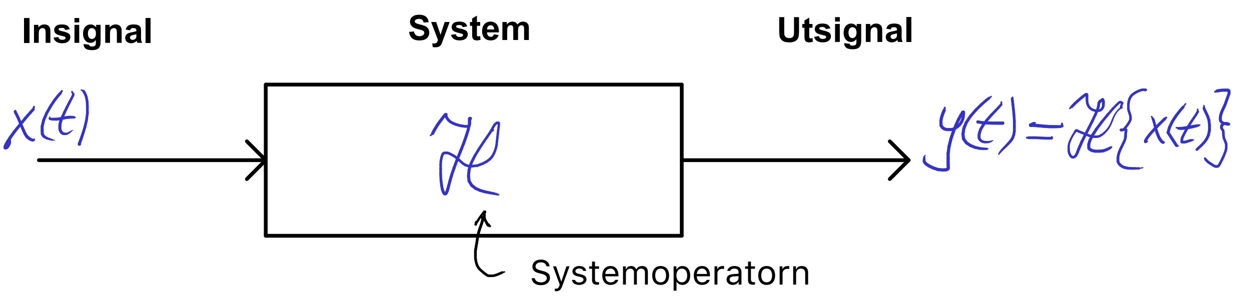
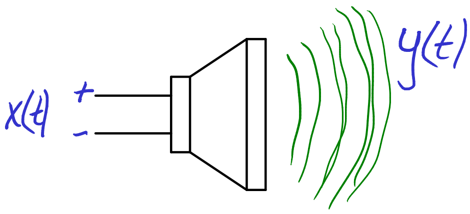
Signaler & System – Föreläsning 1: Inledning, signal- och systemegenskaper

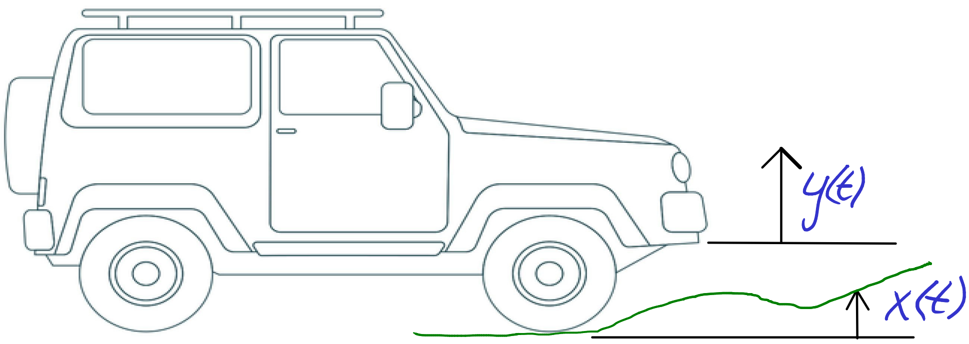


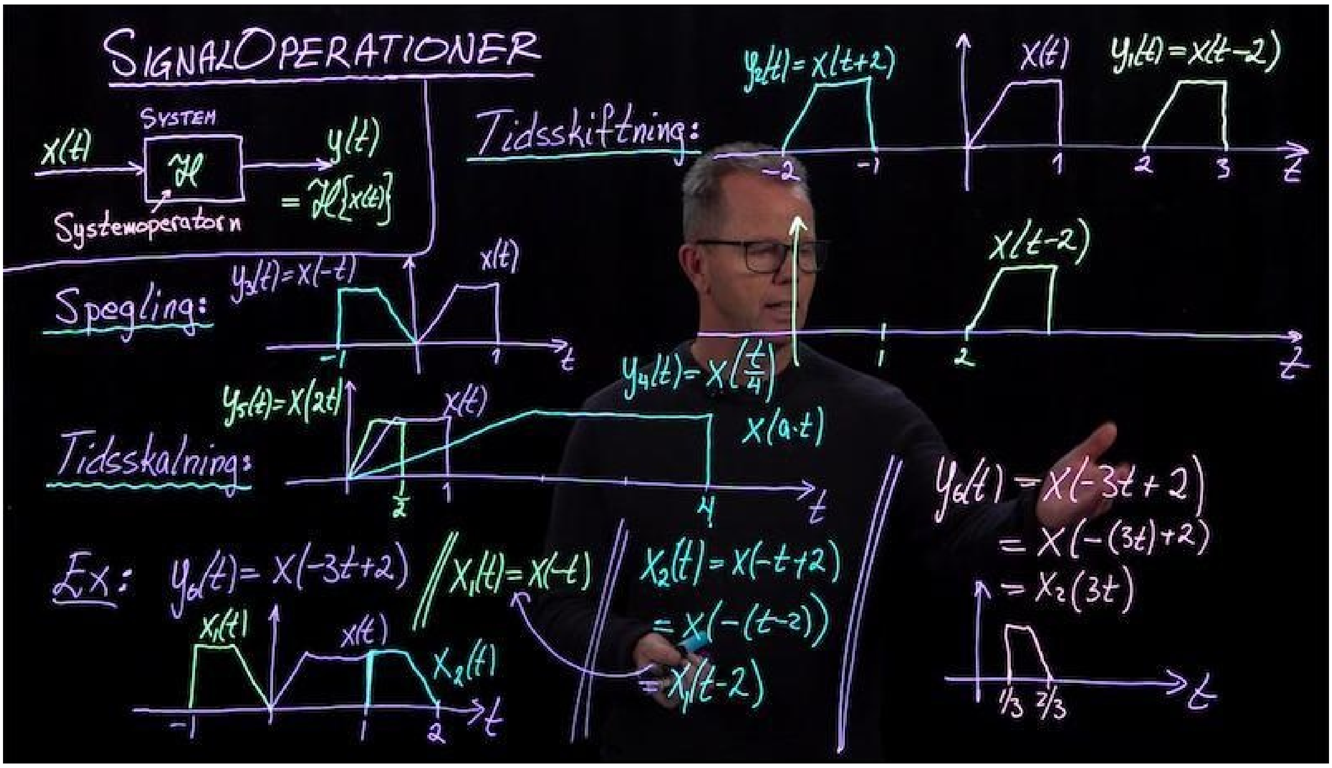
Ett SYSTEM = en matematisk modell av ett fysikaliskt system (alt. en algoritm) som för olika insignaler x genererar olika utsignaler y

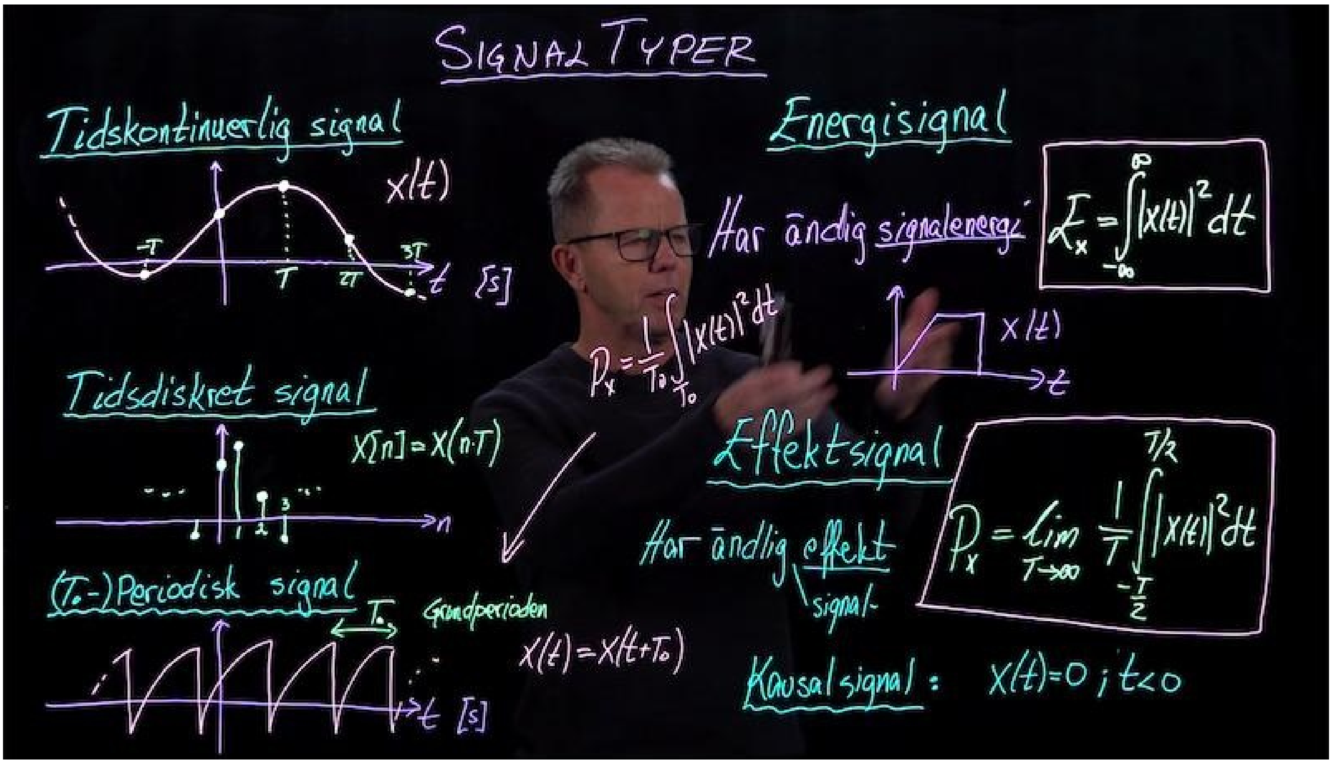
En SIGNAL = en informationsbärande matematisk funktion som representerar en (ofta mätbar) fysikalisk storhet.

Exempel:



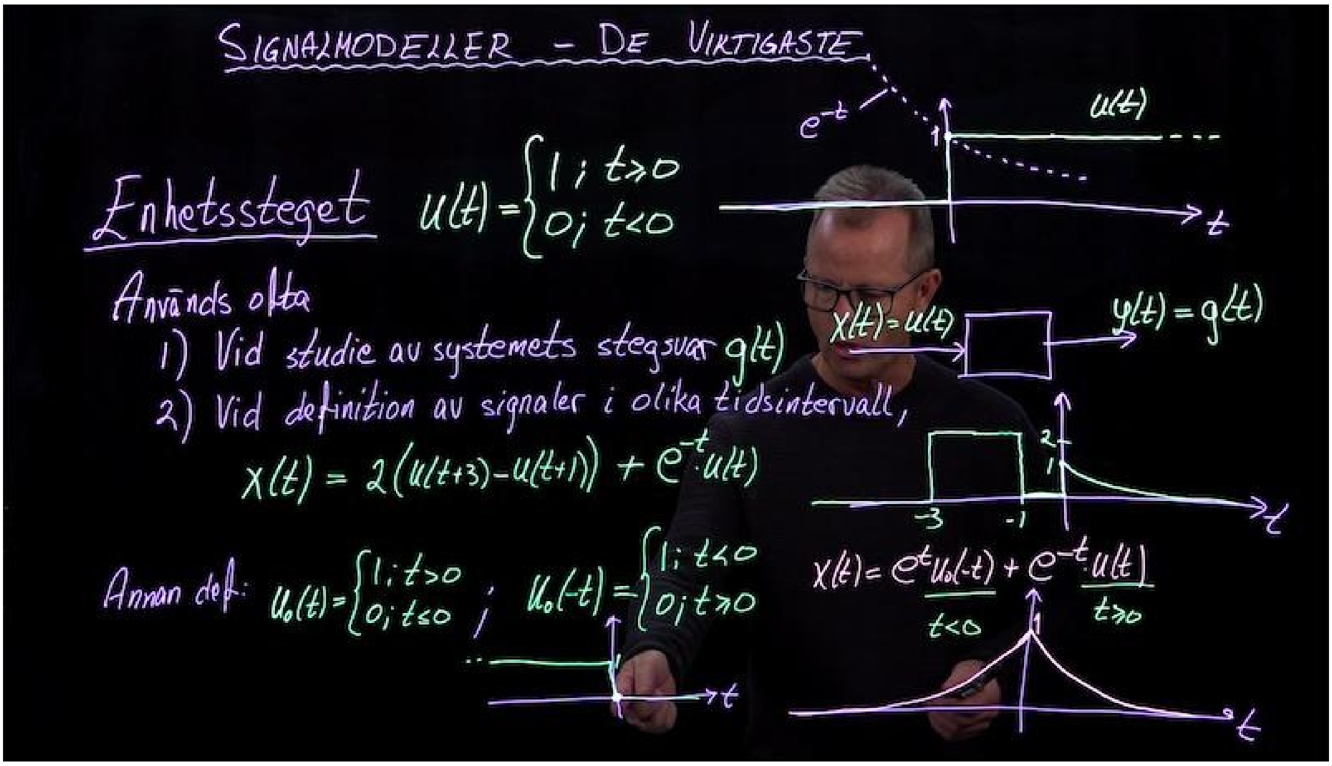


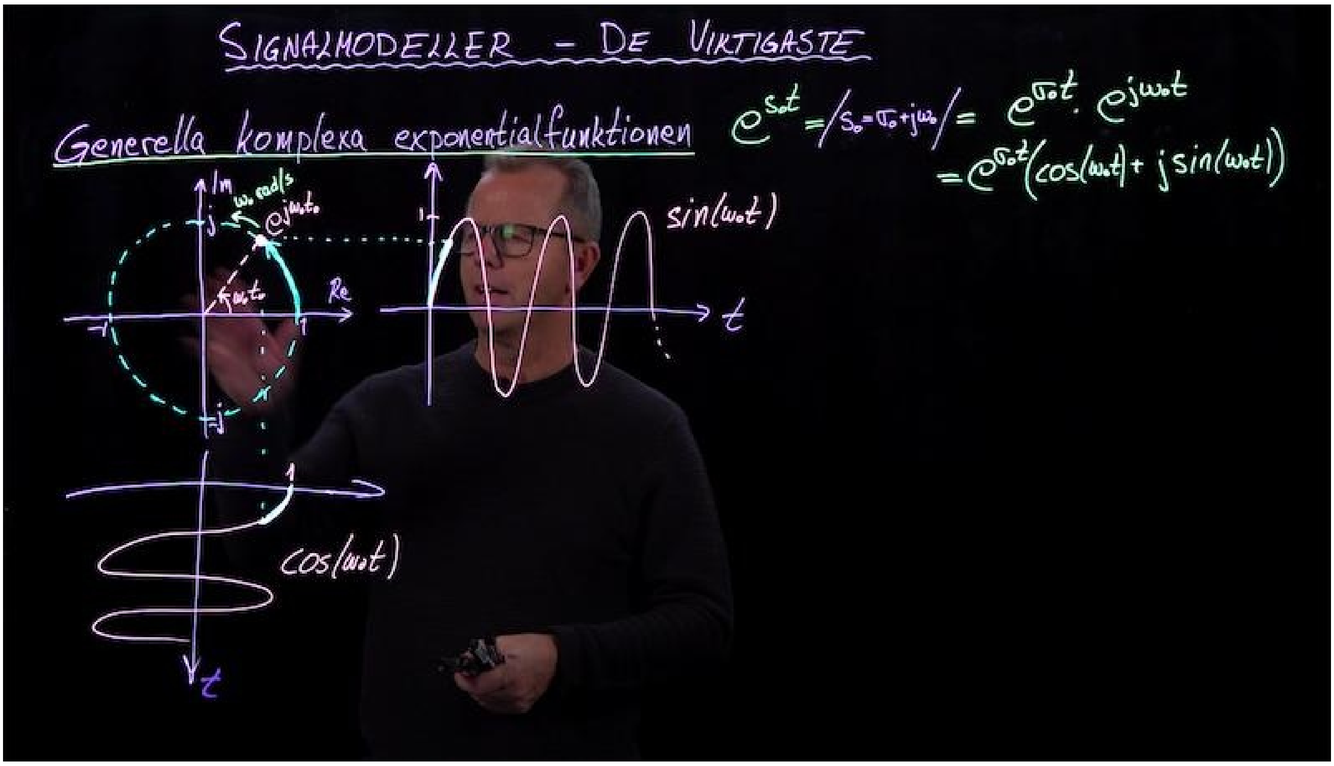

SIGNALOPERATIONER
SYSTEM
x(t)
y(t)
= H{x(t)}
Systemoperatorn
Tidsskiftning:
y2(t) = x(t+2)
x(t)
y1(t) = x(t-2)
x(t-2)
Spegling:
y3(t) = x(-t)
Tidsskalning:
y5(t) = x(2t)
y4(t) = x(t/4)
x(a·t)
Ex: y6(t) = x(-3t+2)
x1(t) = x(-t)
x2(t) = x(-t+2)
= x(-(t-2))
= x1(t-2)
y6(t) = x(-3t+2)
= x(-(3t)+2)
= x2(3t)

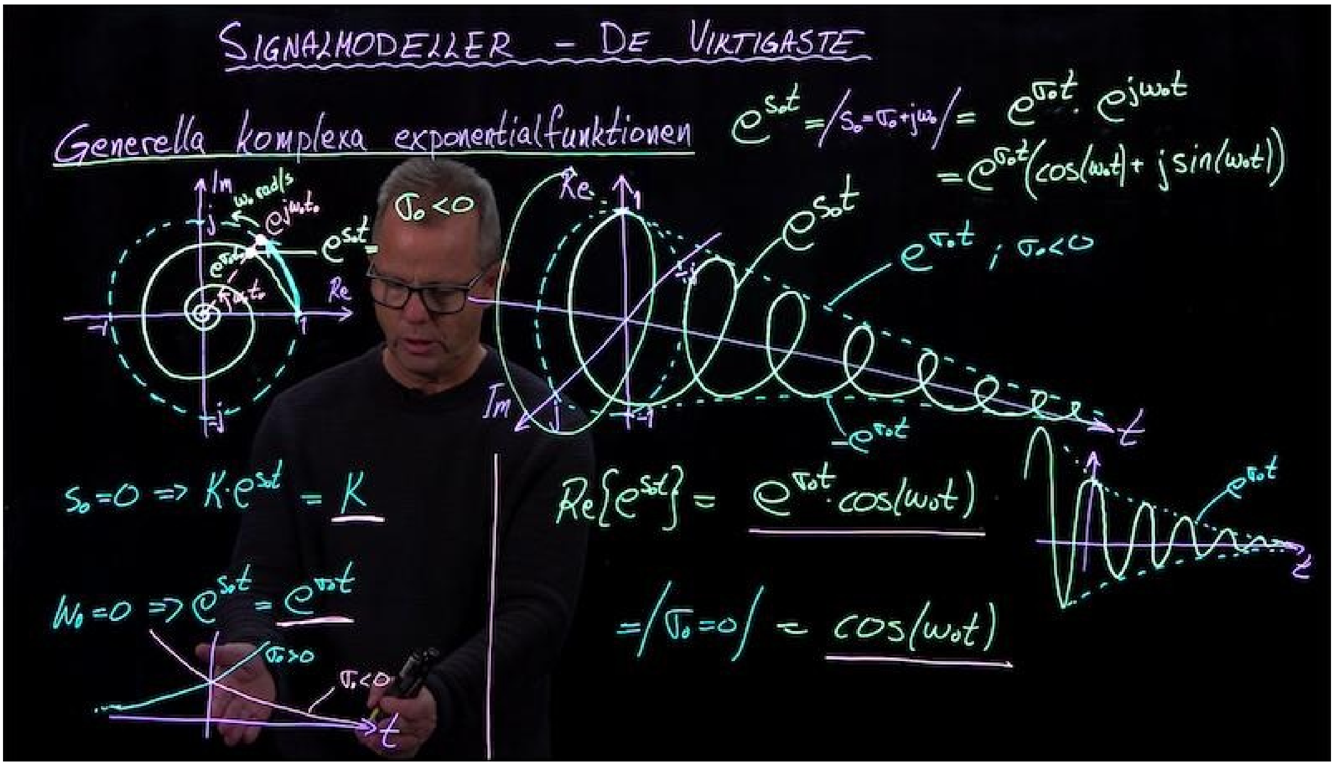


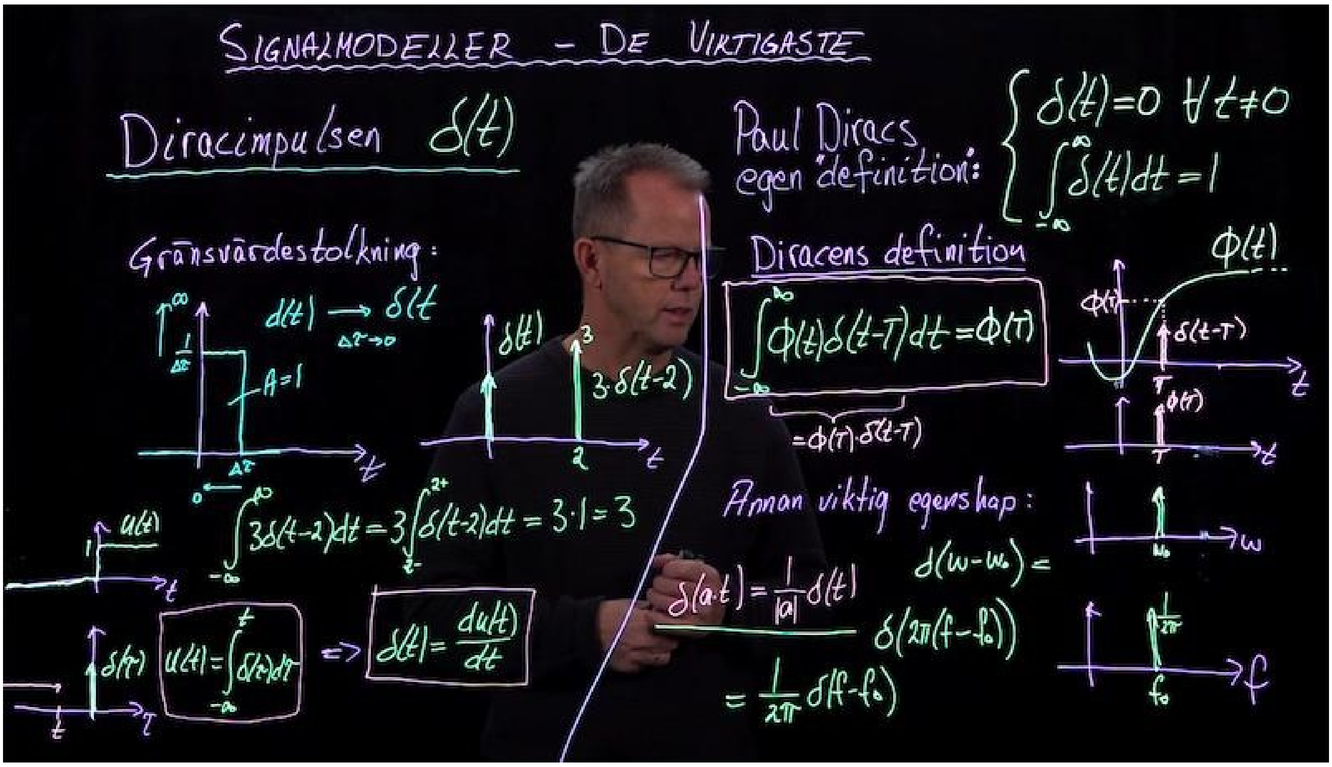
Stationär signal

Transient signal


SIGNALMODELLER - DE VIKTIGASTE
e^{-t}
$u(t)$
Enhetssteget $u(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$
Används ofta
1) Vid studie av systemets stegsvar $g(t)$
$x(t) = u(t)$
$y(t) = g(t)$
2) Vid definition av signaler i olika tidsintervall,
$x(t) = 2\left(u(t+3) - u(t+1)\right) + e^{-t} \cdot u(t)$
Annan def: $u_0(t) = \begin{cases} 1; & t > 0 \\ 0; & t \leq 0 \end{cases}$; $u_0(-t) = \begin{cases} 1; & t < 0 \\ 0; & t \geq 0 \end{cases}$
$x(t) = \underbrace{e^{t} u_0(-t)}_{t<0} + \underbrace{e^{-t} \cdot u(t)}_{t \geq 0}$

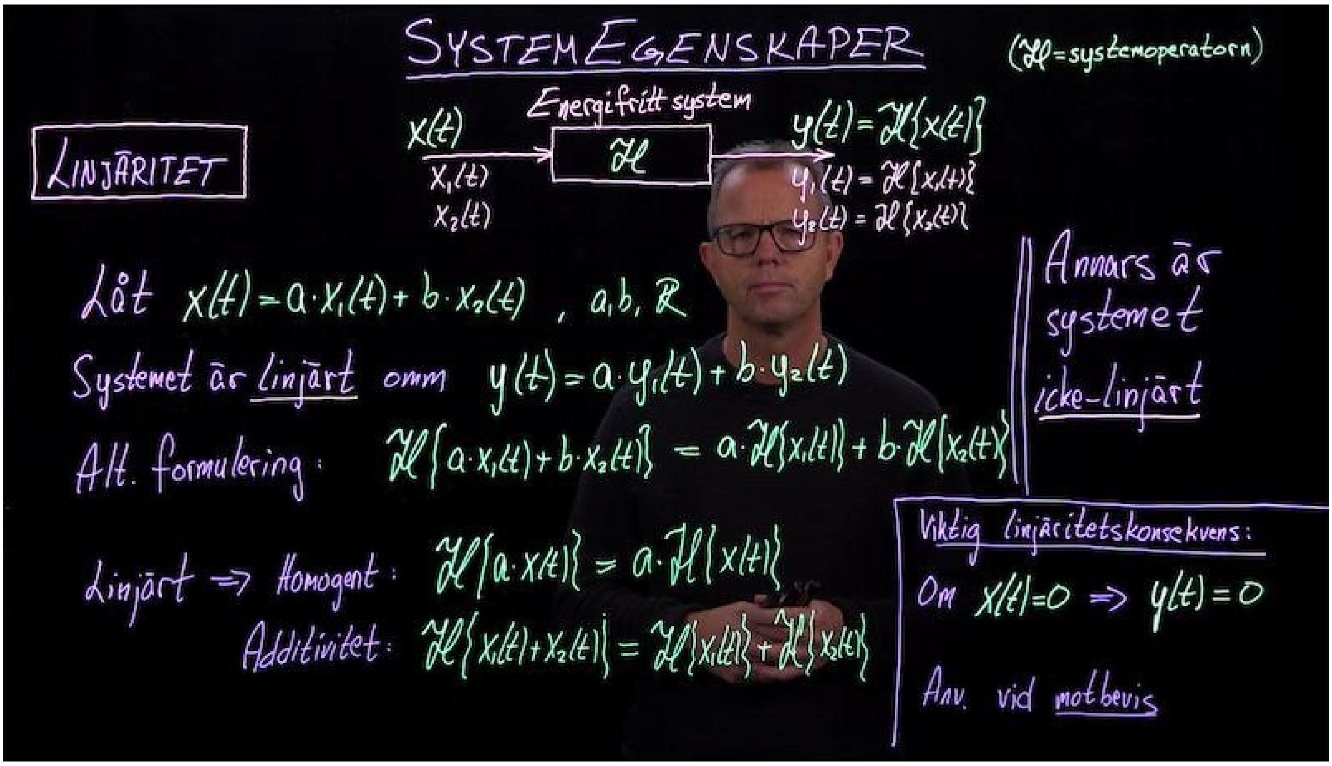

SIGNALMODELLER – DE VIKTIGASTE
Generella komplexa exponentialfunktionen
$e^{s_0 t} = / s_0 = \sigma_0 + j\omega_0 / = e^{\sigma_0 t} \cdot e^{j\omega_0 t}$
$= e^{\sigma_0 t}(\cos(\omega_0 t) + j\sin(\omega_0 t))$
Im
Re
ω_0 rad/s
$e^{j\omega_0 t_0}$
$\omega_0 t_0$
j
-j
-1
1
$\sin(\omega_0 t)$
t
$\cos(\omega_0 t)$
t


SIGNALMODELLER - DE VIKTIGASTE
Generella komplexa exponentialfunktionen
$e^{s_0 t} = / s_0 = \sigma_0 + j\omega_0 / = e^{\sigma_0 t} \cdot e^{j\omega_0 t}$
$= e^{\sigma_0 t}(\cos(\omega_0 t) + j\sin(\omega_0 t))$
$\sigma_0 < 0$
$e^{s_0 t}$
$e^{\sigma_0 t}$; $\sigma_0 < 0$
$-e^{\sigma_0 t}$
Re
Im
t
$s_0 = 0 \Rightarrow K \cdot e^{s_0 t} = K$
$\omega_0 = 0 \Rightarrow e^{s_0 t} = e^{\sigma_0 t}$
$\sigma_0 > 0$
$\sigma_0 < 0$
$Re\{e^{s_0 t}\} = e^{\sigma_0 t} \cdot \cos(\omega_0 t)$
$= / \sigma_0 = 0 / = \cos(\omega_0 t)$

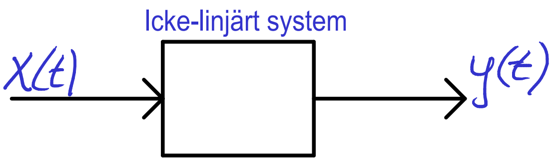


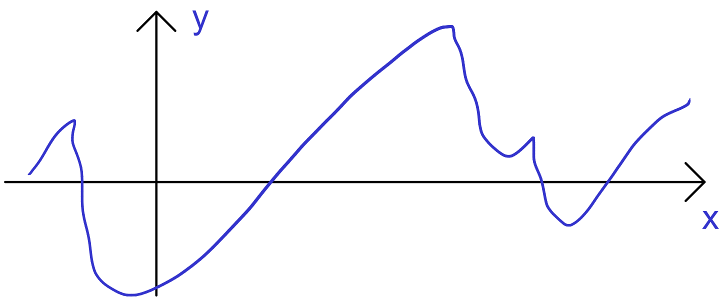
Exempel:

$$\boxed{x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau}$$

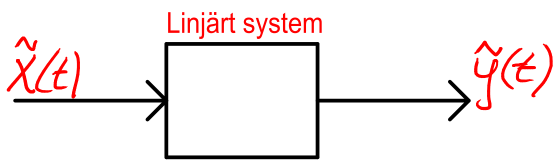


De flesta fysikaliska system är inte linjära.



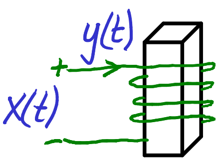


Linjärisera $\Longrightarrow$



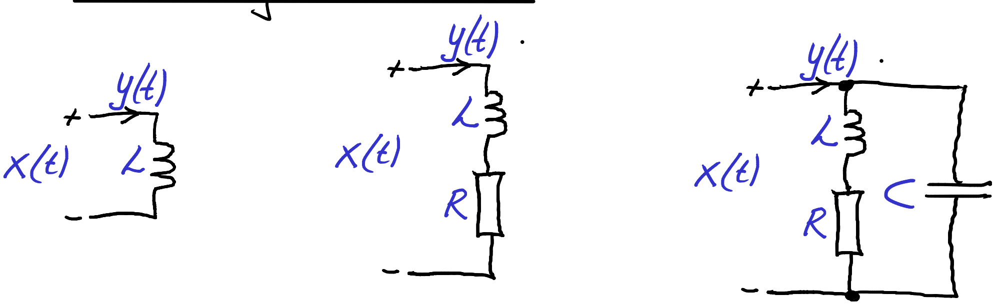
Exempel:

Spole



$\Longrightarrow$

Olika linjära modeller



Exempel – test av linjäritet (betrakta energifritt system)

$y(t) = \mathcal{H}\{x(t)\} \Rightarrow y_n(t) = \mathcal{H}\{x_n(t)\}$

Låt $x(t) = a \cdot x_1(t) + b \cdot x_2(t)$ Om linjärt $\Rightarrow y(t) = a \cdot y_1(t) + b \cdot y_2(t)$, $a, b \in \mathbb{R}$

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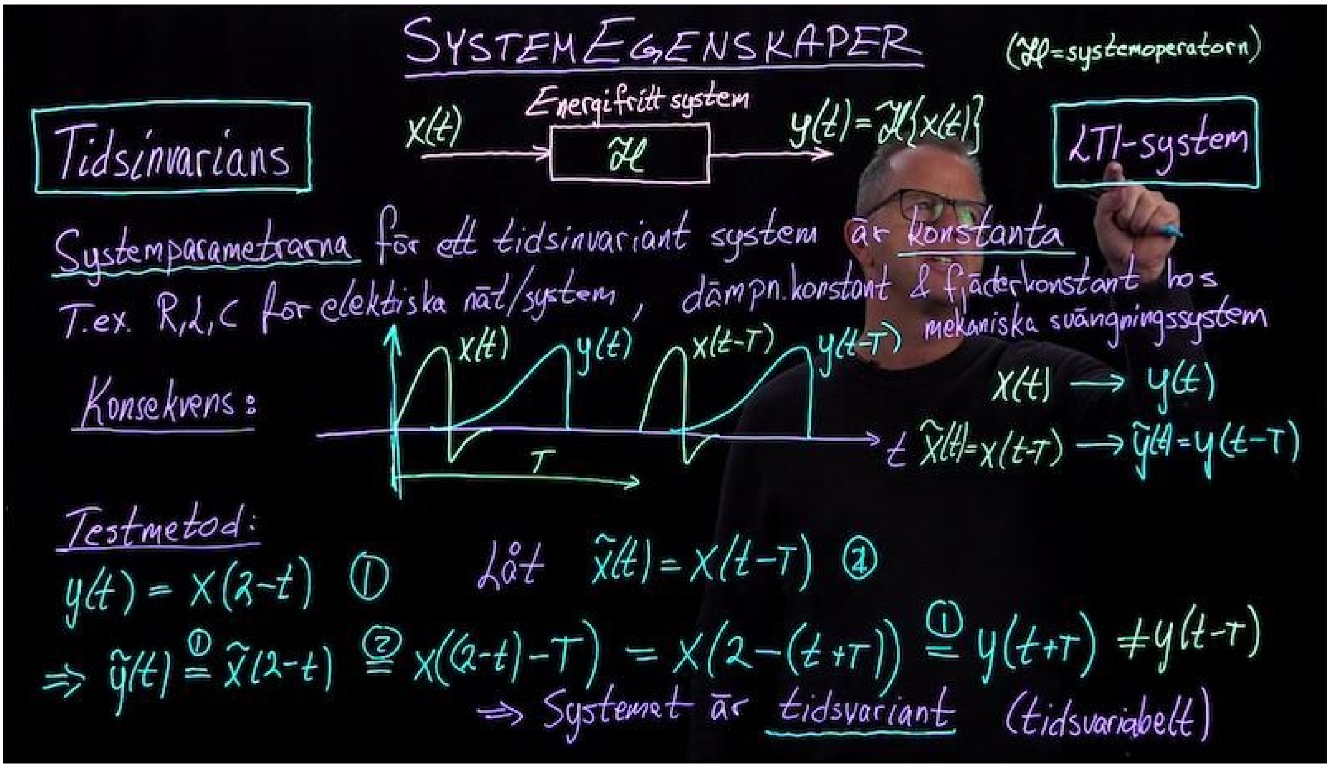
1 $y(t) = t \cdot x(t-4) \Rightarrow y_n(t) = t \cdot x_n(t-4)$

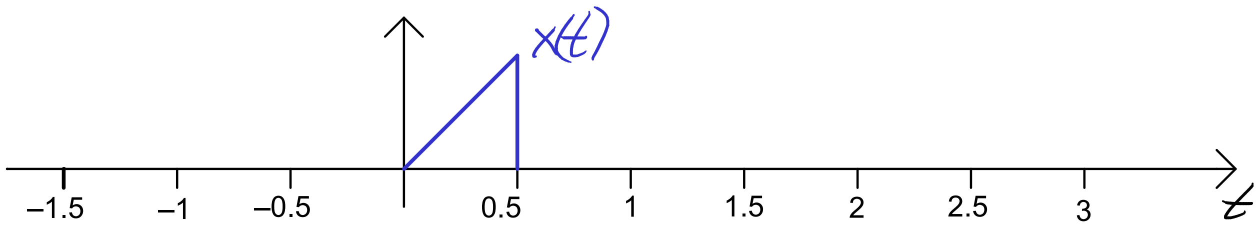
2 $y(t) = 5x(t) + 3 \Rightarrow y_n(t) = 5x_n(t) + 3$

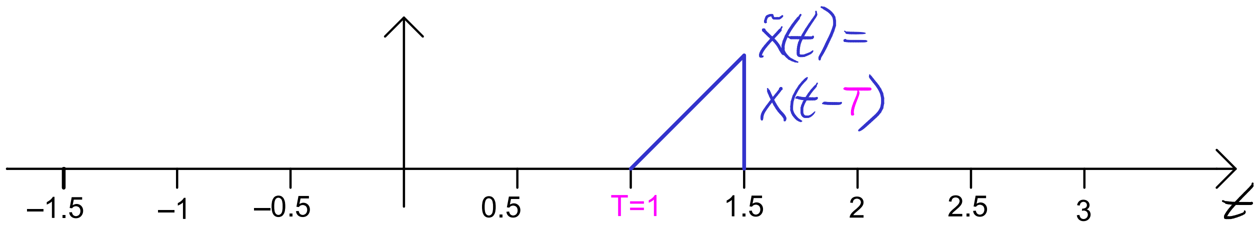
③ $\frac{dy(t)}{dt} + 2y(t) = 3\frac{dx(t)}{dt}$ $\Rightarrow$ $\frac{dy_n(t)}{dt} + 2y_n(t) = 3\frac{dx_n(t)}{dt}$

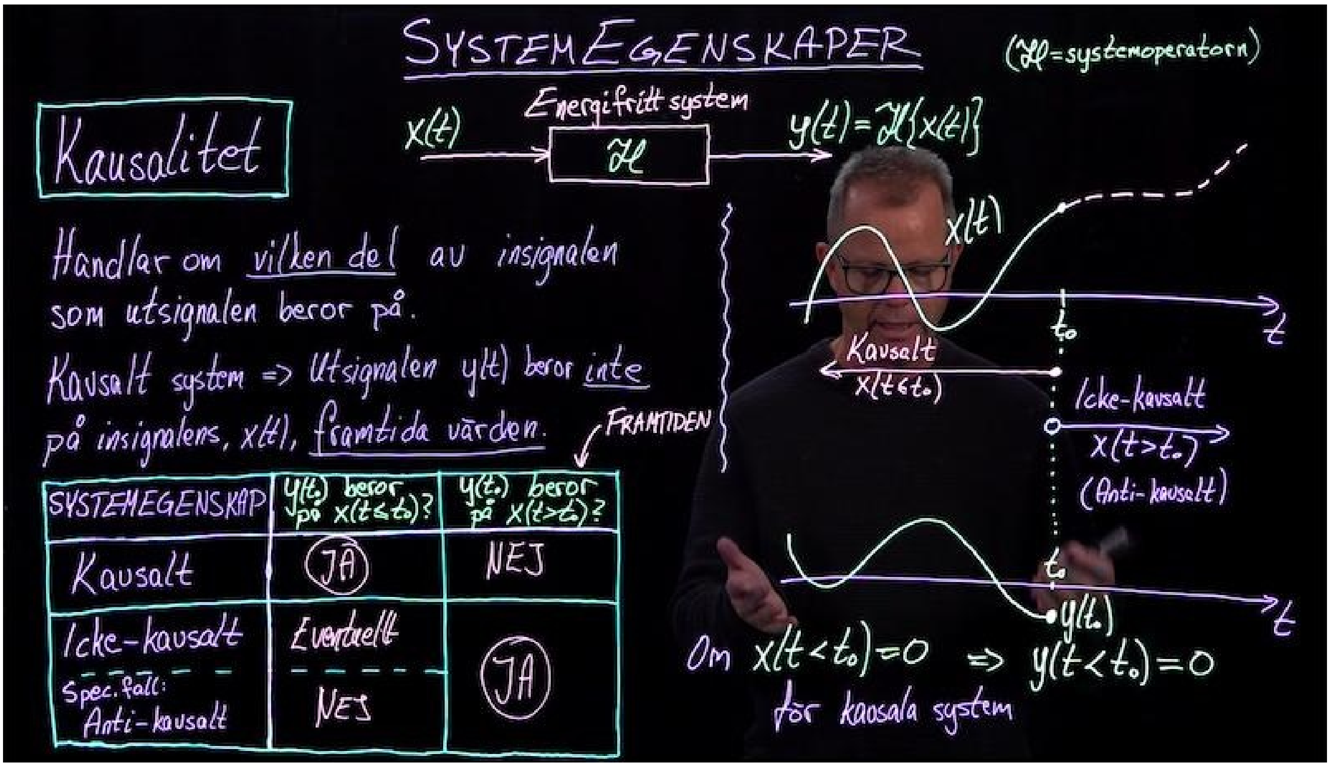
$(y_n' + 2y_n = 3x_n')$



Grafisk illustration av föreläsningsexemplet



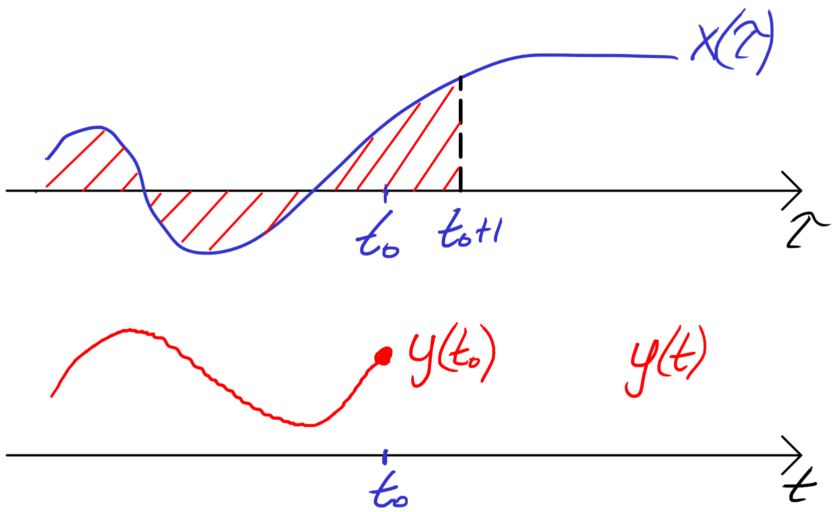




Exempel

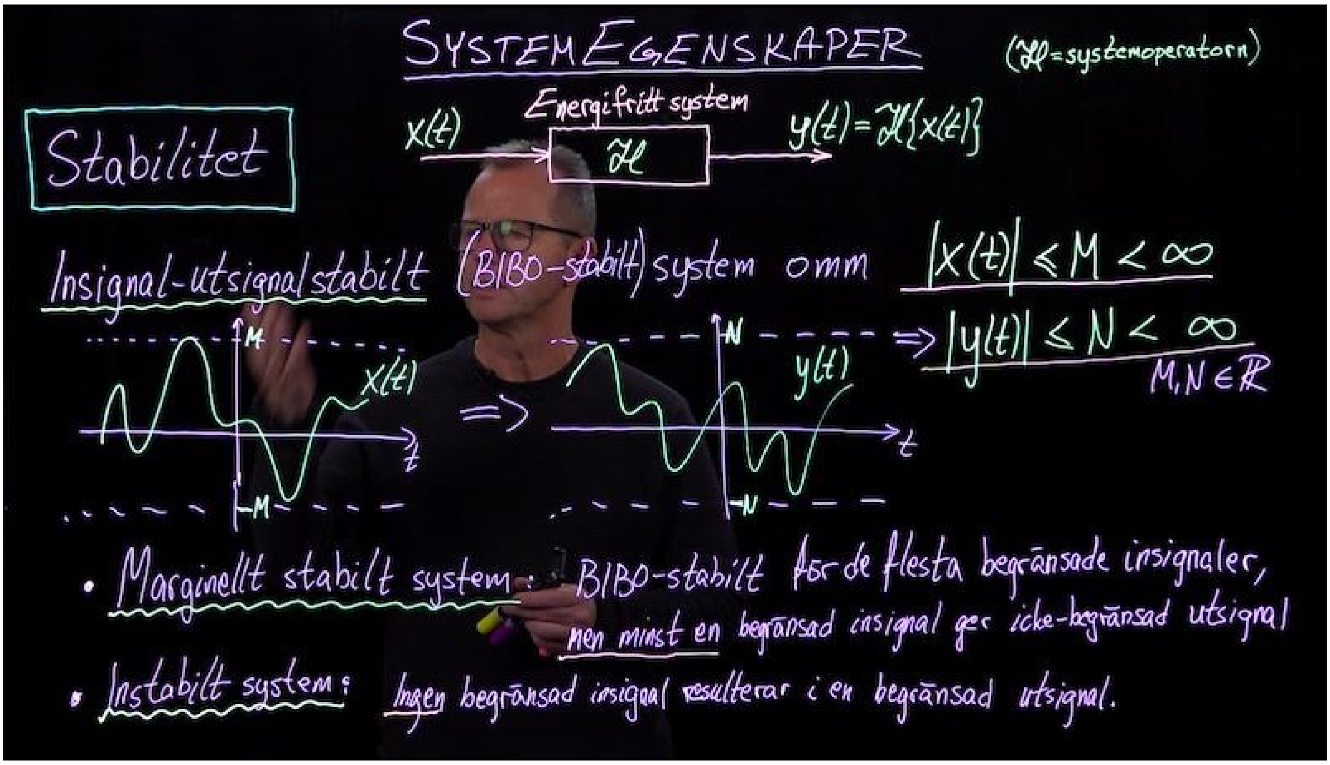
① $y(t) = \sqrt{x(t-2)}$

② $y(t) = \int_{-\infty}^{t+1} x(\tau)\,d\tau$



③ $y(t) = x(t+3)$

④ $y(t) = x(-t)$



Exempel:

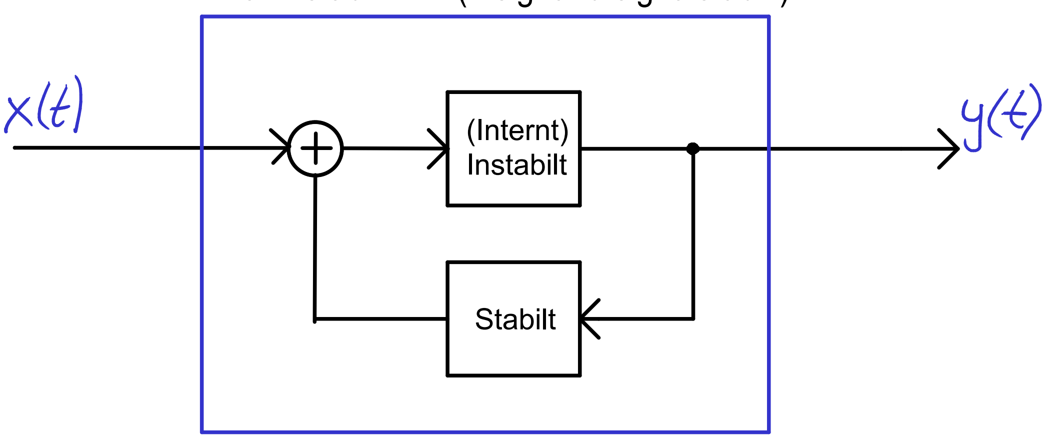
① $y(t) = 5x(t-2) + 3$

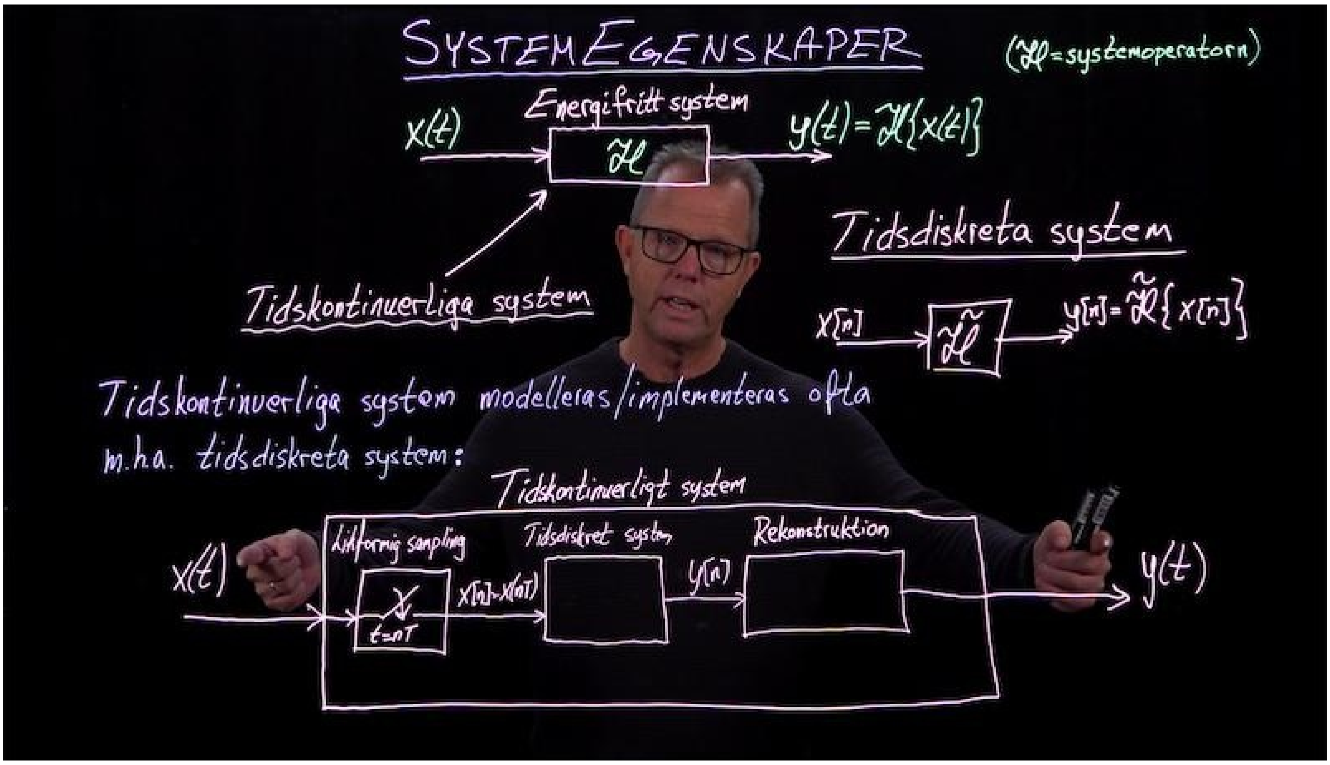
② $y(t) = \dfrac{dx(t-3)}{dt}$

③ $\dfrac{dy(t)}{dt} - y(t) = x(t)$

Stabilisering av instabila system med hjälp av återkoppling (kommer senare i kursen)

Externt stabilt (Insignal-utsignalstabilt)





Tidskontinuerlig signal – Signalen är kontinuerlig längs tidsaxeln

Analog signal – Signalen kan anta alla värden

Tidsdiskret signal – Signalen är diskret längs tidsaxeln

Digital signal – Signalens värden är diskretiserade
(t.e.x. p.g.a. b bitars binär representation)