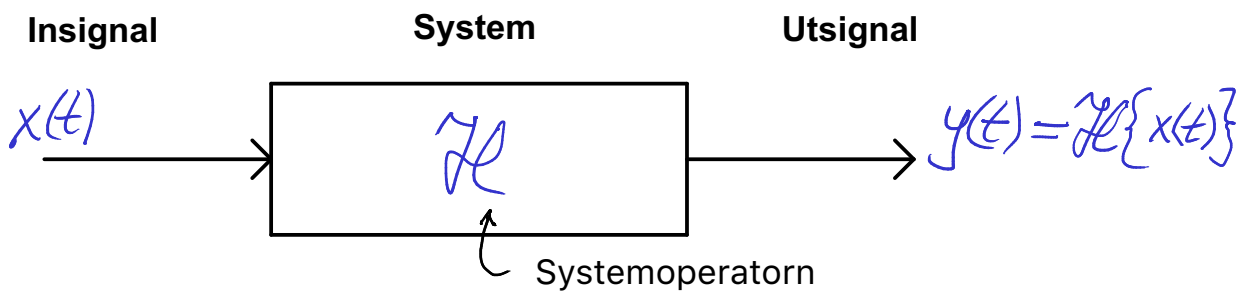


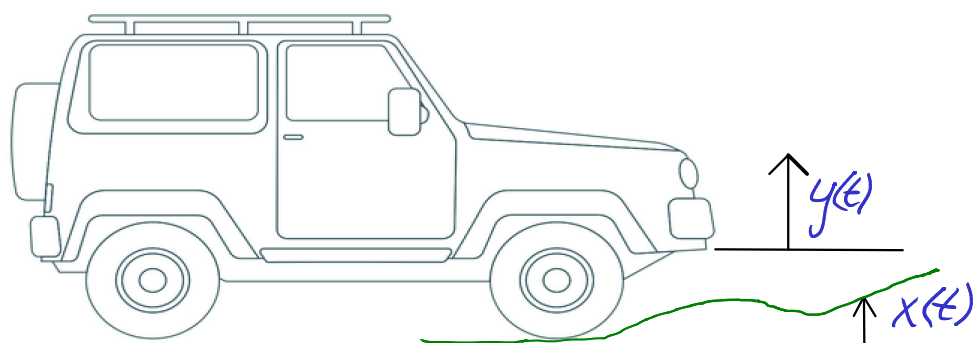
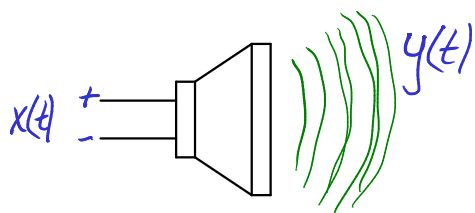
# Signaler & System – Föreläsning 1: Inledning, signal- och systemegenskaper



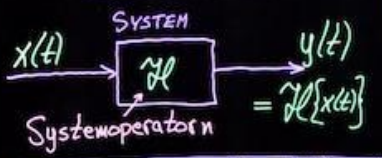
Ett SYSTEM = en matematisk modell av ett fysikaliskt system (alt. en algoritm) som för olika insignaler  $x$  genererar olika utsignaler  $y$

En SIGNAL = en informationsbärande matematisk funktion som representerar en (ofta mätbar) fysikalisk storhet.

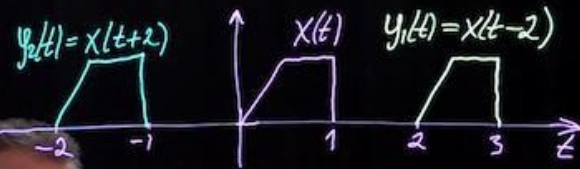
Exempel:



# SIGNALOPERATIONER



Tidsskiftning:



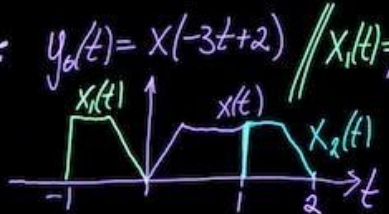
Spejling:



Tidsskalning:

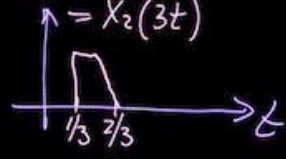


Ex:



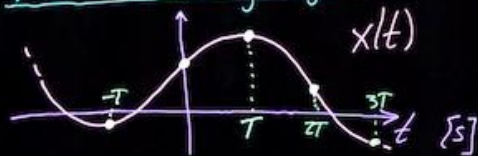
$$\begin{aligned}
 x_1(t) &= x(-t) \\
 x_2(t) &= x(-t+2) \\
 &= x(-(t-2)) \\
 &= x_1(t-2)
 \end{aligned}$$

$$\begin{aligned}
 y_6(t) &= x(-3t+2) \\
 &= x(-(3t)+2) \\
 &= x_2(3t)
 \end{aligned}$$



# SIGNAL TYPER

## Tidskontinuerlig signal



## Energisignal

Har ändlig signalenergi

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

## Tidsdiskret signal



$$P_x = \frac{1}{T_0} \int_{T_0}^{\infty} |x(t)|^2 dt$$



## Effekt signal

Har ändlig effekt signal

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

## (T<sub>0</sub>-)Periodisk signal



$$x(t) = x(t+T_0)$$

Kausal signal:  $x(t) = 0; t < 0$

Stationär signal



Transient signal



# SIGNALMODELLER - DE VIKTIGASTE

Enhetssteget  $u(t) = \begin{cases} 1; & t > 0 \\ 0; & t < 0 \end{cases}$



Används ofta

- 1) Vid studie av systemets stegsvar  $g(t)$
- 2) Vid definition av signaler i olika tidsintervall,

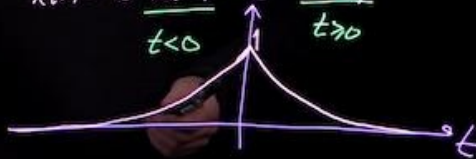
$$x(t) = 2(u(t+3) - u(t+1)) + e^{-t} \cdot u(t)$$



Annars def:  $u_0(t) = \begin{cases} 1; & t > 0 \\ 0; & t < 0 \end{cases}$ ;  $u_0(-t) = \begin{cases} 1; & t < 0 \\ 0; & t > 0 \end{cases}$

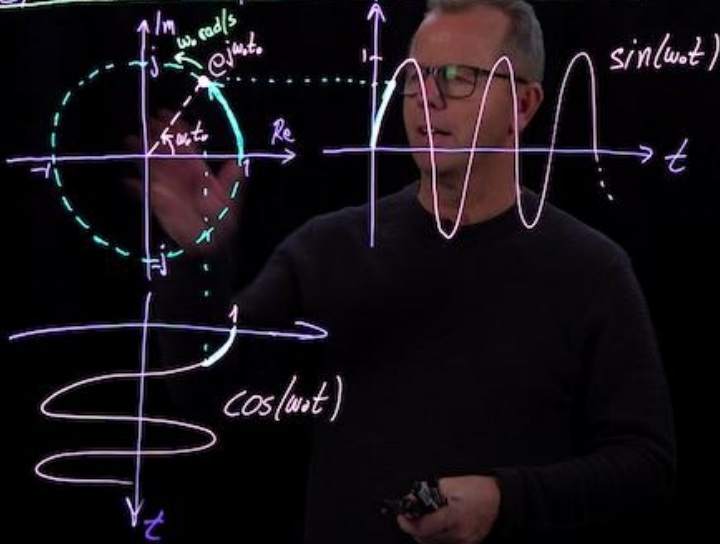


$$x(t) = \begin{cases} e^t u_0(-t) & t < 0 \\ e^{-t} u(t) & t > 0 \end{cases}$$



# SIGNALMODELLER - DE VIKTIGASTE

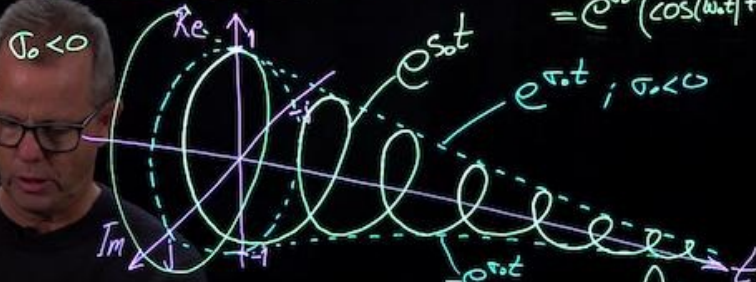
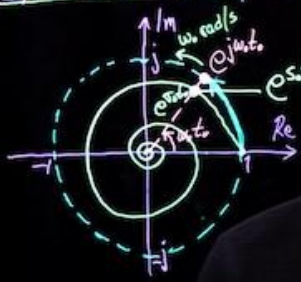
Generella komplexa exponentialfunktioner  $e^{s_0 t} = /s_0 = \sigma_0 + j\omega_0/ = e^{\sigma_0 t} \cdot e^{j\omega_0 t}$   
 $= e^{\sigma_0 t} (\cos(\omega_0 t) + j \sin(\omega_0 t))$



# SIGNALMODELLER - DE VIKTIGASTE

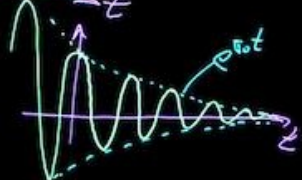
Generella komplexa exponentialfunktioner

$$e^{s_0 t} = /s_0 = \sigma_0 + j\omega_0/ = e^{\sigma_0 t} \cdot e^{j\omega_0 t} = e^{\sigma_0 t} (\cos(\omega_0 t) + j \sin(\omega_0 t))$$



$$s_0 = 0 \Rightarrow K \cdot e^{s_0 t} = K$$

$$\text{Re}\{e^{s_0 t}\} = \underline{e^{\sigma_0 t} \cos(\omega_0 t)}$$



$$\omega_0 = 0 \Rightarrow e^{s_0 t} = e^{\sigma_0 t}$$

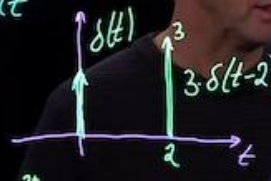
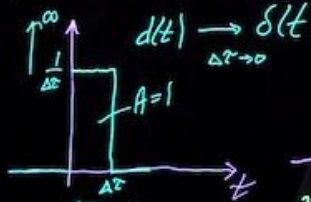
$$= /s_0 = 0/ = \underline{\cos(\omega_0 t)}$$



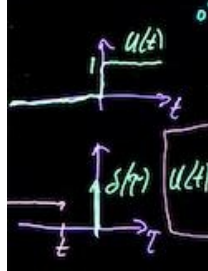
# SIGNALMODELLER - DE VIKTIGASTE

## Diracimpulsen $\delta(t)$

Gränsvärdetolkning:



$$\int_{-\infty}^{\infty} 3\delta(t-2) dt = 3 \int_{-\infty}^{\infty} \delta(t-2) dt = 3 \cdot 1 = 3$$



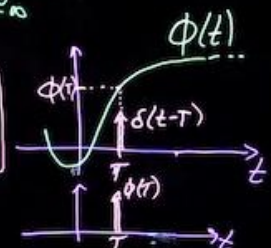
$$\delta(t) = \frac{du(t)}{dt}$$

Paul Diracs egen definition:  $\delta(t) = 0 \forall t \neq 0$   
 $\int_{-\infty}^{\infty} \delta(t) dt = 1$

Diracs definition

$$\int_{-\infty}^{\infty} \phi(t) \delta(t-\tau) dt = \phi(\tau)$$

$$= \phi(\tau) \delta(t-\tau)$$



Annor viktig egenskap:

$$\delta(a \cdot t) = \frac{1}{|a|} \delta(t) \quad d(\omega - \omega_0) = \delta(2\pi(f - f_0)) = \frac{1}{2\pi} \delta(f - f_0)$$



Exempel:

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t-\tau) d\tau$$

# SYSTEMEGENSKAPER

( $\mathcal{L}$  = systemoperatorn)

## LINJÄRITET



Låt  $x(t) = a \cdot x_1(t) + b \cdot x_2(t)$ ,  $a, b, \mathbb{R}$

Systemet är linjärt om  $y(t) = a \cdot y_1(t) + b \cdot y_2(t)$

Alt. formulering:  $\mathcal{L}\{a \cdot x_1(t) + b \cdot x_2(t)\} = a \cdot \mathcal{L}\{x_1(t)\} + b \cdot \mathcal{L}\{x_2(t)\}$

Linjärt  $\Rightarrow$  Homogent:  $\mathcal{L}\{a \cdot x(t)\} = a \cdot \mathcal{L}\{x(t)\}$

Additivitet:  $\mathcal{L}\{x_1(t) + x_2(t)\} = \mathcal{L}\{x_1(t)\} + \mathcal{L}\{x_2(t)\}$

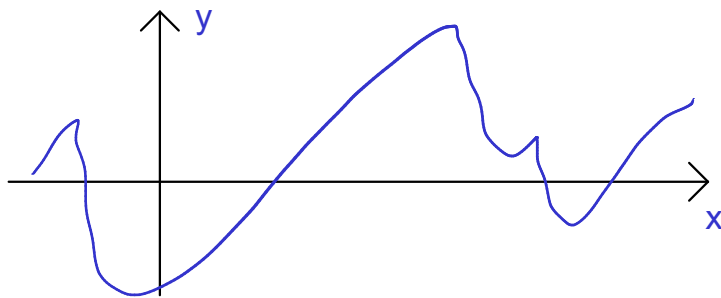
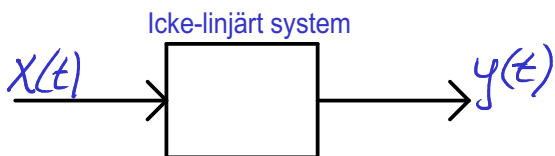
Annars är systemet icke-linjärt

Viktig linjäritetskonsekvens:

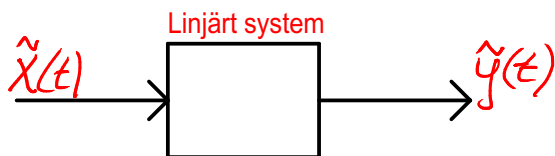
Om  $x(t) = 0 \Rightarrow y(t) = 0$

Anv. vid motbevis

De flesta fysikaliska system är inte linjära.



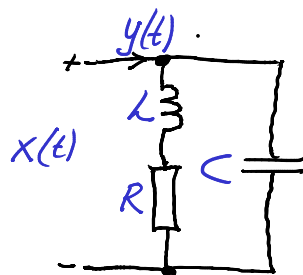
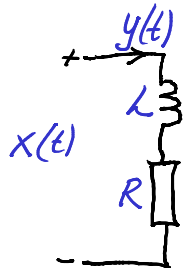
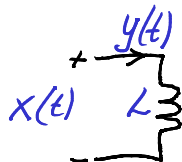
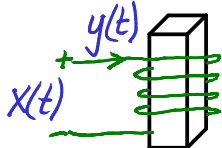
Linjärisera  $\Rightarrow$



Exempel:

Spole

Olika linjära modeller





## Exempel – test av linjäritet

(betrakta energifritt system)

$$y(t) = \mathcal{H}\{x(t)\} \Rightarrow y_n(t) = \mathcal{H}\{x_n(t)\}$$

Låt  $x(t) = a \cdot x_1(t) + b \cdot x_2(t)$  Om linjärt  $\Rightarrow y(t) = a \cdot y_1(t) + b \cdot y_2(t)$ ,  $a, b \in \mathbb{R}$

---

①  $y(t) = t \cdot x(t-4)$   $\Rightarrow y_n(t) = t \cdot x_n(t-4)$

②  $y(t) = 5x(t) + 3$   $\Rightarrow y_n(t) = 5x_n(t) + 3$

$$\textcircled{3} \quad \underline{\frac{dy(t)}{dt} + 2y(t) = 3 \frac{dx(t)}{dt}} \quad \Rightarrow \quad \frac{dy_n(t)}{dt} + 2y_n(t) = 3 \frac{dx_n(t)}{dt}$$
$$(y_n' + 2y_n = 3x_n')$$

# SYSTEM EGENSKAPER

( $\mathcal{A}$  = systemoperatorn)

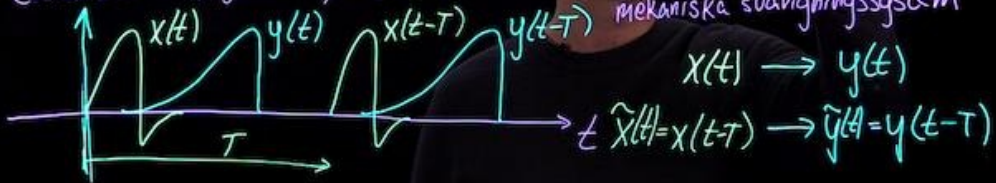
Tidsinvarians



LTI-system

Systemparametrarna för ett tidsinvariant system är konstanta  
 T.ex. R, L, C för elektriska nät/system, dämpn. konstant & fjäderkonstant hos mekaniska svängningssystem

Konsekvens:



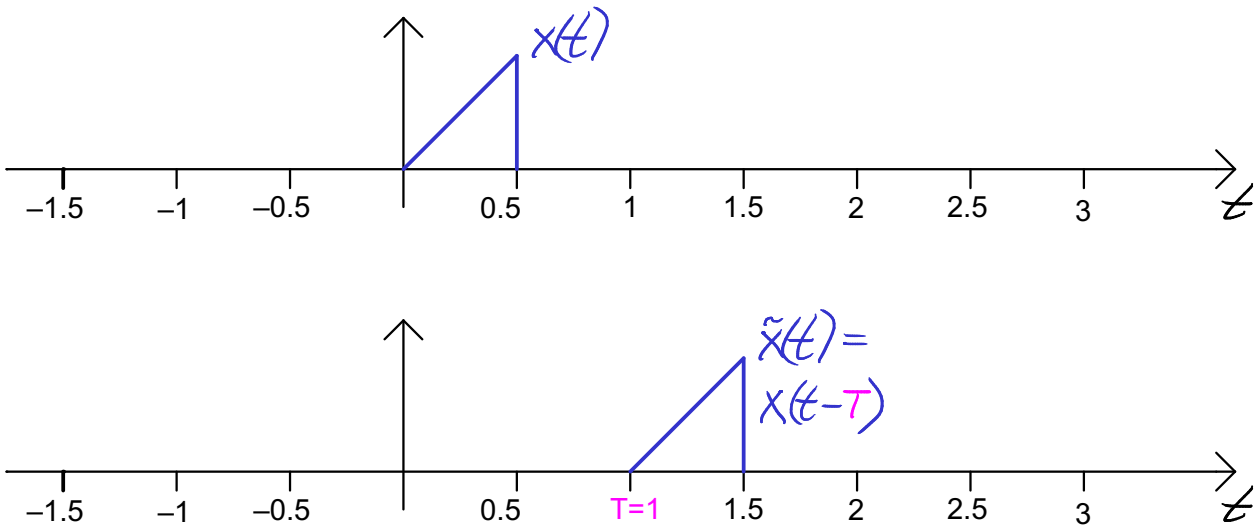
Testmetod:

$$y(t) = x(2-t) \quad \textcircled{1} \quad \text{Låt } \tilde{x}(t) = x(t-T) \quad \textcircled{2}$$

$$\Rightarrow \tilde{y}(t) \stackrel{\textcircled{1}}{=} \tilde{x}(2-t) \stackrel{\textcircled{2}}{=} x(2-t-T) = x(2-(t+T)) \stackrel{\textcircled{1}}{=} y(t+T) \neq y(t-T)$$

$\Rightarrow$  Systemet är tidsvariant (tidsvariabelt)

Grafisk illustration av föreläsningsexemplet



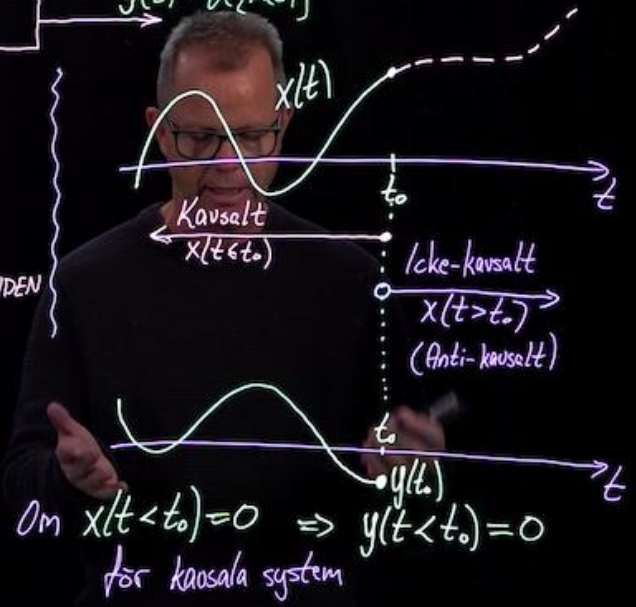
# SYSTEMEGENSKAPER ( $\mathcal{A}$ = systemoperatorn)

## Kausalitet



Handlar om vilken del av insignalen som utsignalen beror på.  
 Kausalt system  $\Rightarrow$  Utsignalen  $y(t)$  beror inte på insignalens,  $x(t)$ , framtidiga värden.

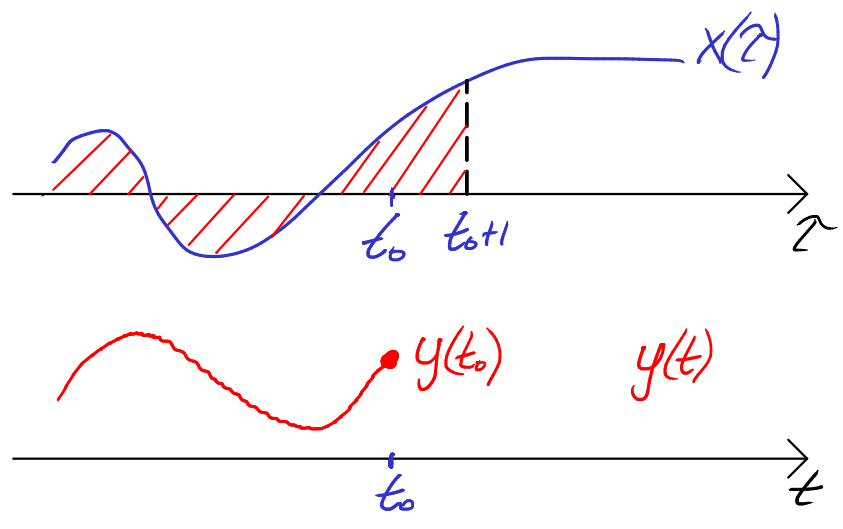
SYSTEMEGENSKAP	$y(t)$ beror på $x(t \leq t_0)$ ?	$y(t_0)$ beror på $x(t > t_0)$ ?
Kausalt	JÄ	NEJ
Icke-kausalt	Eventuellt	JÄ
Spec. fall: Anti-kausalt	NEJ	JÄ



## Exempel

①  $y(t) = \sqrt{x(t-2)}$

②  $y(t) = \int_{-\infty}^{t+1} x(\tau) d\tau$



③  $y(t) = x(t+3)$

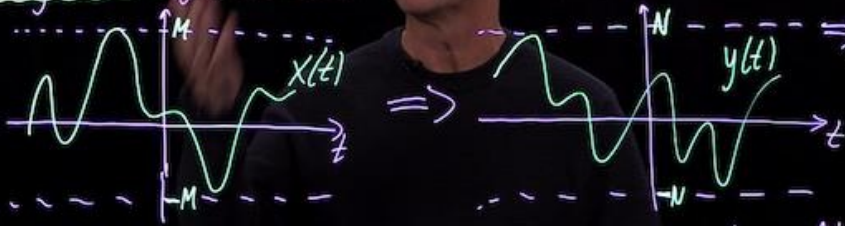
④  $y(t) = x(-t)$

# SYSTEMEGENSKAPER ( $\mathcal{A}$ = systemoperatör)

## Stabilitet



Insignal-utsignalstabil (BIBO-stabil) system omm  $|x(t)| \leq M < \infty$   
 $\Rightarrow |y(t)| \leq N < \infty$   
 $M, N \in \mathbb{R}$



- Marginellt stabilt system: BIBO-stabilt för de flesta begränsade insignaler, men minst en begränsad insignal ger icke-begränsad utsignal
- Instabilt system: Ingen begränsad insignal resulterar i en begränsad utsignal.

## Exempel:

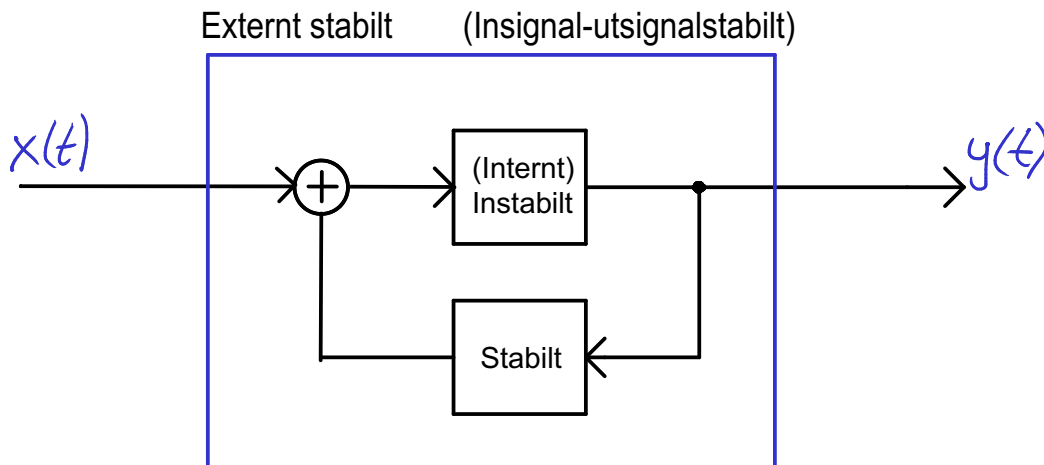
①  $y(t) = 5x(t-2) + 3$

②  $y(t) = \frac{dx(t-3)}{dt}$

③  $\frac{dy(t)}{dt} - y(t) = x(t)$

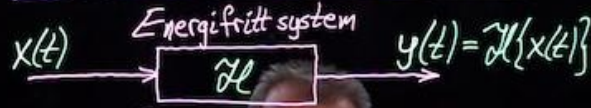
Stabilisering av instabila system med hjälp av återkoppling

(kommer senare i kursen)



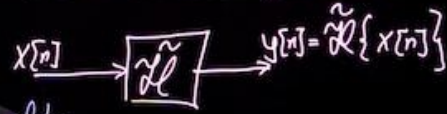
# SYSTEMEGENSKAPER

( $\mathcal{A}$  = systemoperatorn)



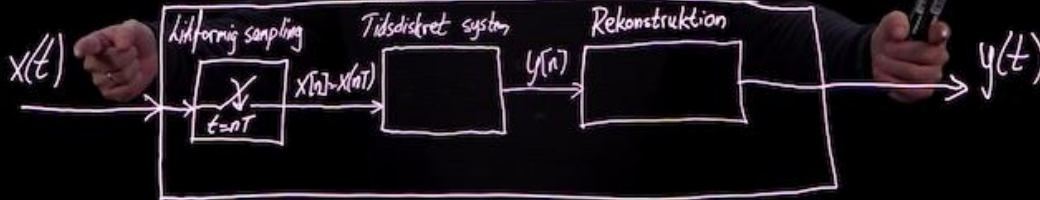
Tidskontinuerliga system

Tidsdiskreta system



Tidskontinuerliga system modelleras/implementeras ofta m.h.a. tidsdiskreta system:

Tidskontinuerligt system



Tidskontinuerlig signal – Signalen är kontinuerlig längs tidsaxeln

Analog signal – Signalen kan anta alla värden

Tidsdiskret signal – Signalen är diskret längs tidsaxeln

Digital signal – Signalens värden är diskretiserade  
(t.e.x. p.g.a. b bitars binär representation)