1.2-2. Refer to Figure S1.2-2.

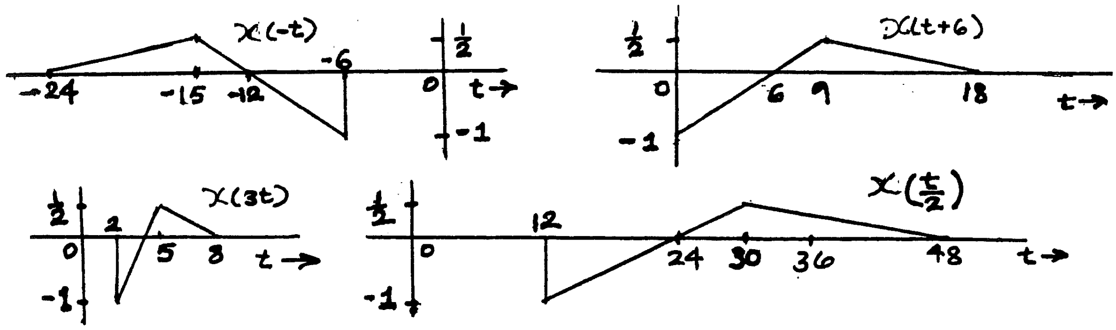


Figure S1.2-1

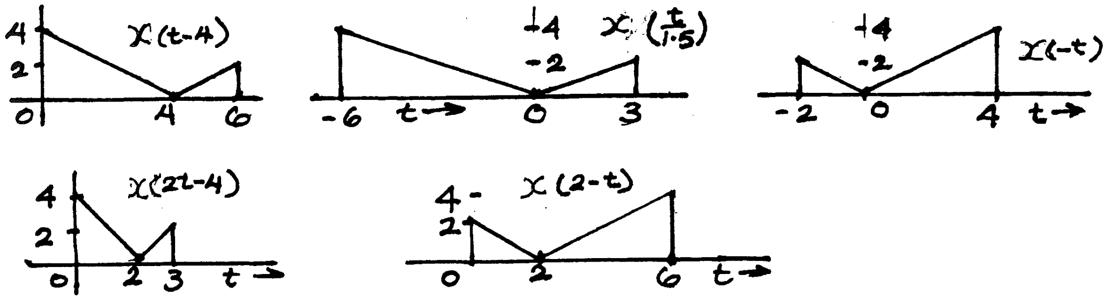


Figure S1.2-2

1.3-5. (a) $E_{y_1} = \int_{-\infty}^{\infty} y_1^2(t)dt = \int_{-\infty}^{\infty} \frac{1}{9}x^2(2t)dt$. Performing the change of variable $t' = 2t$ yields $\int_{-\infty}^{\infty} \frac{1}{9}x^2(t')\frac{dt'}{2} = \frac{E_x}{18}$. Thus,

$$E_{y_1} = \frac{E_x}{18} \approx \frac{1.0417}{18} = 0.0579.$$

(b) Since $y_2(t)$ is just a $(T_{y_2} = 4)$-periodic replication of $x(t)$, the power is easily obtained as

$$P_{y_2} = \frac{E_x}{T_{y_2}} = \frac{E_x}{4} \approx= 0.2604.$$

1.4-2.

$$\begin{aligned} x_1(t) &= (4t+4)[u(t+1) - u(t)] + (-2t+4)[u(t) - u(t-2)] \\ x_2(t) &= t^2[u(t) - u(t-2)] + (2t-8)[u(t-2) - u(t-4)] \end{aligned}$$

1.4-3. Using the fact that $f(x)\delta(x) = f(0)\delta(x)$, we have

(a) 0

(b) $\frac{2}{9}\delta(\omega)$

(c) $\frac{1}{2}\delta(t)$

(d) $-\frac{1}{5}\delta(t-1)$

(e) $\frac{1}{2-j3}\delta(\omega+3)$

(f) $k\delta(\omega)$ (use L' Hôpital's rule)

1.4-4. In these problems remember that impulse $\delta(x)$ is located at $x = 0$. Thus, an impulse $\delta(t-\tau)$ is located at $\tau = t$, and so on.

(b) The impulse is located at $\tau = t$ and $x(\tau)$ at $\tau = t$ is $x(t)$. Therefore

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau = x(t).$$

(a) The impulse $\delta(\tau)$ is at $\tau = 0$ and $x(t-\tau)$ at $\tau = 0$ is $x(t)$. Therefore

$$\int_{-\infty}^{\infty} \delta(\tau)x(t-\tau)\,d\tau = x(t).$$

Using similar arguments, we obtain

(c) 1

(d) $-1/2$

(e) e^3

(f) 5

(g) $x(-1)$

(h) $-e^2$

1.4-10. For sketches, refer to Figure S1.4-10.

(a) $s_{1,2} = \pm j3$

(b) $e^{-3t}\cos 3t = 0.5[e^{-(3+j3)t} + e^{-(3-j3)t}]$. Therefore the frequencies are $s_{1,2} = -3 \pm j3$.

(c) Using the argument in (b), we find the frequencies $s_{1,2} = 2 \pm j3$

(d) $s = -2$

(e) $s = 2$

(f) $5 = 5e^{0t}$ so that $s = 0$.

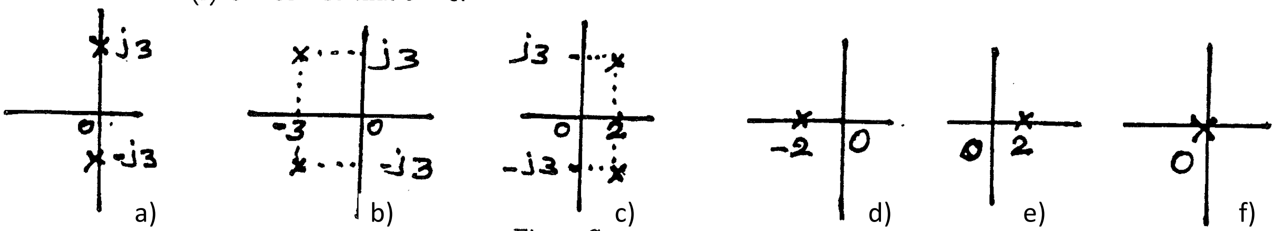


Figure S1.4-10

1.7-1. Only (b), (f), and (h) are linear. All the remaining are nonlinear. This can be verified by using the procedure discussed in Example 1.9.

1.7-2. (a) The system is time-invariant because the input $x(t)$ yields the output $y(t) = x(t-2)$. Hence, if the input is $x(t-T)$, the output is $x(t-T-2) = y(t-T)$, which makes the system time-invariant.

(b) The system is time-varying. The input $x(t)$ yields the output $y(t) = x(-t)$. Thus, the output is obtained by changing the sign of t in $x(t)$. Therefore, when the input is $x(t-T)$, the output is $x(-t-T) = x(-[t+T]) = y(t+T)$, which represents the original output advanced by T (not delayed by T).

(c) The system is time-varying. The input $x(t)$ yields the output $y(t) = x(at)$, which is a scaled version of the input. Thus, the output is obtained by replacing t in the input with at. Thus, if the input is $x(t-T)$ ($x(t)$ delayed by T), the output is $x(at-T) = x(a[t-\frac{T}{a}])$, which is $x(at)$ delayed by T/a (not T). Hence the system is time-varying.

(d) The system is time-varying. The input $x(t)$ yields the output $y(t) = tx(t)$. For the input $x(t-T)$, the output is $tx(t-T)$, which is not $tx(t)$ delayed by T. Hence the system is time-varying.

(e) The system is time-varying. The output is a constant, given by the area under $x(t)$ over the interval $|t| \leq 5$. Now, if $x(t)$ is delayed by T, the output, which is the area under the delayed $x(t)$, is another constant. But this output is not the same as the original output delayed by T. Hence the system is time-varying.

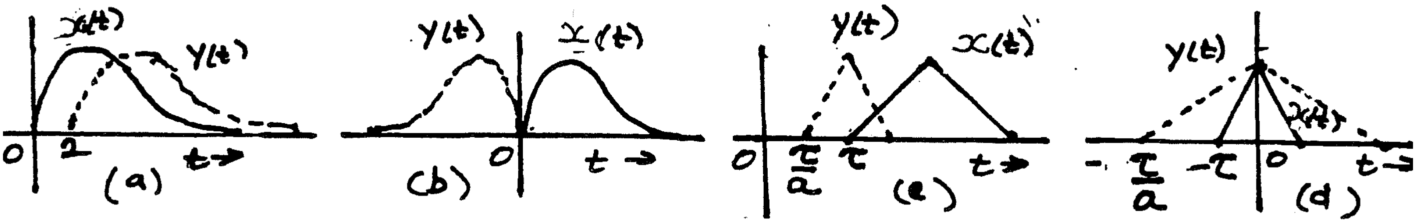
(f) The system is time-invariant. The input $x(t)$ yields the output $y(t)$, which is the square of the second derivative of $x(t)$. If the input is delayed by T, the output is also delayed by T. Hence the system is time-invariant.

1.7-7. (a) $y(t) = x(t-2)$. Thus, the output $y(t)$ always starts after the input by 2 seconds (see Figure S1.7-7a). Clearly, the system is causal.

(b) $y(t) = x(-t)$. The output $y(t)$ is obtained by time inversion in the input. Thus, if the input starts at $t = 0$, the output starts before $t = 0$ (see Figure S1.7-7b). Hence, the system is not causal.

(c) $y(t) = x(at)$, $a > 1$. The output $y(t)$ is obtained by time compression of the input by factor a. Hence, the output can start before the input (see Figure S1.7-7c), and the system is not causal.

(d) $y(t) = x(at)$, $a < 1$. The output $y(t)$ is obtained by time expansion of the input by factor $1/a$. Hence, the output can start before the input (see Figure S1.7-7d), and the system is not causal.



1.7-9. (a) Yes, the system is linear. Begin assuming $y_1(t) = r(t)x_1(t)$ and $y_2(t) = r(t)x_2(t)$. Applying $ax_1(t) + bx_2(t)$ to the system yields $y(t) = r(t)\,(ax_1(t) + bx_2(t)) = ar(t)x_1(t) + br(t)x_2(t) = ay_1(t) + by_2(t)$.

(b) Yes, the system is memoryless. By inspection, it is clear that the system only depends on the current input.

(c) Yes, the system is causal. Since the system is memoryless, the system cannot depend on future values and must be causal.

(d) No, the system is not time-invariant. Since the system function depends on the independent variable t, it is unlikely that the system is time-invariant. To explicitly verify, let $y(t) = r(t)x(t)$. Next, delay $x(t)$ by τ to obtain a new input $x_2 = x(t-\tau)$. Applying $x_2(t)$ to the system yields $y_2(t) = r(t)x_2(t) = r(t)x(t-\tau) \neq r(t-\tau)x(t-\tau) = y(t-\tau)$. Since, the system operator and the time-shift operator do not commute, the system is not time-invariant.

1.7-11. (a) No, the system is not BIBO stable. The system returns the time-delayed derivative, or slope, of the input signal. A square-wave is a bounded signal which, due to point discontinuities, has infinite slope at certain instants in time. Thus, a bounded input may not result in a bounded output, and the system cannot be BIBO stable.

(b) Yes, the system is linear. Begin assuming $y_1(t) = \frac{d}{dt}x_1(t-1)$ and $y_2(t) = \frac{d}{dt}x_2(t-1)$. Applying $ax_1(t) + bx_2(t)$ to the system yields $y(t) = \frac{d}{dt}(ax_1(t-1) + bx_2(t-1)) = a\frac{d}{dt}x_1(t-1) + b\frac{d}{dt}x_2(t-1) = ay_1(t) + by_2(t)$.

(c) No, the system is not memoryless. By inspection, it is clear that the system depends on a past value of the input. For example, at $t = 0$, the output $y(0)$ depends on the time-derivative of $x(-1)$, a past value.

(d) Yes, the system is causal. By inspection, it is clear that the system does not depend on future values.

(e) Yes, the system is time-invariant. To explicitly verify, let $y(t) = \frac{d}{dt}x(t-1)$. Next, delay $x(t)$ by τ to obtain a new input $x_2 = x(t-\tau)$. Applying $x_2(t)$ to the system yields $y_2(t) = \frac{d}{dt}x_2(t) = \frac{d}{dt}x(t-1-\tau) = y(t-\tau)$. Since, the system operator and the time-shift operator commute, the system is time-invariant. In more loose terms, the derivative operator returns the delayed slope of a signal independent of when that signal is applied.

1.8-3. The freebody diagram for the mass M is shown in Figure 1.8-3. From this diagram it follows that

$$M\ddot{y} = B(\dot{x} - \dot{y}) + K(x - y)$$

or

$$(MD^2 + BD + K)y(t) = (BD + K)x(t)$$

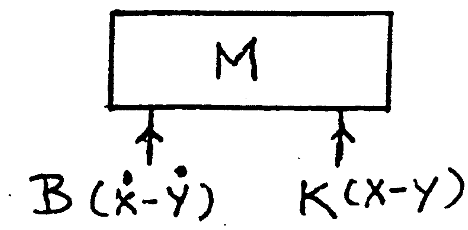


Figure S1.8-3

2.2-1. The characteristic polynomial is $\lambda^2+5\lambda+6$. The characteristic equation is $\lambda^2+5\lambda+6 = 0$. Also $\lambda^2+5\lambda+6 = (\lambda+2)(\lambda+3)$. Therefore the characteristic roots are $\lambda_1 = -2$ and $\lambda_2 = -3$. The characteristic modes are e^{-2t} and e^{-3t}. Therefore

$$y_0(t) = c_1e^{-2t} + c_2e^{-3t}$$

and

$$\dot{y}_0(t) = -2c_1e^{-2t} - 3c_2e^{-3t}$$

Setting $t = 0$, and substituting initial conditions $y_0(0) = 2$, $\dot{y}_0(0) = -1$ in this equation yields

$$\left.\begin{aligned} c_1 + c_2 &= 2 \\ -2c_1 - 3c_2 &= -1 \end{aligned}\right\} \Longrightarrow \quad \begin{aligned} c_1 &= 5 \\ c_2 &= -3 \end{aligned}$$

Therefore

$$y_0(t) = 5e^{-2t} - 3e^{-3t}$$

2.4-7. In this problem, we use Table 2.1 to find the desired convolution.

OBS: Använd INTE tabell för att erhålla $y(t)$. Här skall du **beräkna** $y(t)$ m.h.a. faltningsintegralen!

(a) $y(t) = h(t) * x(t) = e^{-t}u(t) * u(t) = (1-e^{-t})u(t)$

(b) $y(t) = h(t) * x(t) = e^{-t}u(t) * e^{-t}u(t) = te^{-t}u(t)$

(c) $y(t) = e^{-t}u(t) * e^{-2t}u(t) = (e^{-t} - e^{-2t})u(t)$

(d) $y(t) = \sin 3tu(t) * e^{-t}u(t)$

Here we use pair 12 (Table 2.1) with $\alpha = 0$, $\beta = 3$, $\theta = -90°$ and $\lambda = -1$. This yields

$$\phi = \tan^{-1}\left[\frac{-3}{-1}\right] = -108.4°$$

and

$$\begin{aligned} \sin 3t\, u(t) * e^{-t}u(t) &= \frac{(\cos 18.4°)e^{-t} - \cos(3t + 18.4°)}{\sqrt{10}}u(t) \\ &= \frac{0.9486e^{-t} - \cos(3t + 18.4°)}{\sqrt{10}}u(t) \end{aligned}$$

2.4-11. (a) $y(t) = e^{-t}u(t) * e^{-2t}u(t) = (e^{-t} - e^{-2t})u(t)$

(b) $e^{-2(t-3)}u(t) = e^6e^{-2t}u(t)$, and $y(t) = e^6\left[e^{-t}u(t) * e^{-2t}u(t)\right] = e^6(e^{-t} - e^{-2t})u(t)$

(c) $e^{-2t}u(t-3) = e^{-6}e^{-2(t-3)}u(t-3)$. Now from the result in part (a) and the shift property of the convolution [Eq. (2.34)]: $y(t) = e^{-6}\left[e^{-(t-3)}\cancel{u(t)} - e^{-2(t-3)}\right]u(t-3)$

(d) $x(t) = u(t) - u(t-1)$. Now $y_1(t)$, the system response to $u(t)$ is given by

$$y_1(t) = e^{-t}u(t) * u(t) = (1 - e^{-t})u(t)$$

The system response to $u(t-1)$ is $y_1(t-1)$ because of time-invariance property. Therefore the response $y(t)$ to $x(t) = u(t) - u(t-1)$ is given by

$$y(t) = y_1(t) - y_1(t-1) = (1 - e^{-t})u(t) - [1 - e^{-(t-1)}]u(t-1)$$

The response is shown in Figure S2.4-11d.

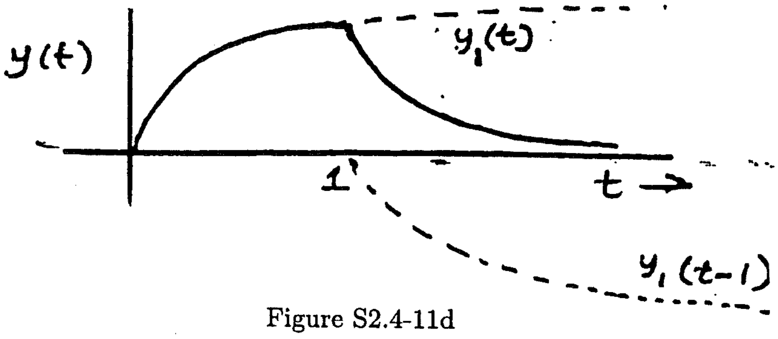


Figure S2.4-11d

2.4-12. (a)

$$\begin{aligned} y(t) &= [-\delta(t) + 2e^{-t}u(t)] * e^{t}u(-t) \\ &= -\delta(t) * e^{t}u(-t) + 2e^{-t}u(t) * e^{t}u(-t) \\ &= -e^{t}u(-t) + [e^{-t}u(t) + e^{t}u(-t)] \\ &= e^{-t}u(t) \end{aligned}$$

(b) Refer to Figure S2.4-12b.

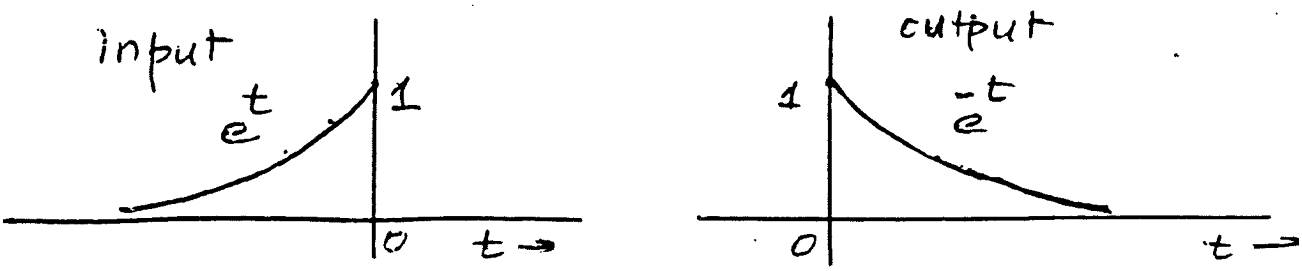


Figure S2.4-12b

2.4-16. For $t < 2\pi$ (see Figure S2.4-16)

$$c(t) = x(t) * g(t) = \int_0^t \sin\tau \, d\tau = 1 - \cos t \qquad 0 \leq t \leq 2\pi$$

For $t \geq 2\pi$, the area of one cycle is zero and

$$x(t) * g(t) = 0 \qquad t \geq 2\pi \quad \text{and} \quad t < 0$$

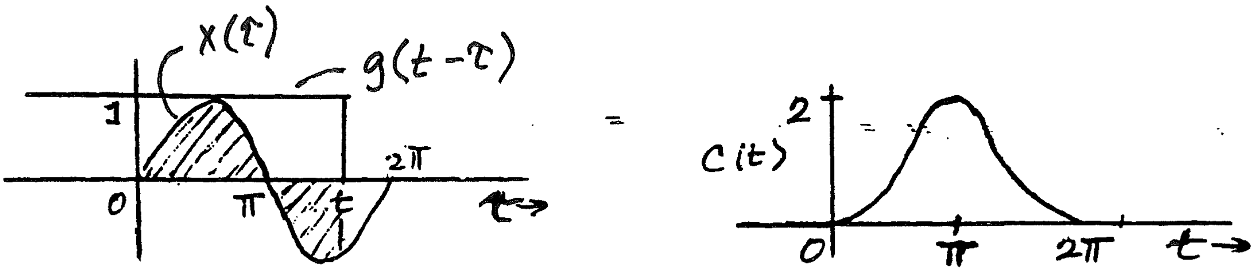


Figure S2.4-16

2.4-17. For $0 \le t \le 2\pi$ (see Figure S2.4-17a)

$$x(t) * g(t) = \int_0^t \sin\tau \, d\tau = 1 - \cos t \qquad 0 \le t \le 2\pi$$

For $2\pi \le t \le 4\pi$ (Figure S2.4-17b)

$$x(t) * g(t) = \int_{t-2\pi}^{2\pi} \sin\tau \, d\tau = \cos t - 1 \qquad 2\pi \le t \le 4\pi$$

For $t > 4\pi$ (also for $t < 0$), $x(t) * g(t) = 0$. Figure S2.4-17c shows $c(t)$.

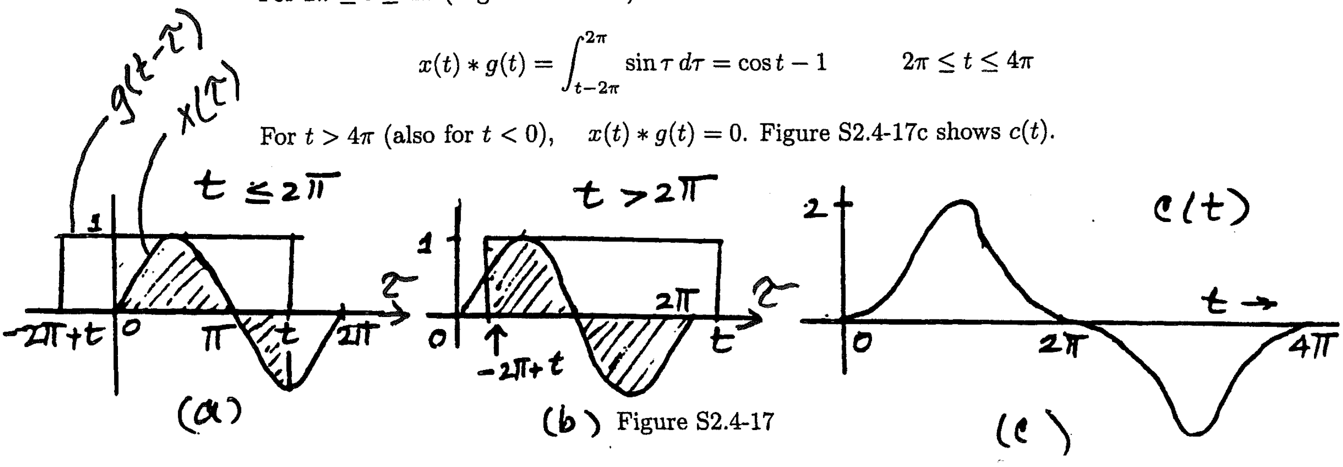


Figure S2.4-17

2.4-28. Since the system step response is $s(t) = e^{-t}u(t) - e^{-2t}u(t)$, the system impulse response is $h(t) = \frac{d}{dt}s(t) = -e^{-t}u(t) + \delta(t) + 2e^{-2t}u(t) - \delta(t) = (2e^{-2t} - e^{-t})u(t)$. The input $x(t) = \delta(t-\pi) - \cos(\sqrt{3})u(t)$ is just a sum of a shifted delta function and a scaled step function. Since the system is LTI, the output is quickly computed using just $h(t)$ and $s(t)$. That is,

$$y(t) = h(t-\pi) - \cos(\sqrt{3})s(t) = (2e^{-2(t-\pi)} - e^{-(t-\pi)})u(t-\pi) - \cos(\sqrt{3})(e^{-t} - e^{-2t})u(t).$$

OBS: Använd $g(t)$ för att beteckna stegsvaret, inte $s(t)$ som i lösningsförslaget för 2.4-28, ovan!

Förtydligad lösning på 2.4-28:

$x(t) = x_1(t) + x_2(t)$ där $x_1(t) = \delta(t-\pi)$, $x_2(t) = \cos\sqrt{3} \cdot u(t)$

$\Rightarrow$

$$y(t) = x(t) * h(t) = x_1(t) * h(t) + x_2(t) * h(t) = \delta(t-\pi) * h(t) + \cos\sqrt{3} \cdot \underbrace{u(t) * h(t)}_{=g(t)}$$

$$= h(t-\pi) + \cos\sqrt{3} \cdot g(t) = \quad \text{/Se svaret ovan!/}$$

2.4-31. Since $h(t)$ is only provided for over $(0 \leq t < 0.5)$, it is not possible to determine with certainty whether or not the system is causal or stable. However, when looking at $h(t)$ the waveform appears to have a DC offset. This apparent DC offset can be very troubling if $h(t)$ is truly an impulse response function. If a DC offset is present, the system is neither causal nor stable. Imagine, a non-causal, unstable heart! Something is probably wrong.

One simple explanation is that a blood-filled heart always has some ventricular pressure. Unless removed, this relaxed-state pressure would likely appear as a DC offset to any measurements. It would likely be most appropriate to subtract this offset when trying to measure the impulse response function.

Another problem is that the impulse response function is most appropriate in the study of linear, time-invariant systems. It is quite unlikely that the heart is either linear or time-invariant. Even if the impulse response could be reliably measured at a particular time, it might not provide much useful information.

2.4-39.

(a) Yes, the system is causal since $h(t) = 0$ for $(t < 0)$.

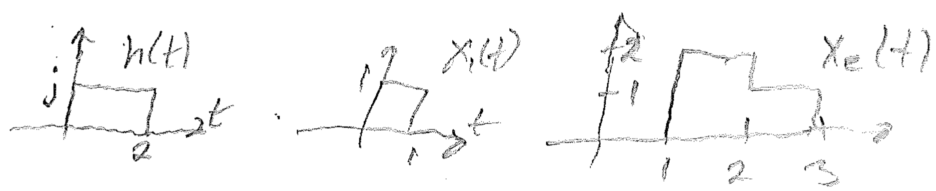
(b) To compute the zero-state response $y_1(t)$, the convolution of two rectangular pulses is required: a pulse of amplitude j and width two and a pulse of amplitude one and a width of one. The convolution involves several regions.
For $t < 0$, $y_1(t) = 0$.
For $0 \leq t < 1$, $y_1(t) = \int_0^t j d\tau = jt$.
For $1 \leq t < 2$, $y_1(t) = \int_{t-1}^t j d\tau = j(t - (t-1)) = j$.
For $2 \leq t < 3$, $y_1(t) = \int_{t-1}^2 j d\tau = j(2 - (t-1)) = j(3-t)$.
For $t \geq 3$, $y_1(t) = 0$.
Thus,

$$y_1(t) = \begin{cases} jt & 0 \leq t < 1 \\ j & 1 \leq t < 2 \\ j(3-t) & 2 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}.$$

(c) To compute $y_2(t)$, first note that $x_2(t) = 2x_1(t-1) + x_1(t-2)$. Using the system properties of linearity and time-invariance, the output $y_2(t)$ is given by

$$y_2(t) = 2y_1(t-1) + y_1(t-2).$$

Anm: $h(t)$ kan också uttryckas som $h(t) = j*(u(t) - u(t-2))$



2.6-1. (a) $\lambda^2 + 8\lambda + 12 = (\lambda + 2)(\lambda + 6)$
Both roots are in LHP. The system is BIBO stable and also asymptotically stable.

(b) $\lambda(\lambda^2 + 3\lambda + 2) = \lambda(\lambda + 1)(\lambda + 2)$
Roots are 0, -1, -2. One root on imaginary axis and none in RHP. The system is BIBO ustable and marginally stable.

(c) $\lambda^2(\lambda^2 + 2) = \lambda^2(\lambda + j\sqrt{2})(\lambda - j\sqrt{2})$
Roots are 0 (repeated twice) and $\pm j\sqrt{2}$. Multiple roots on imaginary axis. The system is BIBO unstable and asymptotically unstable.

(d) $(\lambda + 1)(\lambda^2 - 6\lambda + 5) = (\lambda + 1)(\lambda - 1)(\lambda - 5)$
Roots are -1, 1 and 5. Two roots in RHP. The system is BIBO unstable and asymptotically unstable.

(Anm: Här förutsätts – precis som för övriga system i boken, om inget annat framgår eller sägs – att systemen är kausala!)

2.6-3. (a) Because $u(t) = e^{0t}u(t)$, the characteristic root is 0.

(b) The root lies on the imaginary axis, and the system is marginally stable.

ges även p.g.a. att $\int |h(t)| dt = \infty$ men $|h(t)| < \infty \ \forall t$

(c) $\int_0^\infty h(t)\,dt = \infty$
The system is BIBO unstable.

(d) The integral of $\delta(t)$ is $u(t)$. The system response to $\delta(t)$ is $u(t)$. Clearly, the system is an ideal integrator.

Anm: Här utgår man i lösningen från att det givna impulssvaret är systemets "sanna" impulssvar, dvs. att systemets alla karakteristiska termer (bara en här, dock) finns i $h(t)$. Det är därför vi även kan dra slutsatser om systemets interna stabilitet, inte bara dess externa stabilitet.

2.6-5

(BIBO-stable)

(b) Yes, the system is stable since $\int h(t) = 4 < \infty$.
No, the system is not causal since $h(t) \neq 0$ for ~~all~~ some $t < 0$.

*Anm: I **2.6-5** & **2.5-7** är det systemets externa (insignal-utsignal-)stabilitet som avses!*

2.6-7. Expanding

$$h(t) = \sum_{i=0}^{\infty} (0.5)^i \delta(t - i)$$

yields

$$h(t) = (\delta(t) + 0.5\delta(t-1) + 0.25\delta(t-2) + 0.125\delta(t-3) + \cdots)$$

(a) Yes, the system is causal since $h(t) = 0$ for $t < 0$.

(b) Yes, the system is stable since the impulse response is absolutely integrable. That is, $\int_{-\infty}^{\infty} \sum_{i=0}^{\infty} (0.5)^i \delta(t-i) dt = \sum_{i=0}^{\infty} (0.5)^i \int_{-\infty}^{\infty} \delta(t-i) = \sum_{i=0}^{\infty} (0.5)^i = \frac{1-0}{1-0.5} = 2 < \infty$.

(här menas "BIBO-stable")

6.1-7.

	a	b	c	d	e	f	g	h	i
periodic ?	yes	yes	no	yes	no	yes	yes	yes	yes
ω_0	1	1		π		$\frac{1}{70}$	$\frac{3}{4}$	1	2
period	2π	2π		2		140π	$\frac{8\pi}{3}$	2π	π

6.3-1. (a) $T_0 = 4, \omega_0 = \pi/2$. Also $D_0 = 0$ (by inspection).

$$D_n = \frac{1}{4}\int_{-1}^{1} e^{-j(n\pi/2)t}\,dt - \int_{1}^{3} e^{-j(n\pi/2)t}\,dt = \frac{2}{\pi n}\sin\frac{n\pi}{2} \qquad |n| \geq 1$$

(b) $T_0 = 10\pi$, $\omega_0 = 2\pi/10\pi = 1/5$

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{n}{5}t},$$

where $$D_n = \frac{1}{10\pi}\int_{\pi}^{\pi} e^{-j\frac{n}{5}t}\,dt = \frac{j}{2\pi n}\left(-2j\sin\frac{n\pi}{5}\right) = \frac{1}{\pi n}\sin\left(\frac{n\pi}{5}\right)$$

(c) $T_0 = 2\pi$ sek, $\omega_0 = 1$ rad/s

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jnt}, \quad \text{where, by inspection} \qquad D_0 = 0.5$$

$$D_n = \frac{1}{2\pi}\int_{0}^{2\pi} \frac{t}{2\pi} e^{-jnt}\,dt = \frac{j}{2\pi n}, \qquad \mathbf{n \neq 0\ !}$$

so that $$|D_n| = \frac{1}{2\pi n}, \quad \text{and} \quad \angle D_n = \begin{cases} \frac{\pi}{2} & n > 0 \\ \frac{-\pi}{2} & n < 0 \end{cases}$$

(d) $T_0 = \pi$, $\omega_0 = 2$ and $\boldsymbol{D_0 = 0}$

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2nt},$$

where $$D_n = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{4t}{\pi} e^{-j2nt}\,dt = \frac{-j}{\pi n}\left(\frac{2}{\pi n}\sin\frac{\pi n}{2} - \cos\frac{\pi n}{2}\right)$$

n ≠0 !

(e) $T_0 = 3, \omega_0 = \frac{2\pi}{3}$. $\boldsymbol{D_0 = 1/6}$

$$x(t) = \sum_{n=\infty}^{\infty} D_n\, e^{j\frac{2\pi n}{3}t},$$

where $$D_n = \frac{1}{3}\int_{0}^{1} t\, e^{-j\frac{2\pi n}{3}t}\,dt = \frac{3}{4\pi^2 n^2}\left[e^{-j\frac{2\pi n}{3}}\left(\frac{j2\pi n}{3} + 1\right) - 1\right]$$

n ≠0 !

Therefore

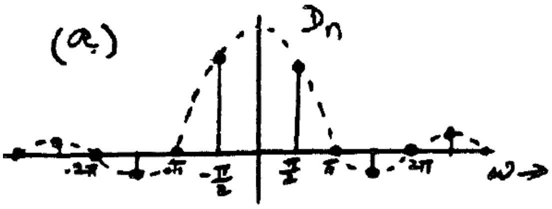
$$|D_n| = \frac{3}{4\pi^2 n^2}\left[\sqrt{2 + \frac{4\pi^2 n^2}{9} - 2\cos\frac{2\pi n}{3} - \frac{4\pi n}{3}\sin\frac{2\pi n}{3}}\right]$$

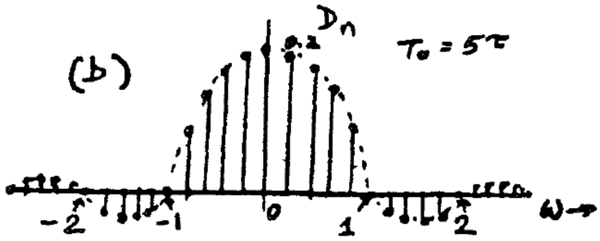
$$\text{and } \angle D_n = \tan^{-1}\left(\frac{\frac{2\pi n}{3}\cos\frac{2\pi n}{3} - \sin\frac{2\pi n}{3}}{\cos\frac{2\pi n}{3} + \frac{2\pi n}{3}\sin\frac{2\pi n}{3} - 1}\right)$$

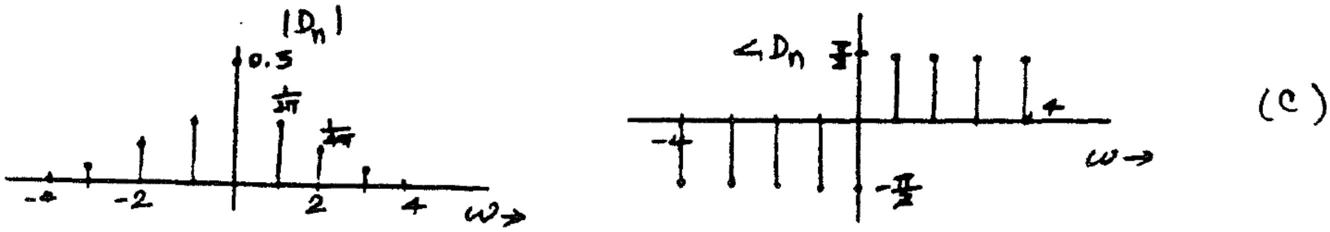
(f) $T_0 = 6$, $\omega_0 = \pi/3$ $D_0 = 0.5$

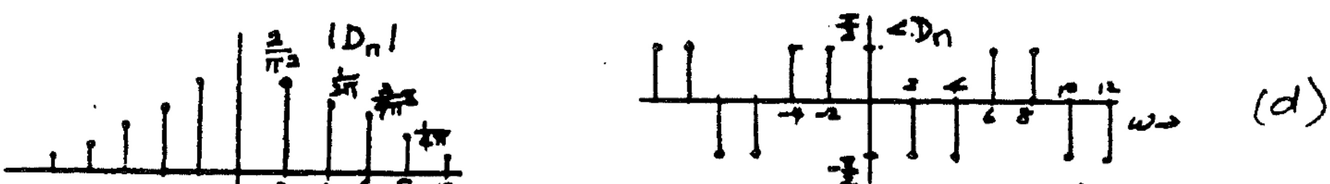
$$x(t) = 0.5 + \sum_{n=-\infty}^{\infty} D_n e^{\frac{j\pi n t}{3}}$$

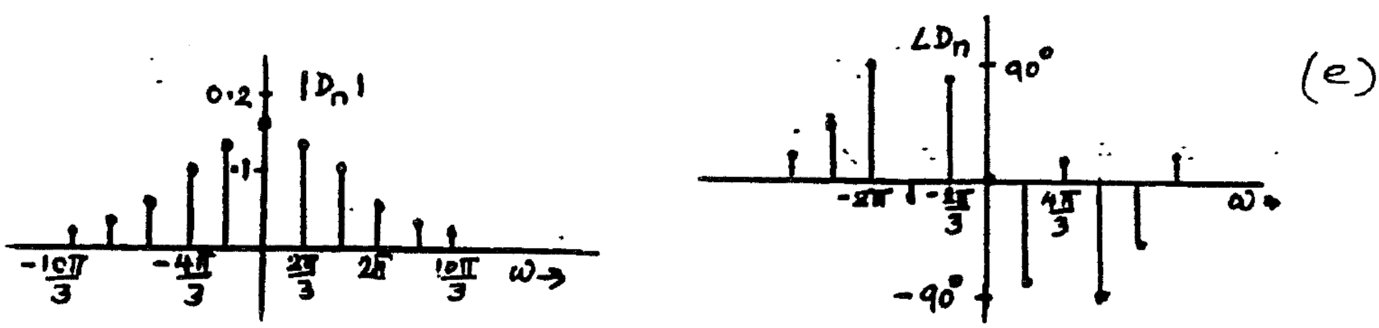
$$\begin{aligned} D_n &= \frac{1}{6}\left[\int_{-2}^{-1}(t+2)e^{-\frac{j\pi n t}{3}}\,dt + \int_{-1}^{1} e^{-\frac{j\pi n t}{3}}\,dt + \int_{1}^{2}(-t+2)e^{-\frac{j\pi n t}{3}}\,dt\right] \\ &= \frac{3}{\pi^2 n^2}\left(\cos\frac{n\pi}{3} - \cos\frac{2\pi n}{3}\right) \qquad n \neq 0\,! \end{aligned}$$

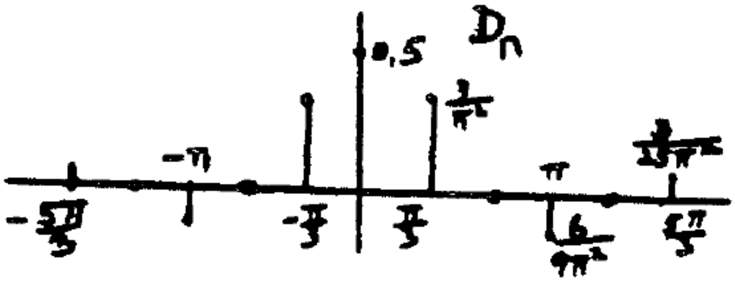












(f)

Tips: Alternativt kan D_n i de olika deluppgifterna i 6.3-1 beräknas utgående från de komplexa fourierseriekoefficienterna till den deriverade signalen, m.h.a. sambandet $D_n = D_{nx'}/jn\omega_0$.

6.3-5. (a) The exponential Fourier series can be expressed with coefficients in Polar form as

$$x(t) = (2\sqrt{2}e^{j\pi/4})e^{-j3t} + 2e^{j\pi/2}e^{-jt} + 3 + 2e^{-j\pi/2}e^{jt} + (2\sqrt{2}e^{-j\pi/4})e^{j3t}$$

From this expression the exponential Spectra are sketched as shown in Figure S6.3-5a.

(b) By inspection of the exponential spectra in Figure S6.3-5a, we sketch the trigonometric spectra as shown in Figure S6.3-5b. From these spectra, we can write the compact trigonometric Fourier series as

$$x(t) = 3 + 4\cos\left(t - \frac{\pi}{2}\right) + 4\sqrt{2}\cos\left(3t - \frac{\pi}{4}\right)$$

(c) Since, the trigonometric series in part (b) is obtained from the exponential series in part (a), the two series are equivalent.

(d) The lowest frequency in the spectrum is 0 and the highest frequency is 3. Therefore the bandwidth is 3 rad/s or $\frac{3}{2\pi}$ Hz.

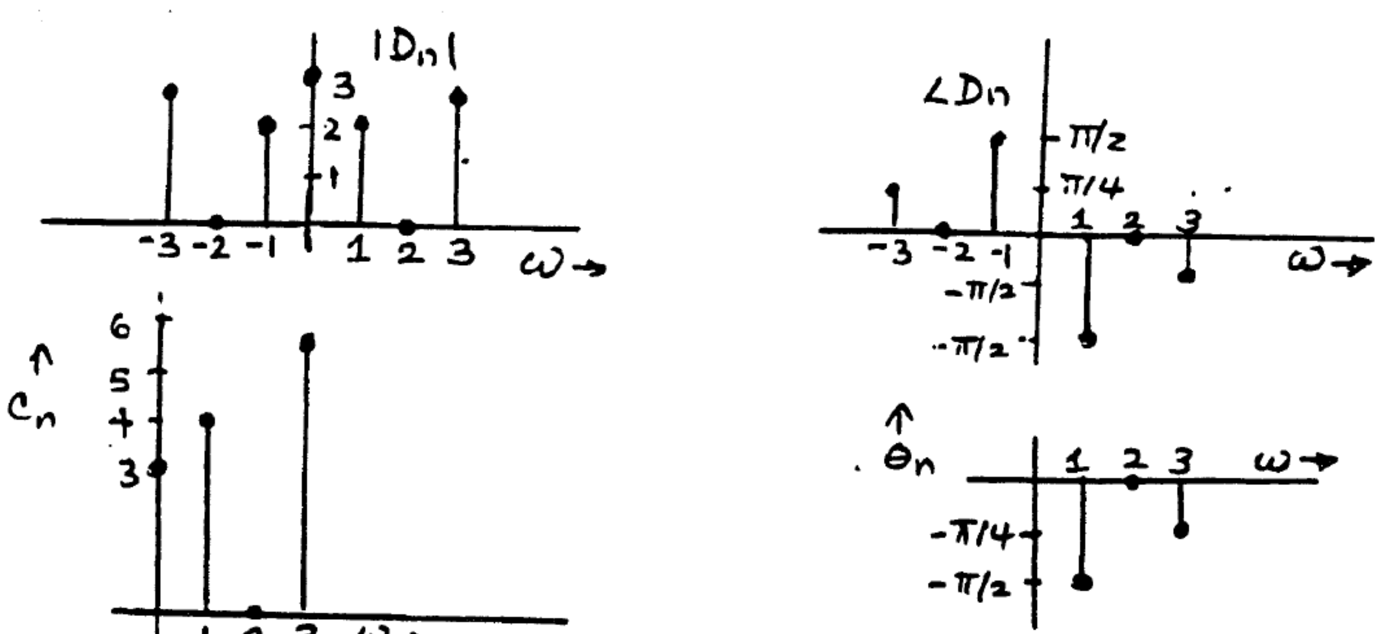


Figure S6.3-5

6.3-7. (a) The exponential Fourier series, as found by inspection of Figure **P6.3-7** is

$$x(t) = -2 + 2e^{j(t+\frac{2\pi}{3})} + 2e^{-j(t+\frac{2\pi}{3})} + e^{j(2t+\frac{\pi}{3})} + e^{-j(2t+\frac{\pi}{3})}$$

(b) To find the corresponding trigonometric series, we consider only the positive frequency components, then double the exponential amplitudes (except for dc, which is kept the same), and maintain the same phase values to obtain the trigonometric spectrum, Figure S6.3-7.

(c) By inspection of the trigonometric spectra

$$x(t) = -2 + 4\cos\left(t + \frac{2\pi}{3}\right) + 2\cos\left(2t + \frac{\pi}{3}\right)$$

(d)

$$\begin{aligned} x(t) &= -2 + 4\cos\left(t + \frac{2\pi}{3}\right) + 2\cos\left(2t + \frac{\pi}{3}\right) \\ &= -2 + 2e^{j(t+\frac{2\pi}{3})} + 2e^{-j(t+\frac{2\pi}{3})} + e^{j(2t+\frac{\pi}{3})} + e^{-j(2t+\frac{\pi}{3})} \end{aligned}$$

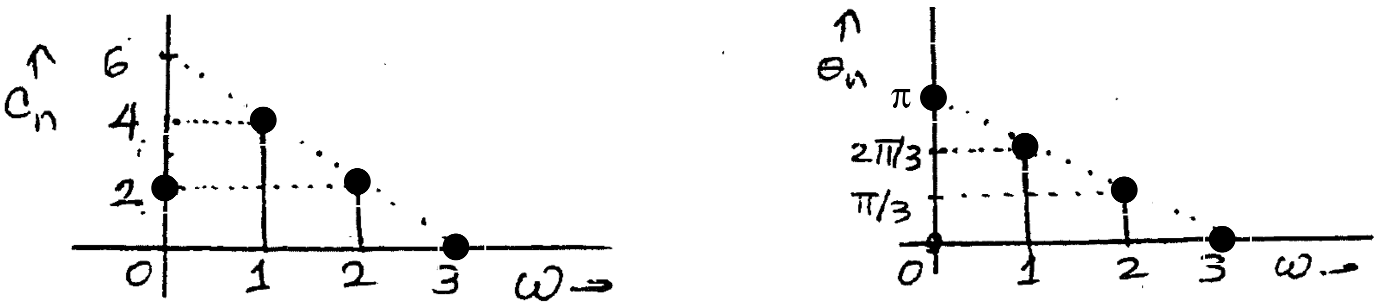


Figure S6.3-7

6.3-8. (a) The period is $T_0 = 8$ and $\omega_0 = \pi/4$. Also $D_0 = 0$ (by inspection), and

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\frac{\pi}{4}t}$$

$$D_n = \frac{1}{8}\left[\int_{-4}^{0}\left(\frac{t}{2}+1\right) e^{-jn(\pi/4)t}\, dt + \int_{0}^{4}\left(-\frac{t}{2}+1\right) e^{-jn(\pi/4)t}\, dt\right] =$$

This yields

$$D_n = \begin{cases} \frac{4}{\pi^2 n^2} & n = \pm 1, \pm 3, \pm 5, \pm 7, \cdots \\ 0 & \text{otherwise} \end{cases}$$

Therefore

$$x(t) = \sum_{\substack{n=1 \\ n\ odd}}^{\infty} \frac{4}{\pi^2 n^2} e^{jn\frac{\pi}{4}t}$$

Fel i lösningen:
Summera från n = –∞!

(b) Observe that $\hat{x}(t)$ is the same as $x(t)$ in Figure P6.3-8a delayed by 2 seconds. Therefore

$$\hat{x}(t) = x(t-2) = \sum_{\substack{n=1 \\ n\ odd}}^{\infty} D_n e^{jn\frac{\pi}{4}(t-2)} = \sum_{\substack{n=1 \\ n\ odd}}^{\infty} D_n e^{-jn\pi/2} e^{jn\frac{\pi}{4}t}$$

Therefore

Fel i lösningen:
Summera från n = –∞!

$$\hat{x}(t) = \sum_{\substack{n=1 \\ n\ odd}}^{\infty} \hat{D}_n e^{jn\frac{\pi}{4}t}$$

where

$$\hat{D}_n = D_n e^{j\frac{n\pi}{2}} = \frac{4}{\pi^2 n^2} e^{-j\frac{n\pi}{2}}$$

(c) Observe that $\tilde{x}(t)$ is the same as $x(t)$ in Figure P6.2-8a time-compressed by a factor 2. Therefore

$$\tilde{x}(t) = x(2t) = \sum_{\substack{n=1 \\ n\ odd}}^{\infty} D_n e^{jn\frac{\pi}{4}(2t)} = \sum_{\substack{n=1 \\ n\ odd}}^{\infty} D_n e^{jn\frac{\pi}{2}t}$$

Therefore

Fel i lösningen:
Summera från n = –∞!

$$\tilde{x}(t) = \sum_{\substack{n=1 \\ n\ odd}}^{\infty} \tilde{D}_n e^{jn\frac{\pi}{2}t}$$

where

$$\tilde{D}_n = D_n = \frac{4}{\pi^2 n^2}$$

6.3-9. (a)

$$\begin{aligned} x(t) &= \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \\ \hat{x}(t) &= x(t-T) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0(t-T)} = \sum_{n=-\infty}^{\infty} (D_n e^{-jn\omega_0 T}) e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} \hat{D}_n e^{jn\omega_0 t} \end{aligned}$$

$$\hat{D}_n = D_n e^{-jn\omega_0 t} \quad \text{so that} \quad |\hat{D}_n| = |D_n|, \quad \text{and} \quad \angle\hat{D}_n = \angle D_n - jn\omega_0 T$$

(b)

$$\begin{aligned} x(t) &= \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \\ \hat{x}(t) &= x(at) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0(at)} \end{aligned}$$

6.3-10. (a) From Exercise E6.1a

$$x(t) = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos n\pi t \qquad -1 \le t \le 1$$

The power of $x(t)$ is

$$P_x = \frac{1}{2}\int_{-1}^{1} t^4\,dt = \frac{1}{5}$$

Moreover, from Parseval's theorem Eq. (6.40)

$$P_x = C_0^2 + \sum_{1}^{\infty}\frac{C_n^2}{2} = \left(\frac{1}{3}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{\pi^2 n^2}\right)^2 = \frac{1}{9} + \frac{8}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{1}{9} + \frac{8}{90} = \frac{1}{5}$$

(b) If the N-term Fourier series is denoted by $w(t)$, then

$$w(t) = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{N-1}\frac{(-1)^n}{n^2}\cos n\pi t \qquad -1 \le t \le 1$$

The power P_x is required to be $99\% P_x = 0.198$. Therefore

$$P_x = \frac{1}{9} + \frac{8}{\pi^4}\sum_{n=1}^{N-1}\frac{1}{n^4} = 0.198$$

For $N = 1$, $P_x = 0.1111$; for $N = 2$, $P_x = 0.19323$, For $N = 3$, $P_x = 0.19837$, which is greater than 0.198. Thus, $N = 3$.

6.3-11. (a) From Exercise E6.1b

$$x(t) = \frac{2A}{\pi}(-1)^{n+1}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\pi t \qquad -\pi \le t \le \pi$$

The power of $x(t)$ is

$$P_x = \frac{1}{2}\int_{-1}^{1}(At)^2\,dt = \frac{A^2}{3}$$

Moreover, from Parseval's theorem [Eq. (6.40)]

$$P_x = C_0^2 + \sum_{1}^{\infty}\frac{C_n^2}{2} = \frac{1}{2}\sum_{1}^{\infty}\frac{4A^2}{\pi^2 n^2} = \frac{2A^2}{\pi^2}\sum_{1}^{\infty}\frac{1}{n^2} = \frac{A^2}{3}$$

(b) If the N-term Fourier series is denoted by $w(t)$, then

$$w(t) = \frac{2A}{\pi}(-1)^{n+1}\sum_{n=1}^{N}\frac{1}{n}\sin n\pi t \qquad -\pi \le t \le \pi$$

The power P_w is required to be no less than $0.90\frac{A^2}{3} = 0.3A^2$. Therefore

$$P_w = \frac{1}{2}\sum_{1}^{N} \frac{4A^2}{\pi^2 n^2} \geq 0.3A^2$$

For $N = 1$, $P_w = 0.2026A^2$; for $N = 2$, $P_w = 0.2533A^2$, for $N = 5$, $P_w = 0.29658A^2$, for $N = 6$, $P_w = 0.30222A^2$, which is greater than $0.3A^2$. Thus, $N = 6$.

6.3-12. The power of a rectified sine wave is the same as that of a sine wave, that is, 1/2. Thus $P_x = 0.5$. Let the $2N+1$ term truncated Fourier series be denoted by $\hat{x}(t)$. The power $P_{\hat{x}}$ is required to be no less than $0.9975P_x = 0.49875$. Using the Fourier series coefficients in Exercise E6.5, we have

$$P_{\hat{x}} = \sum_{n=-N}^{N} |D_n|^2 = \frac{4}{\pi^2}\sum_{n=-N}^{N} \frac{1}{(1-4n^2)^2} \geq 0.49875$$

Direct calculations using the above equation gives $P_{\hat{x}} = 4/\pi^2 = 0.4053$ for $N = 0$ (only dc), $P_{\hat{x}} = 0.49535$ for $N = 1$ (3 terms), and $P_{\hat{x}} = 0.49895$ for $N = 2$ (5 terms). Thus, a 5-term Fourier series yields a signal whose power is 99.79% of the power of the rectified sine wave. The power of the error in the approximation of $x(t)$ by $\hat{x}(t)$ is only 0.21% of the signal power P_x.

6.4-1. Period $T_0 = \pi$, and $\omega_0 = 2$, and

$$H(\omega) = \frac{j\omega}{(-\omega^2+3)+j2\omega}, \qquad \text{and from Eq. (6.30b)} \qquad D_n = \frac{0.504}{1+j4n}$$

Therefore, $$y(t) = \sum_{n=-\infty}^{\infty} D_n H(n\omega_0)e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} \frac{j1.008n}{(1+j4n)(3-4n^2+j4n)}e^{j2nt}$$

Anm: Figur 6.2a, som uppgiftstexten hänvisar till, hör till Exempel 6.1, där författaren bl.a. beräknar a_n och b_n. Då erhålls, från ekv. 6.30b på sid. 625, $D_n = 0.5(a_n - jb_n)$ = uttrycket ovan.

Eftersom vi i kursen *inte* använder oss av den allmänna trigonometriska formen, så bör du i stället beräkna D_n direkt, enligt ekv. 6.29b.

6.4-2. (a)

$$\begin{aligned}\cos 5t \sin 3t &= \frac{1}{2}[\sin 8t - \sin 2t] \\ &= \frac{1}{4j}\left[e^{j8t} - e^{-j8t} - e^{j2t} + e^{-j2t}\right]\end{aligned}$$

(<u>Alternativt:</u> utveckla
$\cos(5t)\sin(3t) = (e^{j5t}+e^{-j5t})/2*(e^{j3t}-e^{-j3t})/2j$)

$$= \frac{1}{4}\left[e^{j(8t-\frac{\pi}{2})} + e^{-j(8t-\frac{\pi}{2})} + e^{j(2t+\frac{\pi}{2})} + e^{-j(2t+\frac{\pi}{2})}\right]$$

This is the desired exponential Fourier series.

(b) There are four spectral components at $\omega = \pm 8$ and ± 2. The phases are either $\frac{\pi}{2}$ or $-\frac{\pi}{2}$, as shown in the spectrum in Figure S6.4-2b.

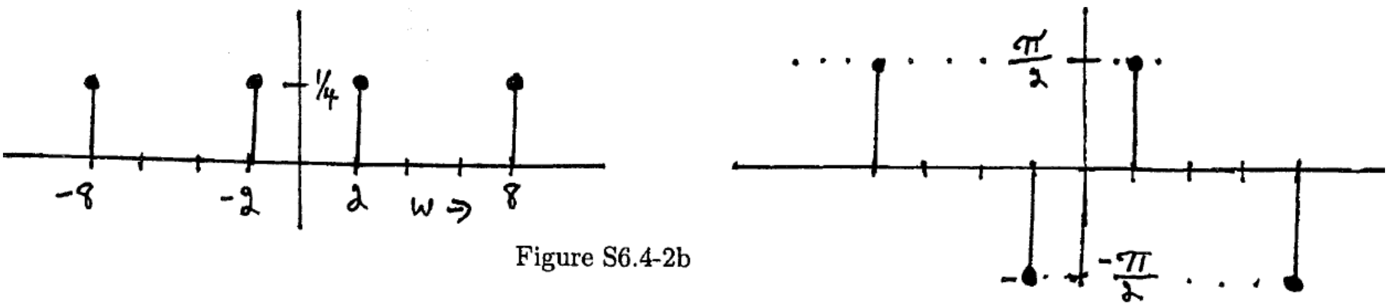


Figure S6.4-2b

(c) Since none of the spectral components of $x(t)$ appear in the pass-band of the filter, the output is $y(t) = 0$.

6.4-3.

$$D_n = \int_0^1 e^{-t} e^{-jn\omega_0 t}\,dt = \frac{(e-1)(1-j2\pi n)}{e(1+4\pi^2 n^2)}$$

The frequency function of the R-C circuit is

$$H(\omega) = \frac{1}{1+\left(\frac{1}{j\omega}\right)} = \frac{j\omega}{j\omega+1}$$

The input $x(t)$ can be expressed as a Fourier series

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{(e-1)(1-j2\pi n)}{e(1+4\pi^2 n^2)} e^{j2\pi nt}$$

Hence the output $y(t)$ is given by

$$\begin{aligned} y(t) &= \sum_{n=-\infty}^{\infty} D_n H(2\pi n) e^{j2\pi nt} \\ &= \sum_{n=-\infty}^{\infty} \frac{(e-1)(1-j2\pi n)(j2\pi n)}{e(1+4\pi^2 n^2)(j2\pi n+1)} e^{j2\pi nt} \end{aligned}$$

Anm: Uttrycket för utsignalens komplexa fourierseriekoefficienter bör även förenklas…