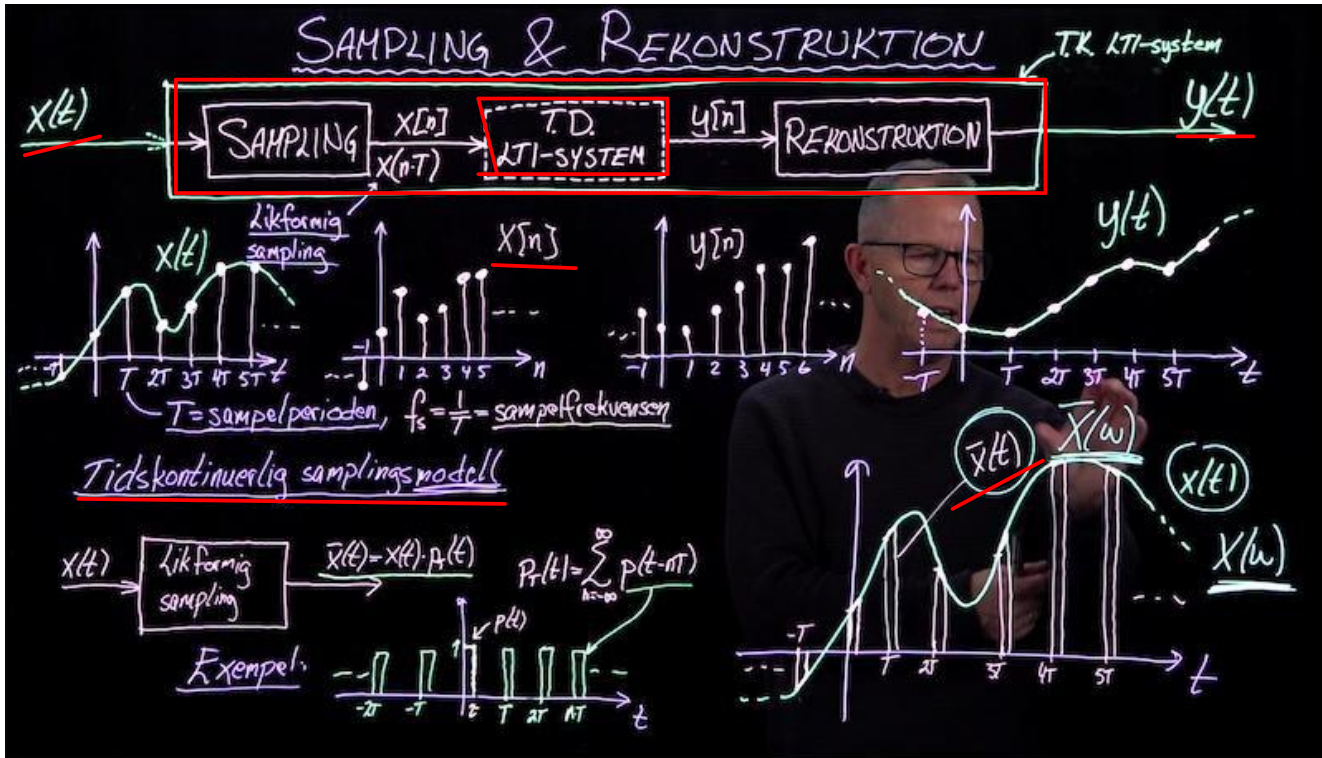
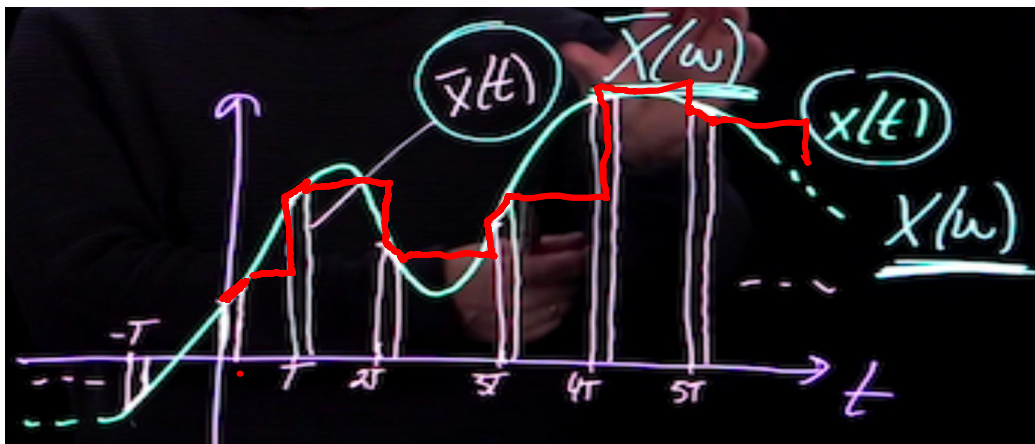
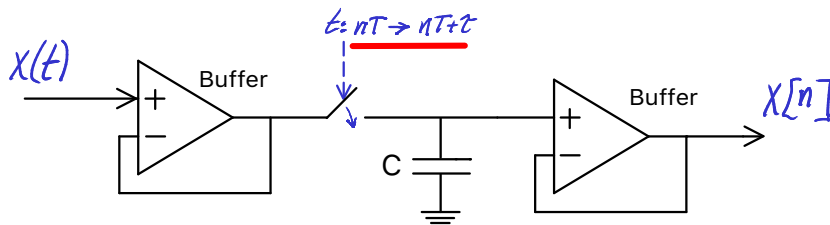


# Signaler & System – Föreläsning 6: Sampling & Rekonstruktion

VIDEO 1



Sampling i praktiken – Sample & Hold:



# SAMPLING & REKONSTRUKTION

$X(t) \rightarrow$  **LIKFORMIG SAMPLING**  $\rightarrow X[n] = X(nT)$   
 $\bar{X}(t) = X(t) \cdot p_T(t)$

Ideal sampling:  $p_T(t) = \delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$

$\mathcal{F}\{\bar{X}(t)\} = X(\omega) \cdot \mathcal{F}\{\delta_T(t)\} = X(\omega) \sum_{n=-\infty}^{\infty} D_n e^{jn\omega T}$   
 =  $\mathcal{F}\left\{ \sum_{n=-\infty}^{\infty} D_n X(\omega) e^{jn\omega T} \right\} = \sum_{n=-\infty}^{\infty} D_n \mathcal{F}\{X(\omega) e^{jn\omega T}\}$   
 =  $\sum_{n=-\infty}^{\infty} D_n X(\omega - n\omega_s)$

dar  $D_n = \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) e^{-jn\omega T} dt = \frac{1}{T} e^{-jn\omega T} = \frac{1}{T}$

**Poissons Summationsformel**

$$\bar{X}(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

Det flesta grundläggande samband finns i formelsamlingen – du behöver främst förstå dem och veta när och hur de ska användas:

### 3.2.4 Sampling & Rekonstruktion

**Likformig sampling** med sampelperiod  $T$ :  $x[n] = x(t)|_{t=nT} = x(nT)$

- Tidskontinuerlig representation:  $\bar{x}(t) = x(t) p_T(t) = \sum_{n=-\infty}^{\infty} x(nT) p(t - nT)$
- Frekvenstransformation:  $\Omega = \omega T = \frac{\omega}{f_s}$ ,  $\theta = fT = \frac{f}{f_s}$
- Ideal sampling:  $p(t) = \delta(t) \Rightarrow$

Poissons summationsformel:  $\bar{X}(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$

Alternativa former:  $X[\Omega] = \frac{1}{T} \sum_{n=-\infty}^{\infty} X\left(\frac{\Omega - n2\pi}{T}\right)$ ,  $X[\theta] = \frac{1}{T} \sum_{n=-\infty}^{\infty} X\left(\frac{\theta - n}{T}\right)$

**Rekonstruktion** m.h.a. PAM:  $y(t) = \sum_{n=-\infty}^{\infty} x[n] h(t - nT) \Leftrightarrow Y(\omega) = X[\omega T] H(\omega)$

- Ideal rekonstruktion:  $h(t) = \text{sinc}\left(\frac{\pi t}{T}\right) = \text{sinc}_N\left(\frac{t}{T}\right)$

15. Ideal sampling	$\sum_{n=-\infty}^{\infty} x(nT) \delta(t - nT)$	$\frac{1}{T} \sum_{n=-\infty}^{\infty} X\left(\omega - n \frac{2\pi}{T}\right)$
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**Tabell 2** Egenskaper hos fouriertransformen för tidskontinuerliga funktioner.

# SAMPLING & REKONSTRUKTION

**LIRFORMIG SAMPLING**  
 $X(t) \rightarrow X[n] = X(nT)$   
 $\bar{X}(t) = X(t) \cdot p(t)$

**IDEAL SAMPLING**  
 $\bar{X}(w) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(w - n\omega_s)$

**REKONSTRUKTION**  
 $\bar{X}(t) \rightarrow y(t) = (\bar{X} * h)(t) \Rightarrow Y(w) = \bar{X}(w) H(w)$

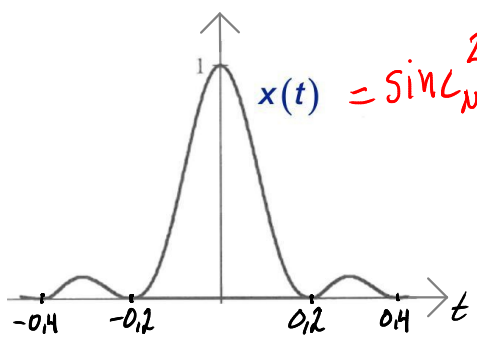
**Praktiskt rekonstruktionsfilter**

**Samplingssteomet**  
 Låt  $x(t)$  vara en bandbegränsad signal med bandbredd  $B$  Hz, som samplas med sampelfrekvensen  $f_s = \frac{1}{T}$ , vilket ger  $x[n] = x(nT)$ . Om  $f_s > 2B$  så kan  $x(t)$  återskapas från  $x[n]$ .

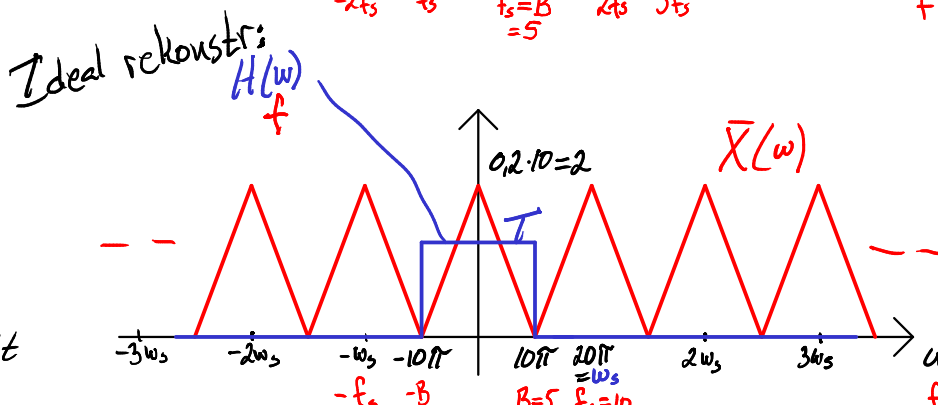
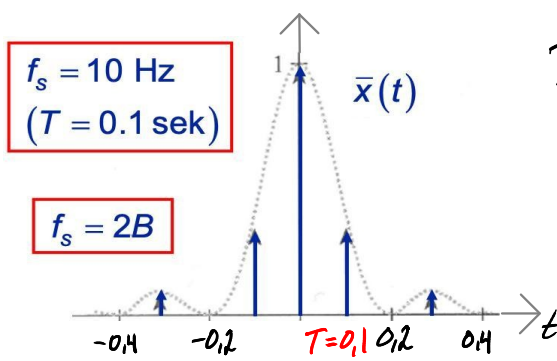
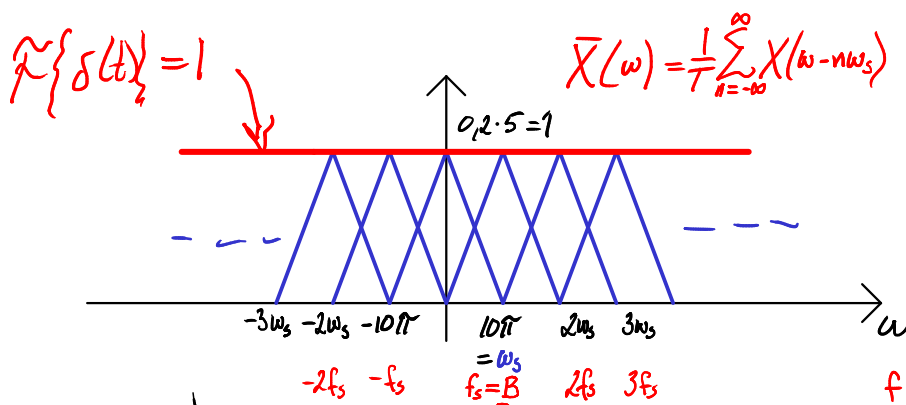
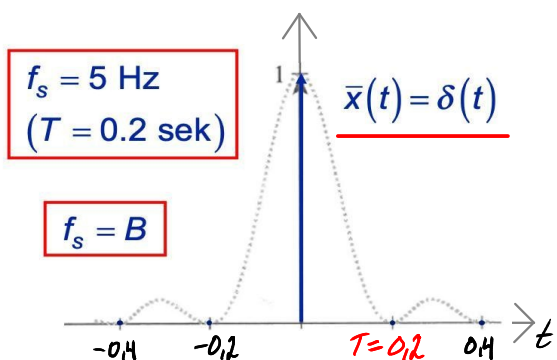
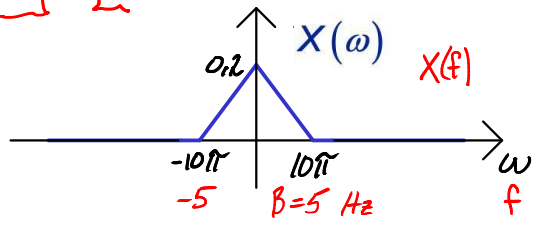
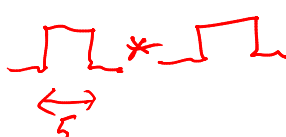
$f_s - B > B$   
 $\Rightarrow f_s > 2B$

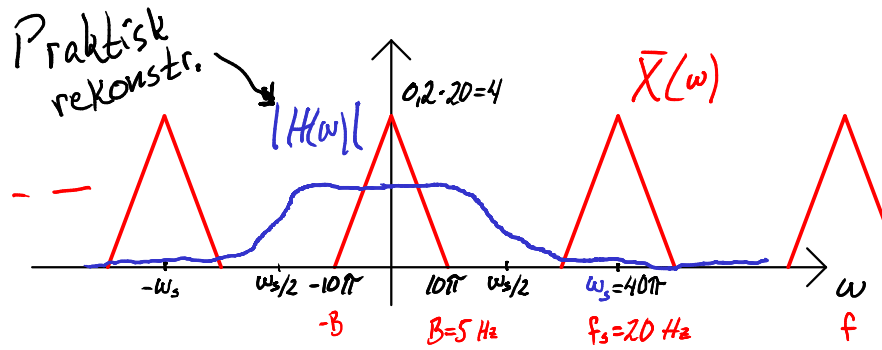
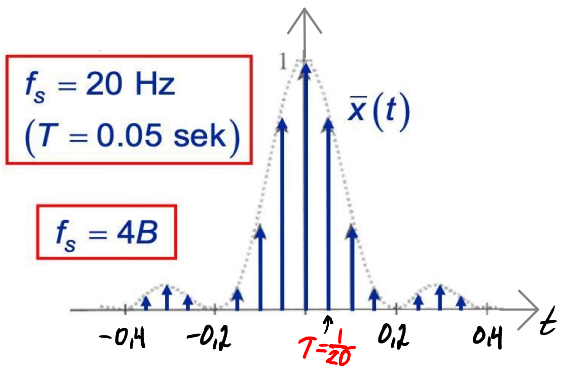
Lasses hjälpgraf under föreläsningen

Samplingsexempel med olika sampelfrekvenser  $f_s$ :



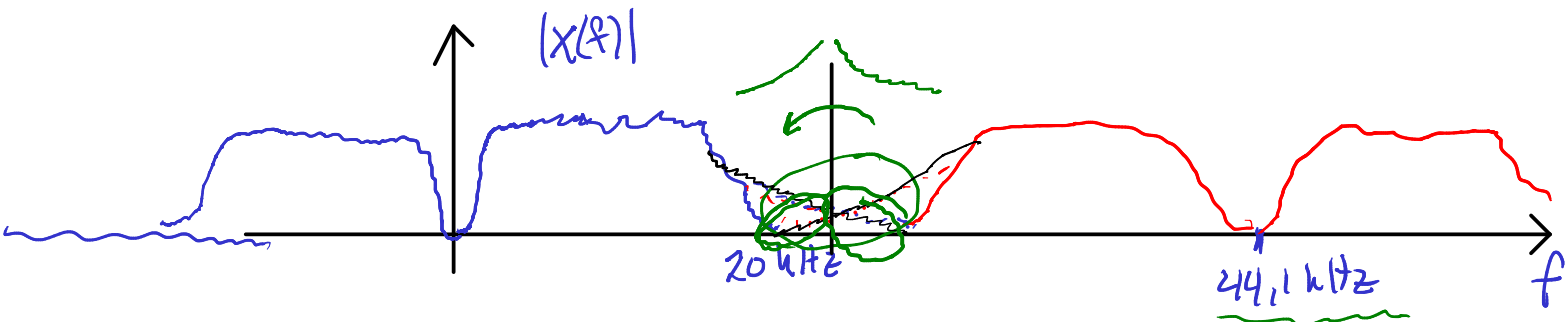
Signalens bandbredd:  $W = 10\pi$  rad/s  
 $B = 5$  Hz





Ex audiosignaler

Musik på CD är sampled med  
 $f_s = 44,1 \text{ kHz}$ , 16 bitar/sampel



Vanlig:  $f_s = 48 \text{ kHz}$

PAL  
 NTSC

$f_s < 2B \Rightarrow$  Undersampling

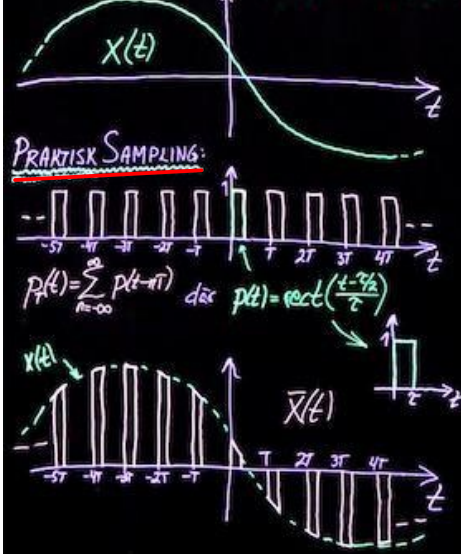
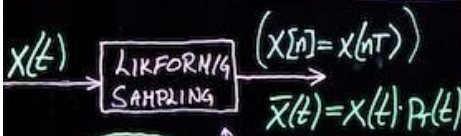
$\Rightarrow$  Vikning (eng: aliasing)

$f_s \gg 2B \Rightarrow$  Översampling

$\Rightarrow$  Enklare rekonstr.krets

Här följer en demonstration i Matlab, som illustrerar vikning vid sampling av en cosinussignal.  
 ([con2dis](#) i toolboxen [spfirst](#), som finns på kurswebbsidan under "Intressanta länkar")

# PRAKTISK SAMPLING & REKONSTRUKTION

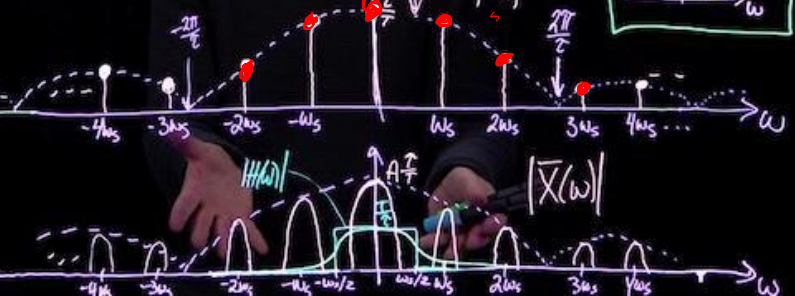


$$\bar{X}(\omega) = \sum_{n=-\infty}^{\infty} D_n X(\omega - n\omega_s)$$

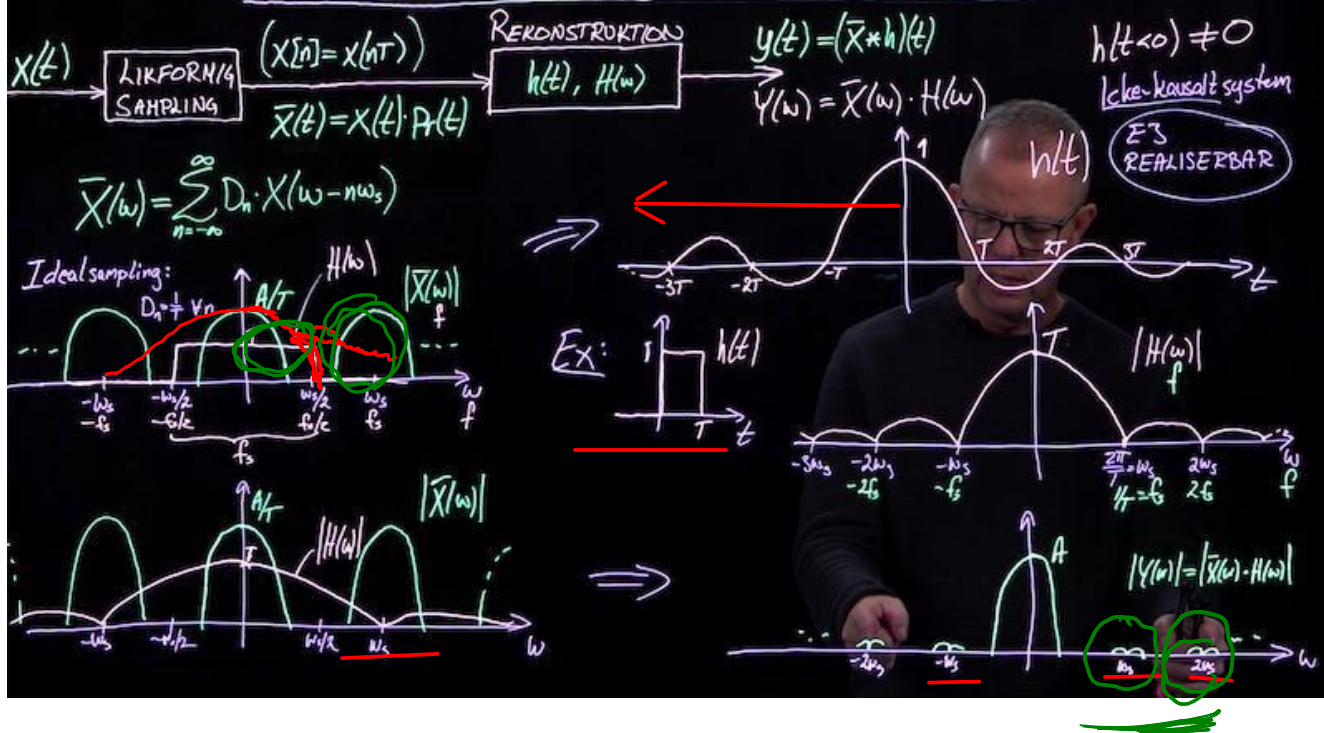
Här:  $D_n = \frac{1}{T} \int_{-T/2}^{T/2} p(t) e^{jn\omega_s t} dt = \frac{1}{T} \mathcal{F}\{p(t)\} \Big|_{\omega = n\omega_s} = \frac{1}{T} \int_{-T/2}^{T/2} \text{rect}\left(\frac{t-T/2}{T}\right) e^{jn\omega_s t} dt$

$= \frac{1}{T} \int_{-T/2}^{T/2} \text{sinc}\left(\frac{\omega T}{2\pi}\right) e^{jn\omega_s t} dt \Big|_{\omega = n\omega_s} = \frac{2\pi}{T} \text{sinc}\left(\frac{n\pi}{T}\right)$

$$\Rightarrow |D_n| = \frac{2\pi}{T} \text{sinc}\left(\frac{n\pi}{T}\right)$$



# PRAKTISK SAMPLING & REKONSTRUKTION



Ideal och Praktisk rekonstruktion

Rekonstruktion

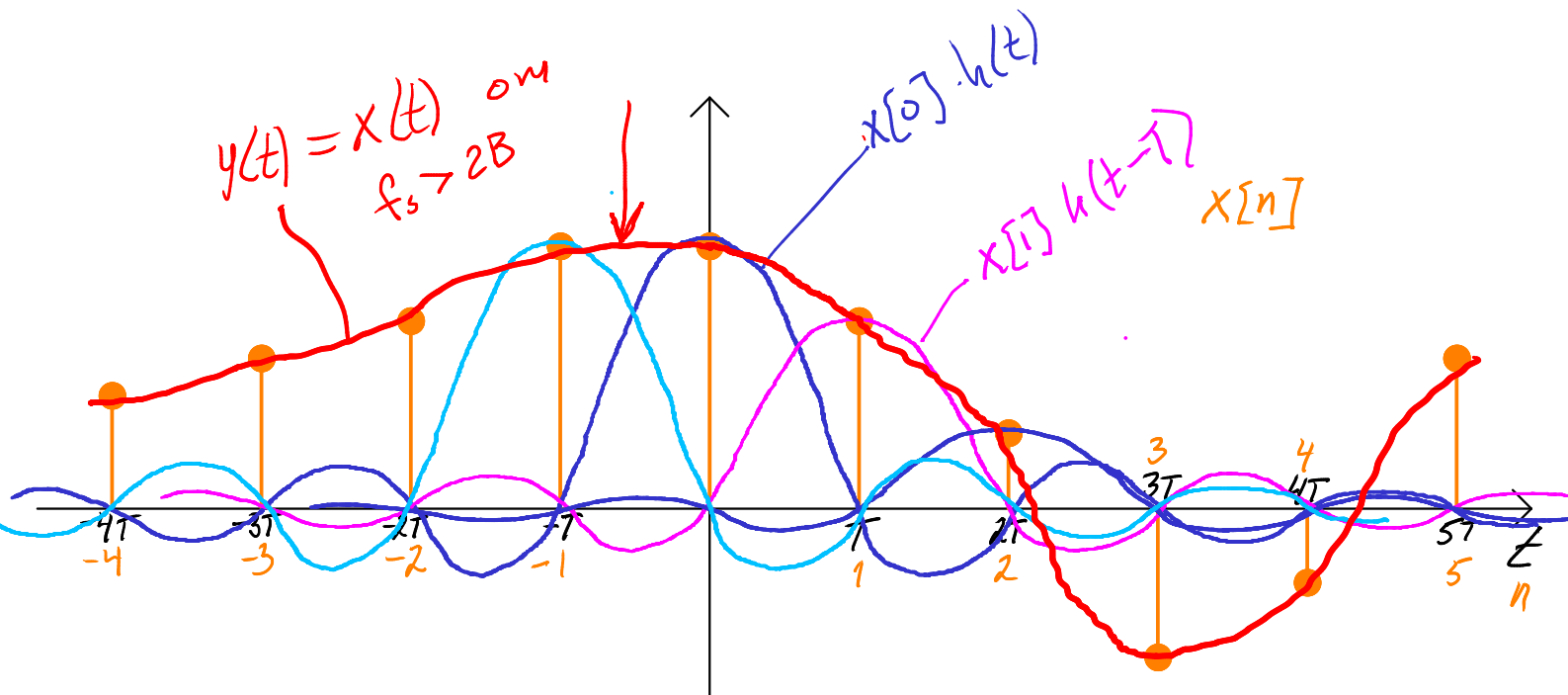
$x[n]$   $\rightarrow$   $h(t), H(w)$   $\rightarrow$   $y(t) = (\bar{x} * h)(t) = \bar{x}(t) * h(t)$

$\bar{x}(t) = x(t) p(t)$

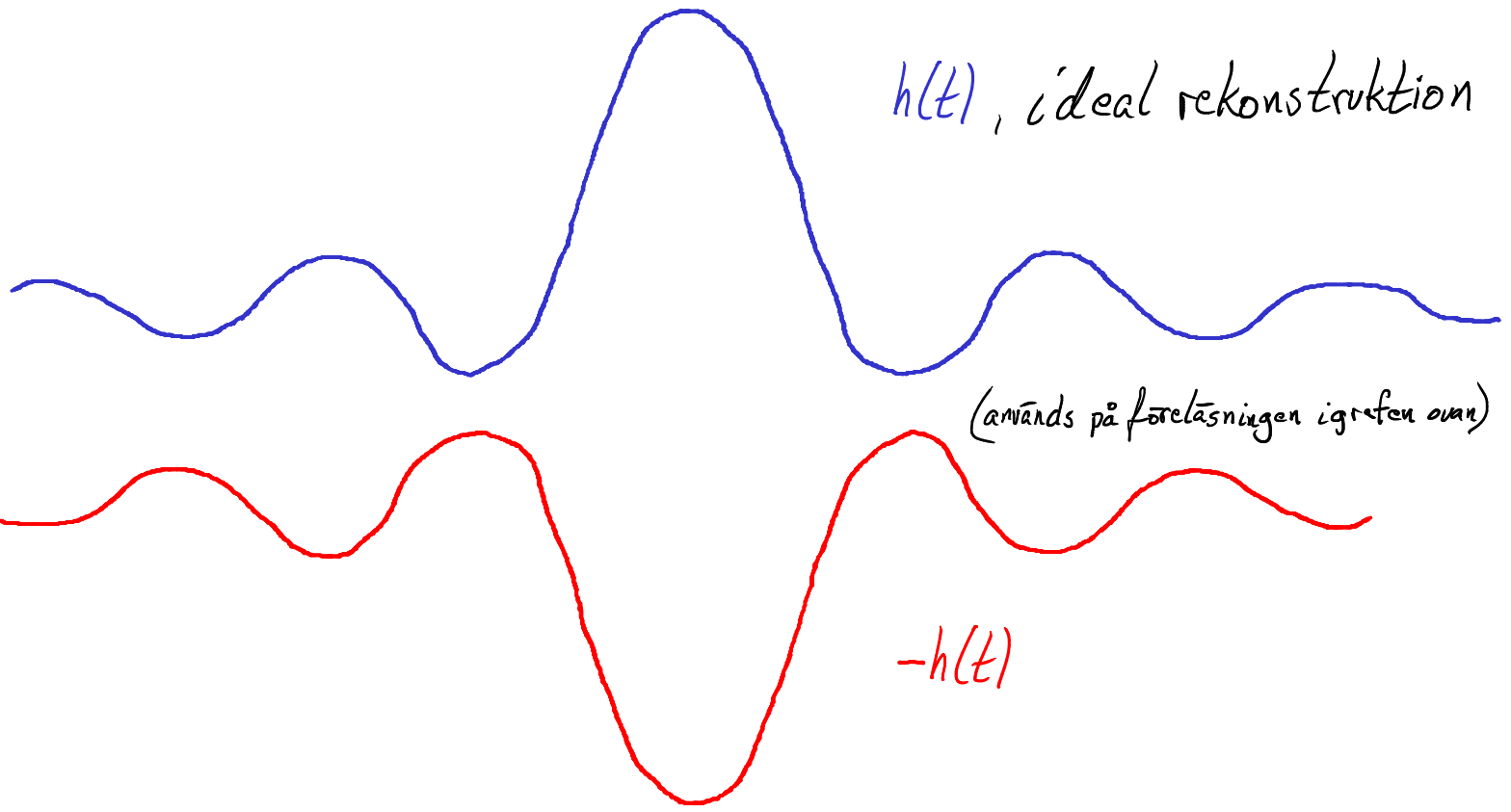
$$= \left( \sum_{n=-\infty}^{\infty} x(t) p(t - nT) \right) * h(t) = \sum_{n=-\infty}^{\infty} x[n] \underbrace{(\delta(t - nT) * h(t))}_{= h(t - nT)} = \sum_{n=-\infty}^{\infty} x[n] h(t - nT)$$

*vid ideal sampling*  
 $x(t) \delta(t - nT) = x(nT) \delta(t - nT) = x[n] \delta(t - nT)$

$\Leftrightarrow$  Pulsamplitudmodulering (PAM)



$h(t)$ , ideal rekonstruktion

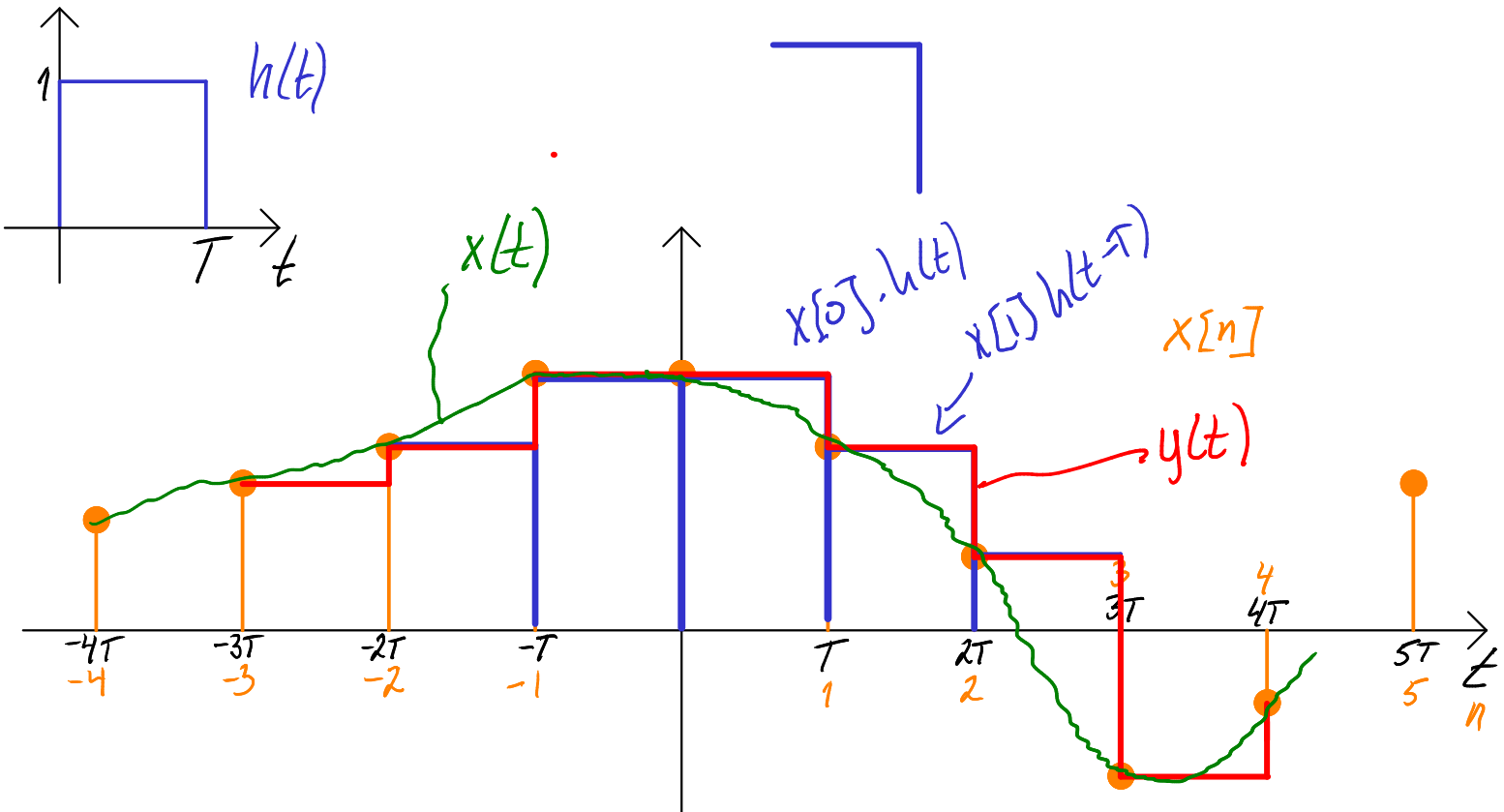


(används på föreläsningen i grafen ovan)

$-h(t)$

Praktisk rekonstruktion – två exempel på interpolationsfunktion vid PAM, dvs. rekonstruktionsfiltrets impulssvar:

Exempel 1 (från videon):



Exempel 2:

