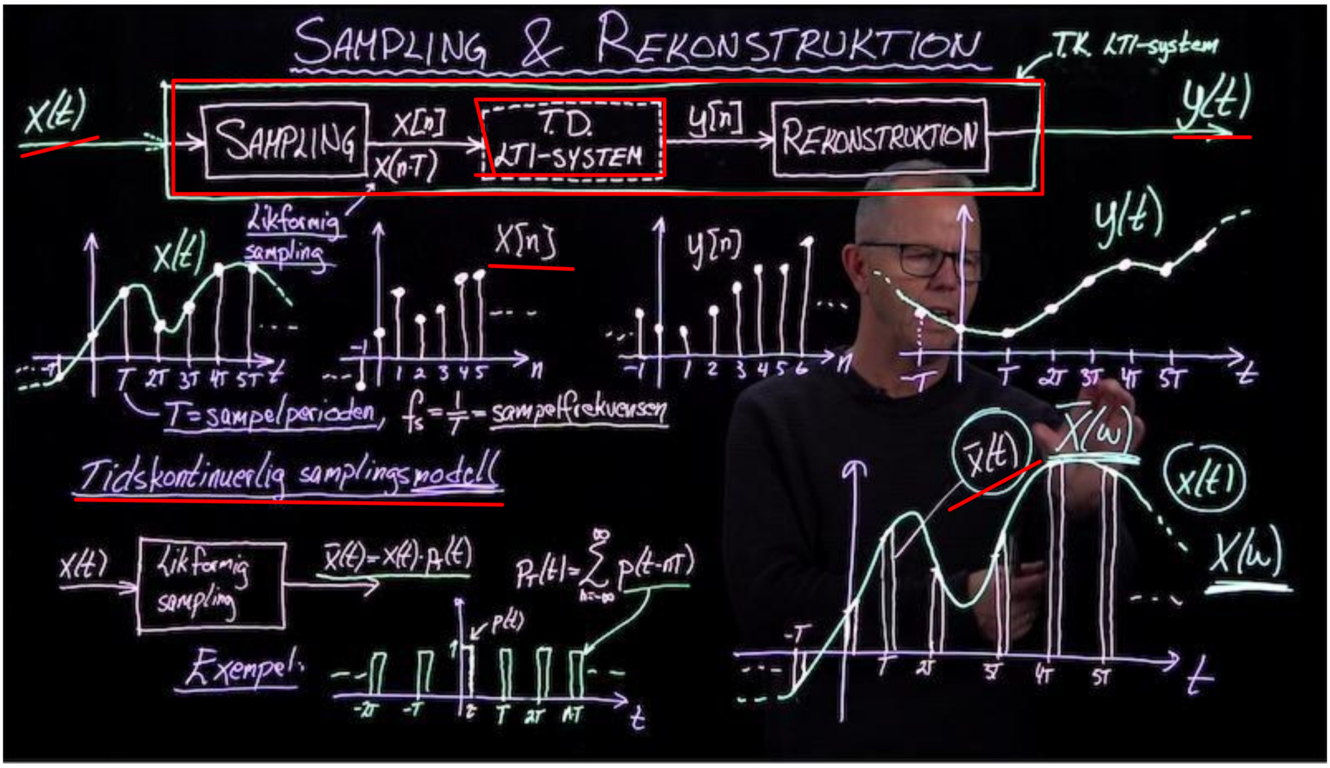
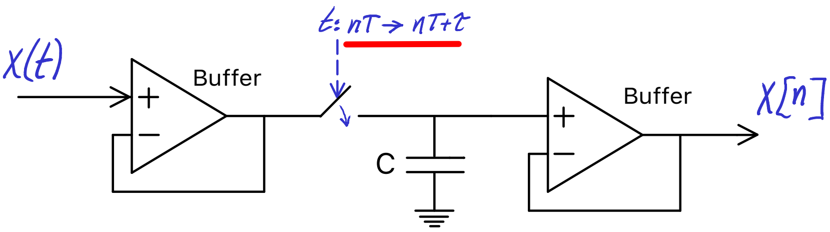
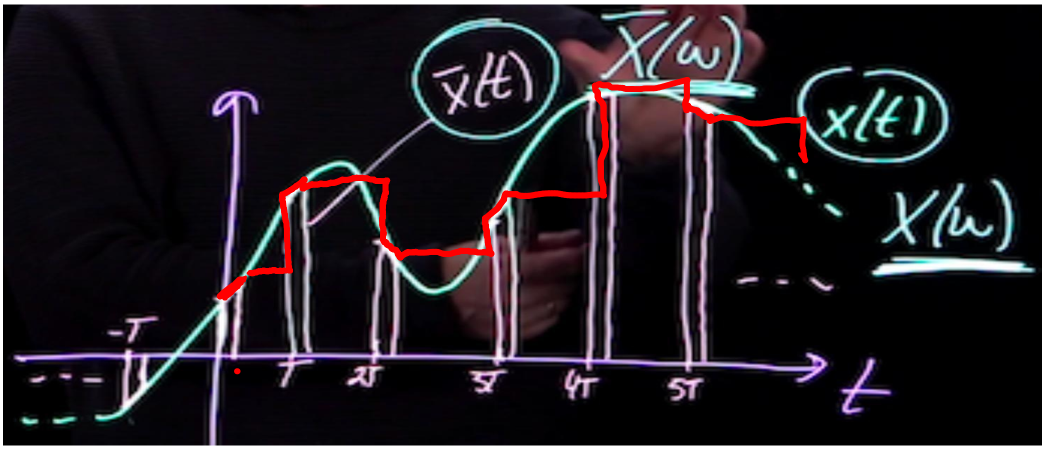
Signaler & System – Föreläsning 6: Sampling & Rekonstruktion

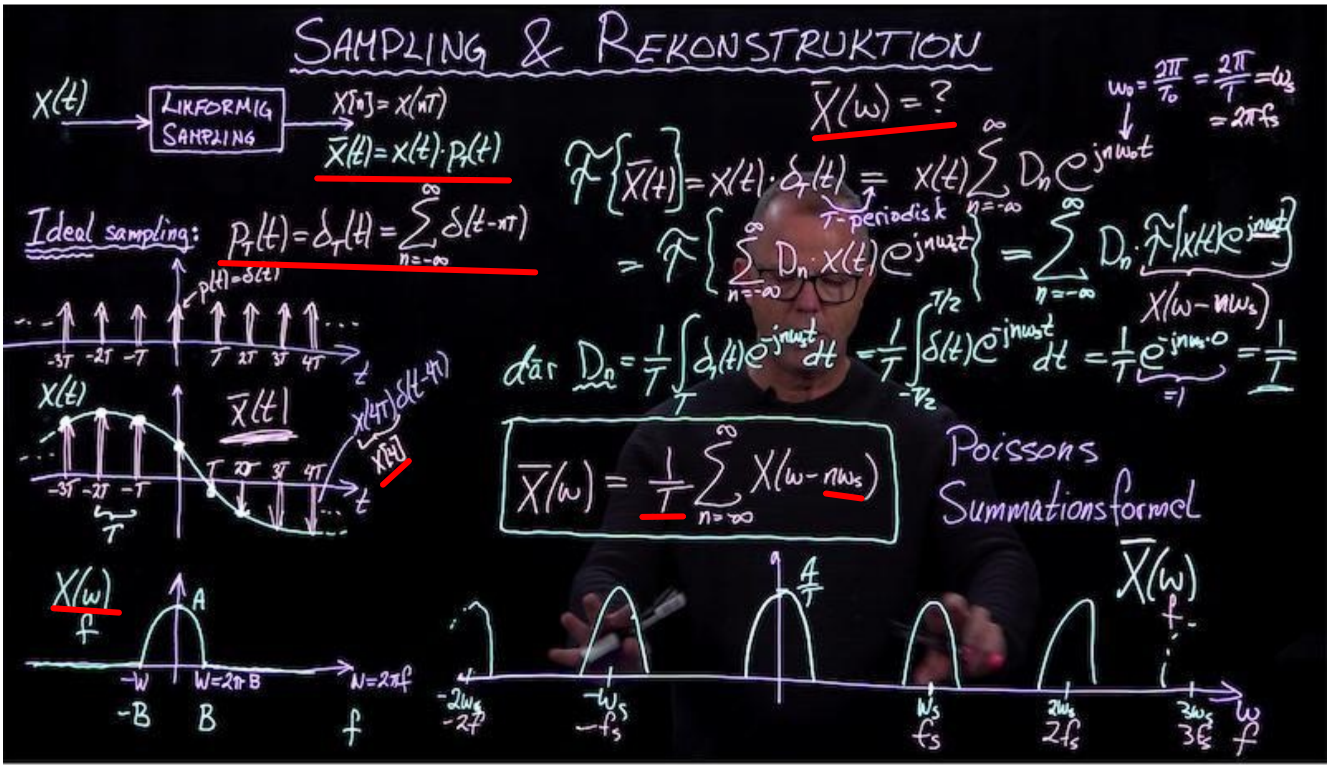


VIDEO 1

Sampling i praktiken – Sample & Hold:







VIDEO 1

Det flesta grundläggande samband finns i formelsamlingen –
du behöver främst förstå dem och veta när och hur de ska användas:

3.2.4 Sampling & Rekonstruktion

Likformig sampling med sampelperiod T: $x[n] = x(t)\big|_{t=nT} = x(nT)$

- Tidskontinuerlig representation: $\bar{x}(t) = x(t)\,p_T(t) = \sum_{n=-\infty}^{\infty} x(nT)\,p(t-nT)$
- Frekvenstransformation: $\Omega = \omega T = \dfrac{\omega}{f_s}$, $\theta = fT = \dfrac{f}{f_s}$
- Ideal sampling: $p(t) = \delta(t) \quad \Rightarrow$

  Poissons summationsformel: $\bar{X}(\omega) = \dfrac{1}{T}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$

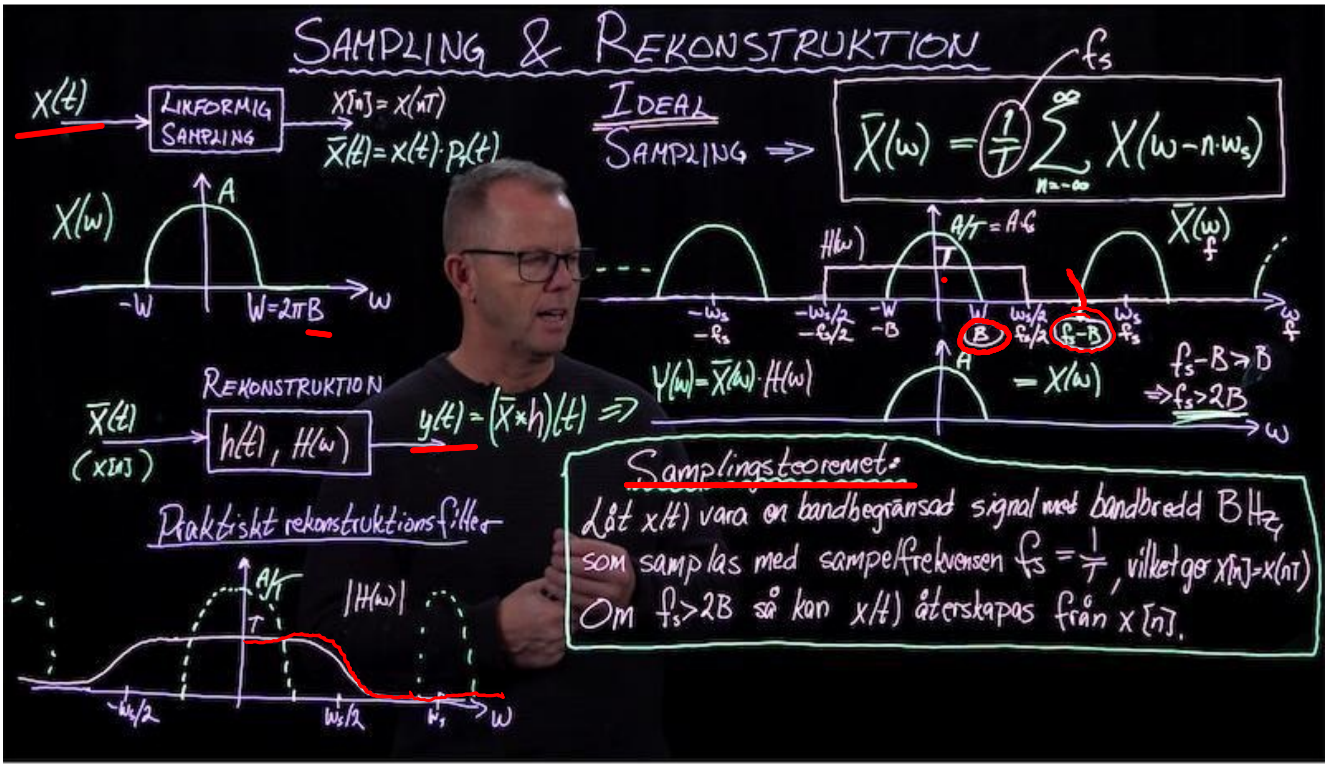
  Alternativa former: $X[\Omega] = \dfrac{1}{T}\sum_{n=-\infty}^{\infty} X\left(\dfrac{\Omega - n2\pi}{T}\right)$, $X[\theta] = \dfrac{1}{T}\sum_{n=-\infty}^{\infty} X\left(\dfrac{\theta - n}{T}\right)$

Rekonstruktion m.h.a. PAM: $y(t) = \sum_{n=-\infty}^{\infty} x[n]\,h(t-nT) \quad \Leftrightarrow \quad Y(\omega) = X[\omega T]\,H(\omega)$

- Ideal rekonstruktion: $h(t) = \mathrm{sinc}\left(\dfrac{\pi t}{T}\right) = \mathrm{sinc}_N\left(\dfrac{t}{T}\right)$

15. Ideal sampling	$\sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$	$\dfrac{1}{T}\sum_{n=-\infty}^{\infty} X\left(\omega - n\dfrac{2\pi}{T}\right)$

Tabell 2 Egenskaper hos fouriertransformen för tidskontinuerliga funktioner.



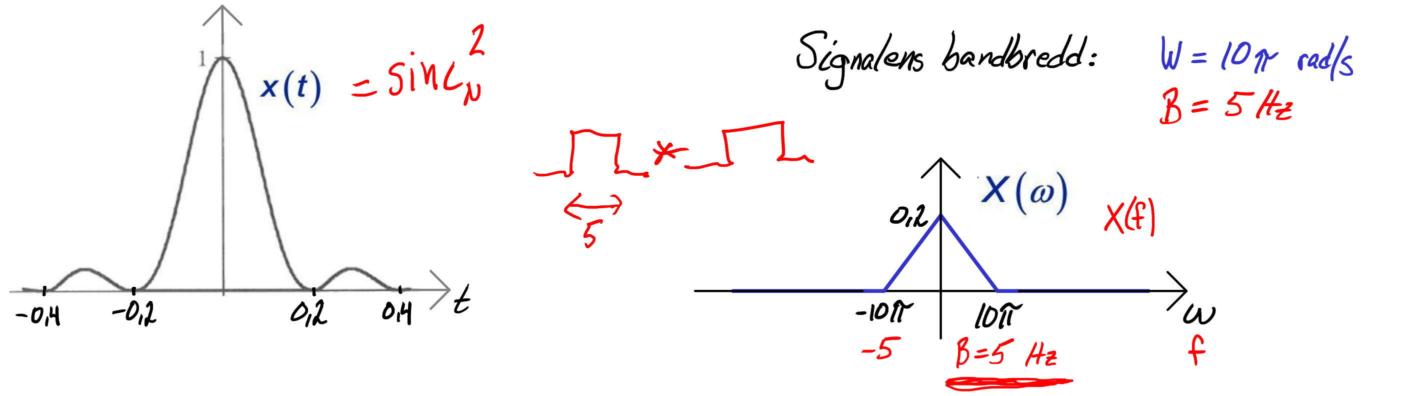
VIDEO 1

$f_s - B > B$

$\Rightarrow f_s > 2B$

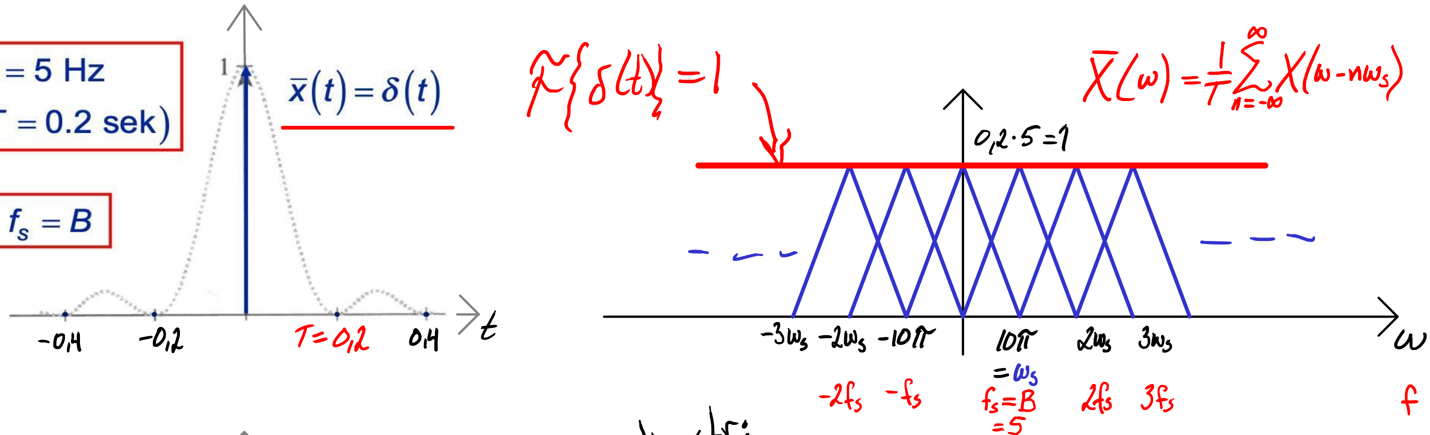
Lasses hjälpgraf under föreläsningen

Samplingsexempel med olika sampelfrekvenser f_s:



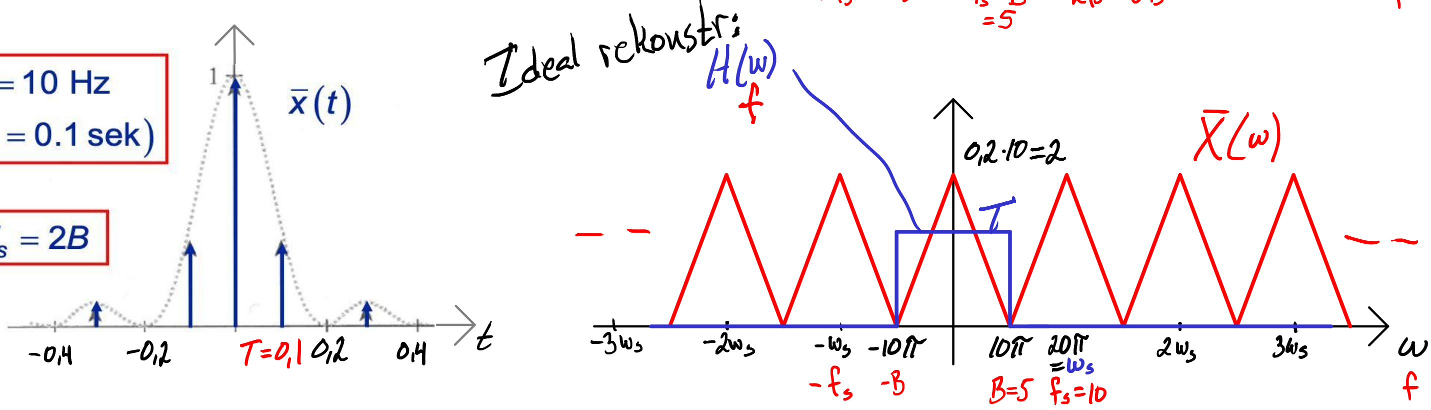
$f_s = 5$ Hz
($T = 0.2$ sek)

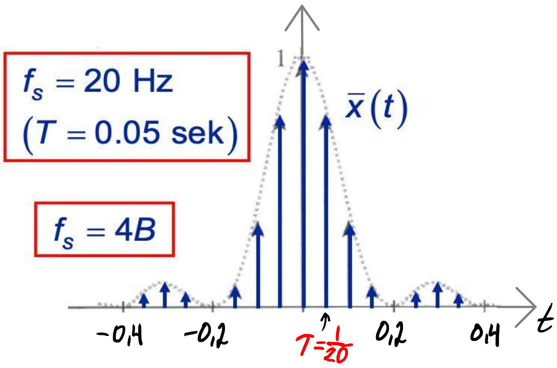
$f_s = B$

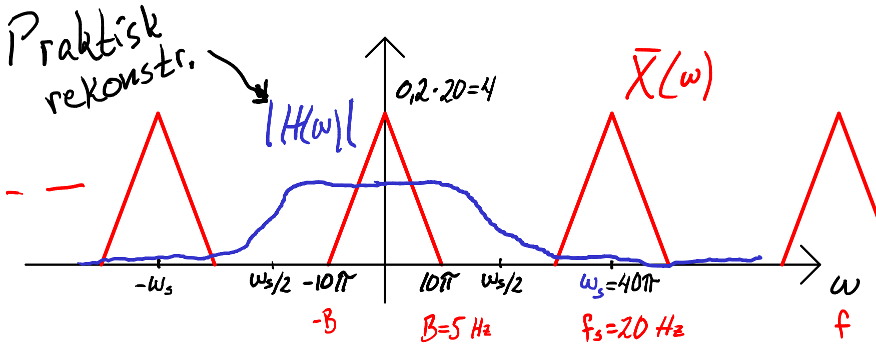


$f_s = 10$ Hz
($T = 0.1$ sek)

$f_s = 2B$



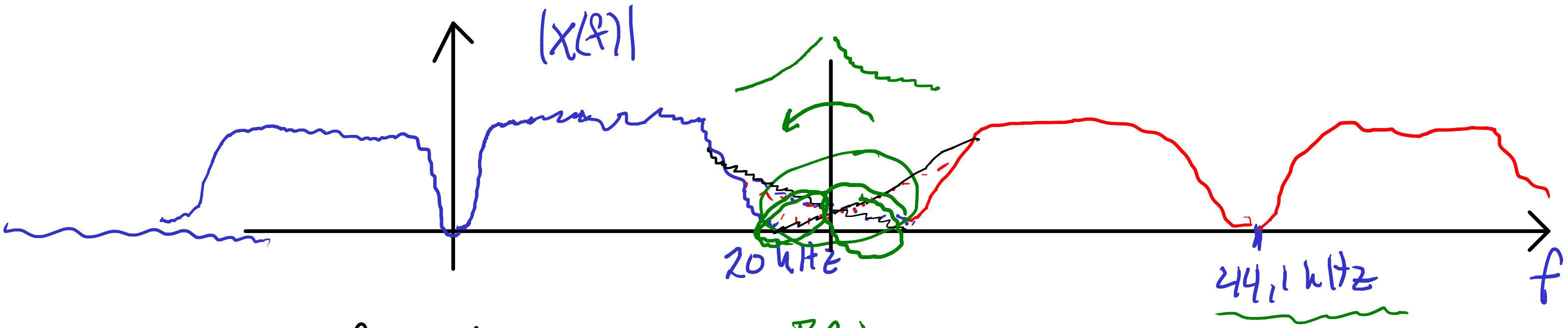




Ex audiosignaler

Musik på CD är samplad med

$f_s = 44{,}1$ kHz, 16 bitar/sampel



PAL
NTSC

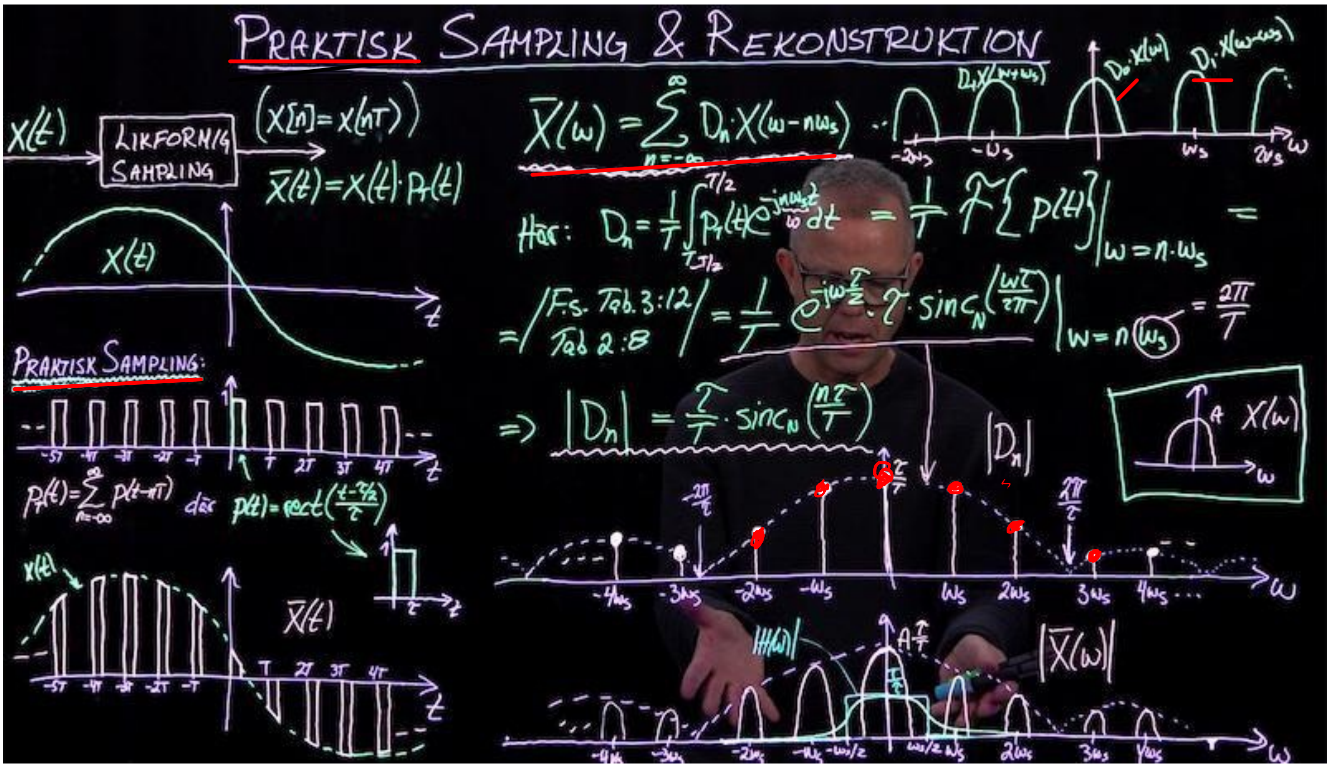
Vanlig: $f_s = 48$ kHz

$f_s < 2B \Rightarrow$ Undersampling $\Rightarrow$ Vikning (eng: aliasing)

$f_s \gg 2B \Rightarrow$ Översampling $\Rightarrow$ Enklare rekonstr.krets

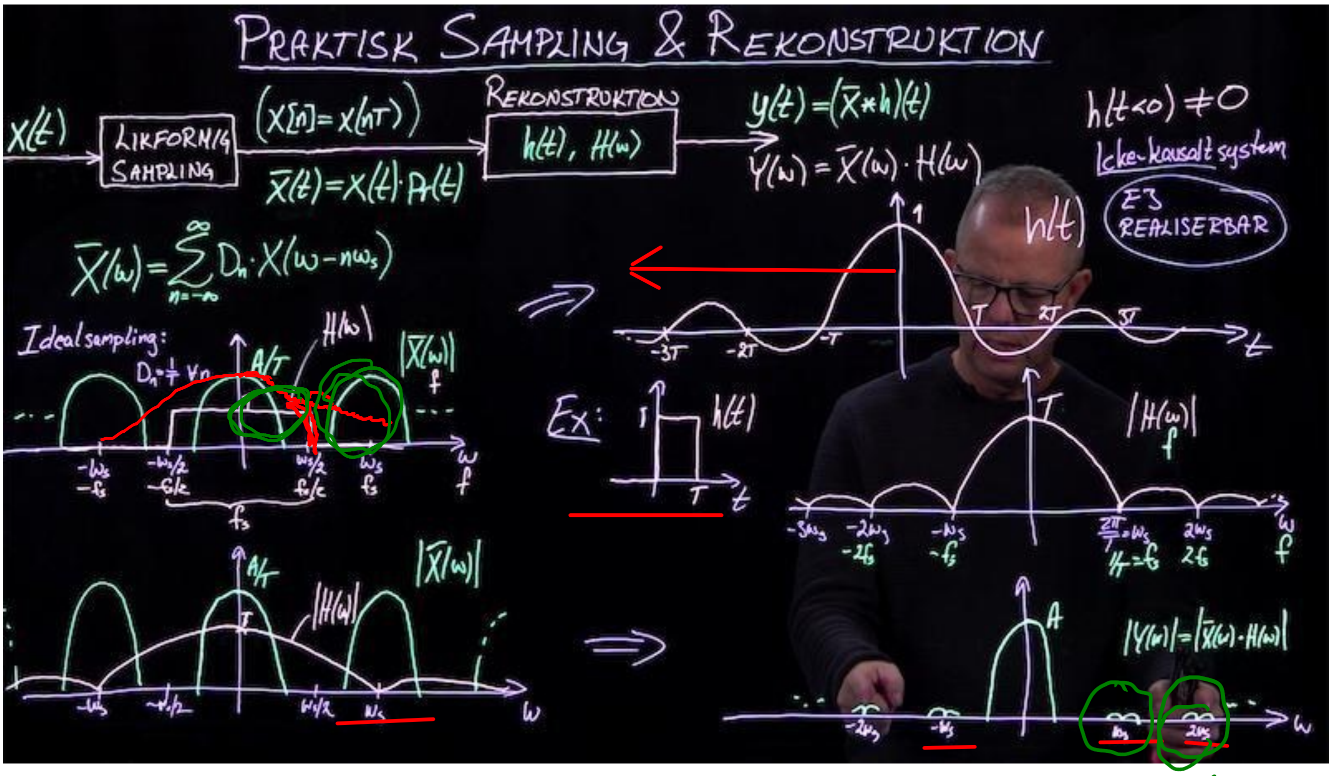
Här följer en demonstration i Matlab, som illustrerar vikning vid sampling av en cosinussignal. (con2dis i toolboxen spfirst, som finns på kurswebbsidan under "Intressanta länkar")

VIDEO 2



Praktisk sampling

VIDEO 2



Ideal och Praktisk rekonstruktion

Rekonstruktion

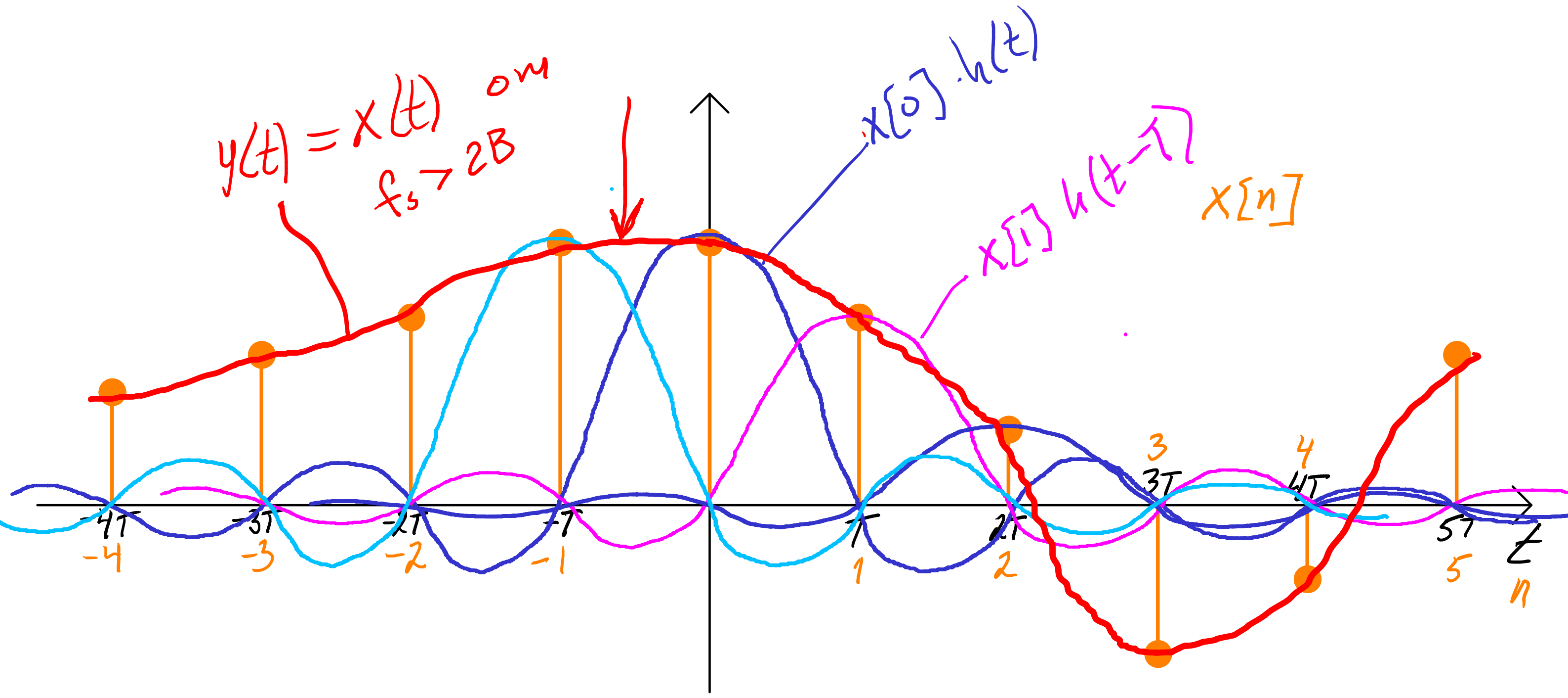
$$x[n] \quad \bar{x}(t) = x(t)\,p_T(t) \longrightarrow \boxed{h(t),\ H(\omega)} \longrightarrow y(t) = (\bar{x} * h)(t) = \bar{x}(t) * h(t)$$

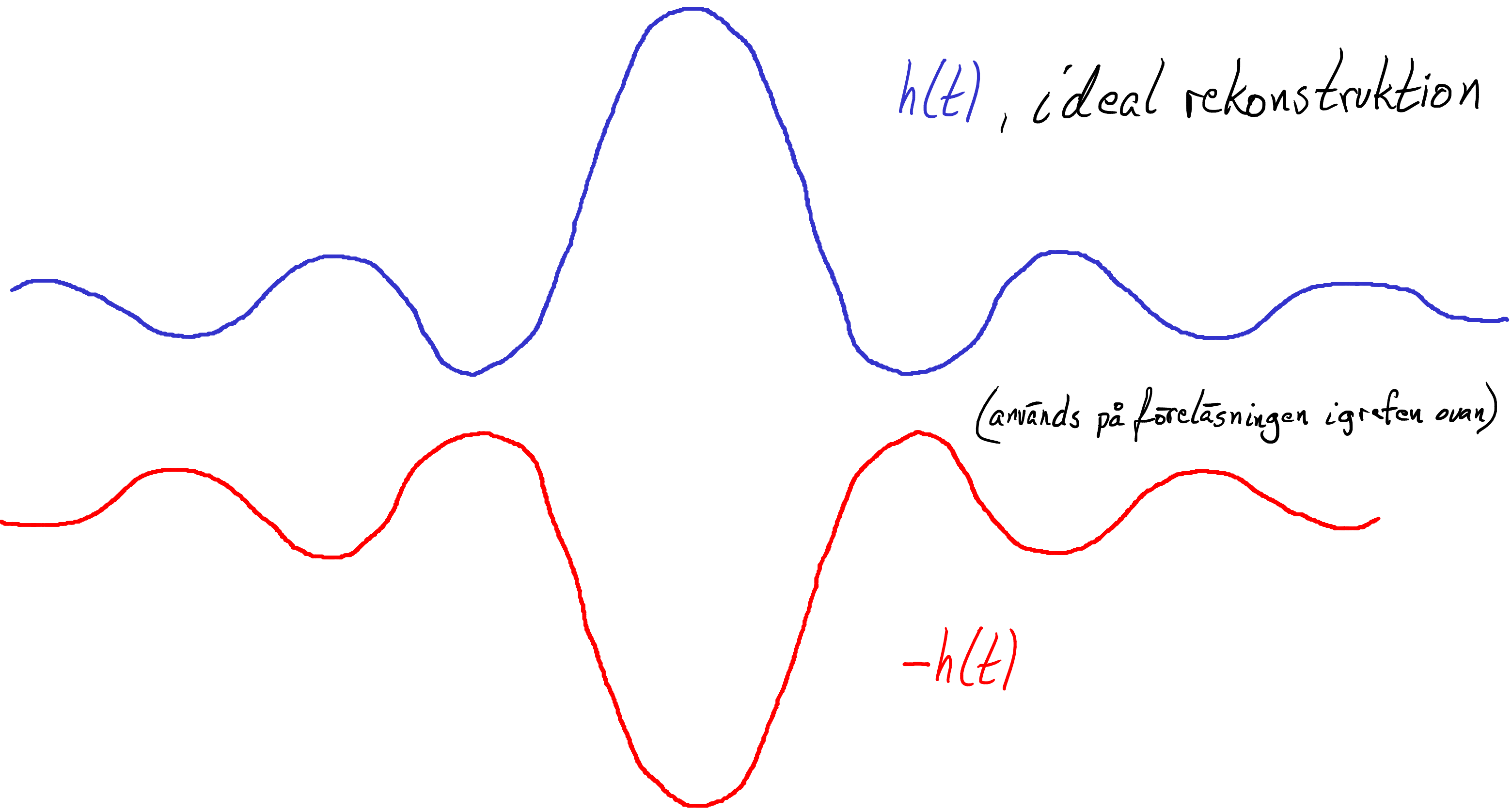
$$= \left(\sum_{n=-\infty}^{\infty} \underbrace{x(t)\,p(t-nT)}\right) * h(t) = \sum_{n=-\infty}^{\infty} x[n]\left(\underbrace{\delta(t-nT) * h(t)}_{=h(t-nT)}\right) = \sum_{n=-\infty}^{\infty} x[n]\,h(t-nT)$$

$x(t)\,\delta(t-nT)$ vid ideal sampling

$= x(nT)\,\delta(t-nT) = x[n]\,\delta(t-nT)$

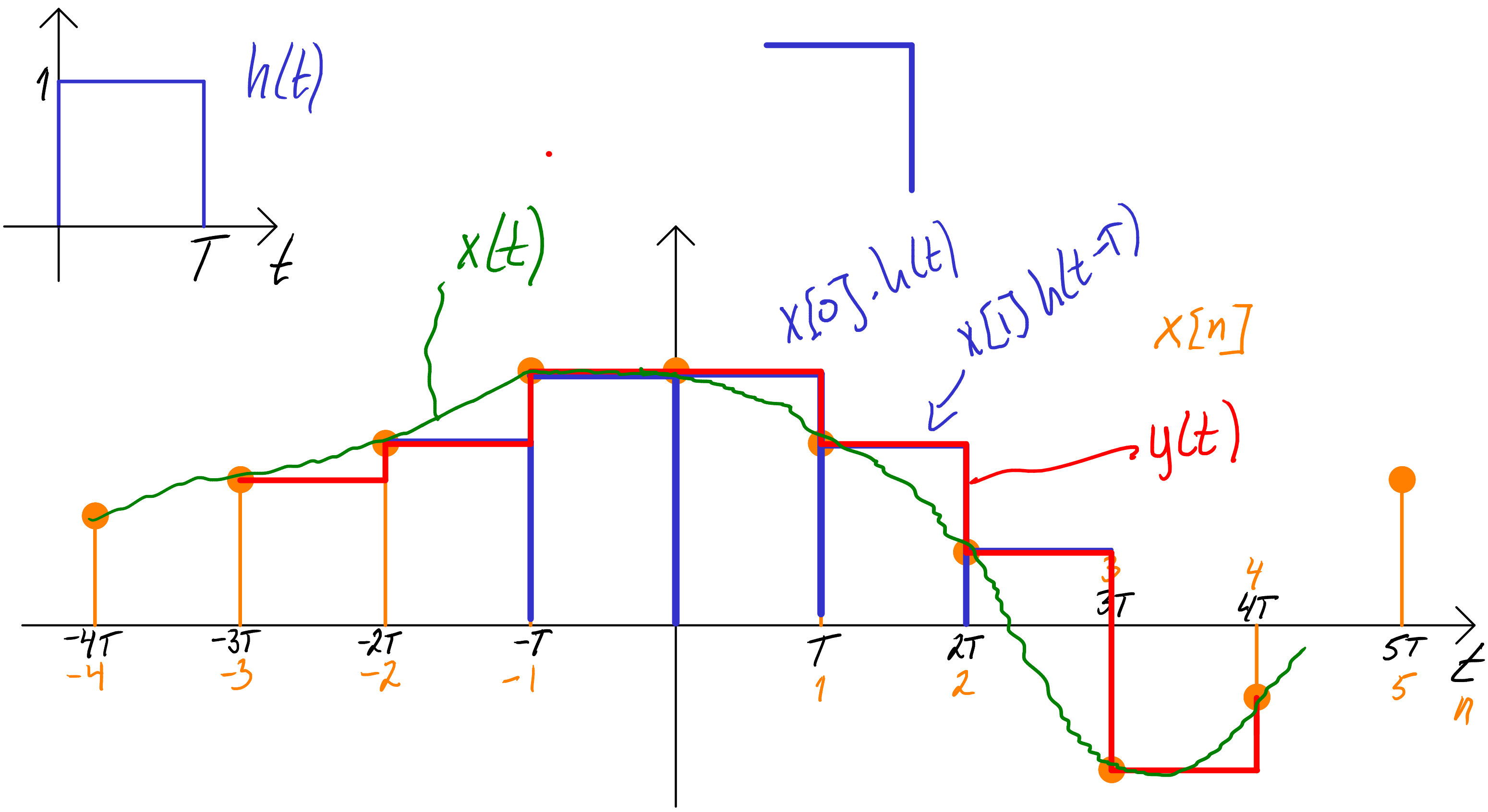
$\Leftrightarrow$ Pulsamplitudmodulering (PAM)



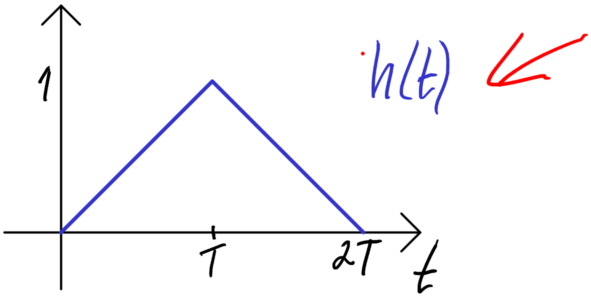


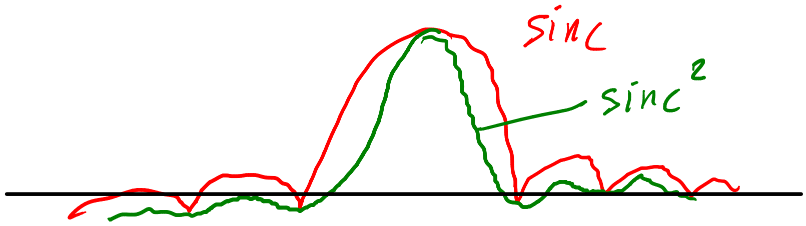
<u>Praktisk rekonstruktion</u> – två exempel på interpolationsfunktion vid PAM, dvs. rekonstruktionsfiltrets impulssvar:

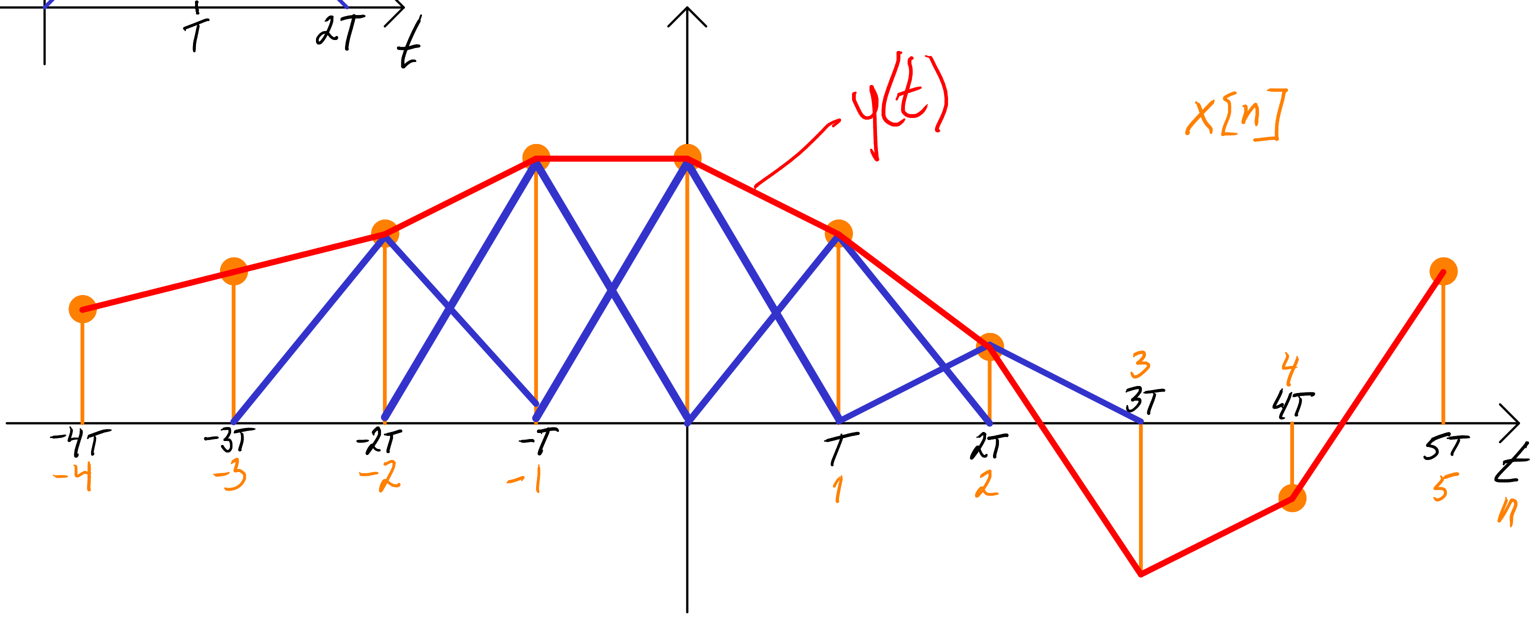
Exempel 1 (från videon):



Exempel 2:


1
T
2T
t
h(t)


sinc
sinc^2


y(t)
x[n]
-4T -3T -2T -T T 2T 3T 4T 5T
-4 -3 -2 -1 1 2 3 4 5
t
n