Lösningar till de inledande uppgifterna 9.2-X och 9.3-X finns i ett separat dokument (se även le-webbsidan för TSDT84).

9.4-1.

$$\begin{aligned}
X(\Omega) &= \frac{1}{1+0.5e^{-j\Omega}} = \frac{e^{j\Omega}}{e^{j\Omega}+0.5} \\
Y(\Omega) &= X(\Omega)H(\Omega) = \frac{e^{j\Omega}\left(e^{j\Omega}+0.32\right)}{\left(e^{j\Omega}+0.5\right)\left(e^{j\Omega}+0.8\right)\left(e^{j\Omega}+0.2\right)} \\
\frac{Y(\Omega)}{e^{j\Omega}} &= \frac{e^{j\Omega}+0.32}{\left(e^{j\Omega}+0.5\right)\left(e^{j\Omega}+0.8\right)\left(e^{j\Omega}+0.2\right)} \\
&= \frac{2}{e^{j\Omega}+0.5} - \frac{8/3}{e^{j\Omega}+0.8} + \frac{2/3}{e^{j\Omega}+0.2} \\
Y(\Omega) &= 2\frac{e^{j\Omega}}{e^{j\Omega}+0.5} - \frac{8}{3}\frac{e^{j\Omega}}{e^{j\Omega}+0.8} + \frac{2}{3}\frac{e^{j\Omega}}{e^{j\Omega}+0.2} \\
y[n] &= \left[2\left(-0.5\right)^n - \frac{8}{3}\left(-0.8\right)^n + \frac{2}{3}\left(-0.2\right)^n\right]u[n]
\end{aligned}$$

9.4-2.

$$Y[\Omega] = H[\Omega]X[\Omega] = \left[\frac{e^{j\Omega}+0.32}{(e^{j\Omega}+0.2)(e^{j\Omega}+0.8)}\right]\left[\pi\delta(\Omega) + \frac{e^{j\Omega}}{e^{j\Omega}-1}\right]$$

We use the fact that $f(x)\delta(x) = f(0)\delta(x)$ to obtain

$$Y(\Omega) = \frac{1.32\pi}{2.16}\delta(\Omega) + \frac{e^{j\Omega}(e^{j\Omega}+0.32)}{(e^{j\Omega}-1)(e^{j\Omega}+0.2)(e^{j\Omega}+0.8)}$$

Using partial fraction expansion, we obtain

$$Y(\Omega) = \frac{1.32\pi}{2.16}\delta(\Omega) + \frac{1.32}{2.16}\frac{e^{j\Omega}}{e^{j\Omega}-1} - \frac{1}{6}\frac{e^{j\Omega}}{e^{j\Omega}+0.2} - \frac{4}{9}\frac{e^{j\Omega}}{e^{j\Omega}+0.8}$$

and

$$y[n] = 0.611u[n] - \left[\frac{1}{6}(-0.2)^n + \frac{4}{9}(-0.8)^n\right]u[n]$$

9.4-3.

$$\begin{aligned}
X(\Omega) &= \frac{e^{j\Omega}}{e^{j\Omega}-0.8} - \frac{2e^{j\Omega}}{e^{j\Omega}-2} \\
Y(\Omega) &= X(\Omega)H(\Omega) = \frac{e^{j2\Omega}}{\left(e^{j\Omega}-0.5\right)\left(e^{j\Omega}-0.8\right)} - \frac{2e^{j2\Omega}}{\left(e^{j\Omega}-0.5\right)\left(e^{j\Omega}-2\right)} \\
\Rightarrow \frac{Y[\Omega]}{e^{j\Omega}} &= \frac{-5/3}{e^{j\Omega}-0.5} + \frac{8/3}{e^{j\Omega}-0.8} + \frac{2/3}{e^{j\Omega}-0.5} - \frac{8/3}{e^{j\Omega}-2} \\
&= \frac{-1}{e^{j\Omega}-0.5} + \frac{8/3}{e^{j\Omega}-0.8} - \frac{8/3}{e^{j\Omega}-2} \\
\Rightarrow Y(\Omega) &= -\frac{e^{j\Omega}}{e^{j\Omega}-0.5} + \frac{8}{3}\frac{e^{j\Omega}}{e^{j\Omega}-0.8} - \frac{8}{3}\frac{e^{j\Omega}}{e^{j\Omega}-2} \\
\Rightarrow y[n] &= \left[-\left(0.5\right)^n + \frac{8}{3}\left(0.8\right)^n\right]u[n] + \frac{8}{3}\left(2\right)^n u\left[-\left(n+1\right)\right]
\end{aligned}$$

9.4-4. (a) When $x[n] = \delta[n]$, the output is $h[n]$, given by (assuming causal accumulator)

$$h[n] = \sum_{k=0}^{n} \delta[k] = u[n]$$

and

$$H(\Omega) = \frac{e^{j\Omega}}{e^{j\Omega} - 1} + \pi\delta(\Omega)$$

(b) If this accumulator is used as a digital processor for the digital integrator, discussed in example 3.7, and the input is $x(t) = u(t)$, then the sampled $x(t)$ yields the digital input $x[n] = x(nT) = u[n]$

9.4-5. (a)

$$\text{sinc}\left(\frac{\pi n}{2}\right) \Longleftrightarrow 2\text{rect}\left(\frac{\Omega}{\pi}\right)$$

Hence the output corresponding to this input is

$$Y(\Omega) = 2\text{rect}^2\left(\frac{\Omega}{\pi}\right)e^{-j2\Omega} = 2\text{rect}\left(\frac{\Omega}{\pi}\right)e^{-j2\Omega}$$

Therefore

$$y[n] = \text{sinc}\left[\frac{\pi(n-2)}{2}\right]$$

(b)

$$\text{sinc}(\pi n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

Hence

$$\text{sinc}(\pi n) = \delta[n] \Longleftrightarrow 1$$

The output corresponding to this input is

$$Y(\Omega) = \text{rect}\left(\frac{\Omega}{\pi}\right)e^{-j2\Omega}$$

and

$$y[n] = \frac{1}{2}\text{sinc}\left[\frac{\pi(n-2)}{2}\right]$$

(c)

$$\text{sinc}^2\left(\frac{\pi n}{4}\right) \Longleftrightarrow 4\Delta\left(\frac{\Omega}{\pi}\right)$$

The output corresponding to this input is

$$\begin{aligned} Y(\Omega) &= 4\Delta\left(\frac{\Omega}{\pi}\right)\text{rect}\left(\frac{\Omega}{\pi}\right)e^{-j2\Omega} \\ &= 4\Delta\left(\frac{\Omega}{\pi}\right)e^{-j2\Omega} \end{aligned}$$

and

$$y(n) = \text{sinc}^2\left[\frac{\pi(n-2)}{4}\right]$$

9.4-6. (a) Let

$$y[n] = (-1)^n x[n] = e^{-j\pi n} x[n]$$

Use of frequency-shifting property [Eq. (9.45)] yields

$$Y(\Omega) = X(\Omega - \pi)$$

(b) Figure S9.4-6a shows $\gamma^n u[n]$ and $(-1)^n \gamma^n u[n]$. The spectra for $(-1)^n \gamma^n u[n]$ are the same as those for $\gamma^n u[n]$ (Figure S9.4-6 b and c) but shifted by π, as shown in Figure S9.4-6 b over the fundamental band $|\Omega| \leq \pi$.

(c)

$$h_{LP}[n] = \frac{\Omega_c}{\pi}\text{sinc}(\Omega_c n)$$

The frequency response of $(-1)^n h_{LP}[n] = \frac{\Omega_c}{\pi}(-1)^n \text{sinc}(\Omega_c n)$ is $\text{rect}\left(\frac{\Omega}{2\Omega_c}\right)$ frequency-shifted by π is $\text{rect}\left(\frac{\Omega-\pi}{2\Omega_c}\right)$, as shown in Figure S9.4-6c. It is clear that this is a highpass filter.

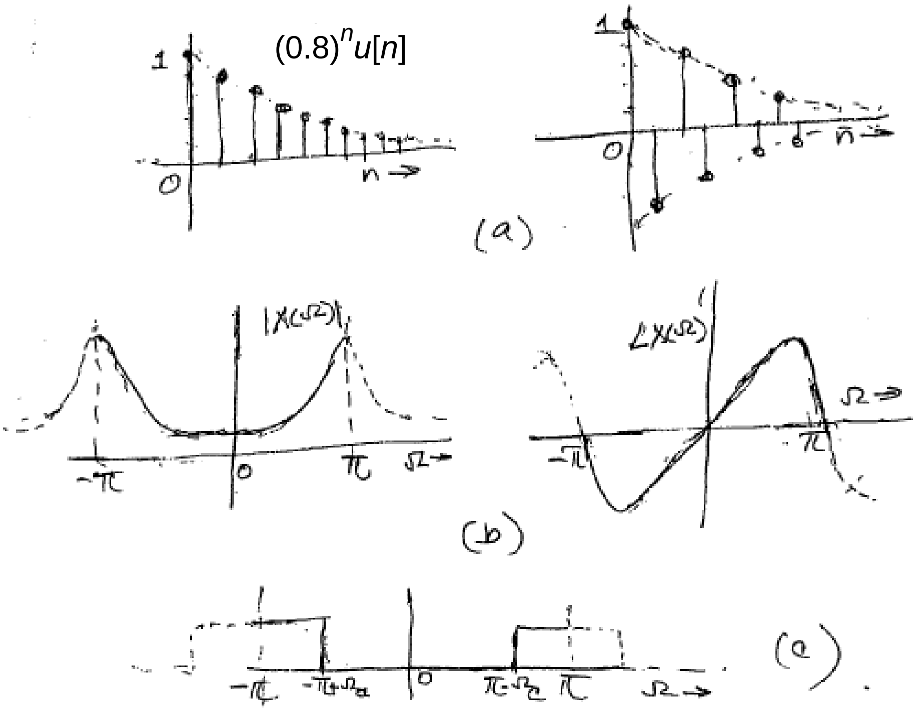

Figure S9.4-6

9.4-7.

$$w[n] = (-1)^n x[n]$$

Inför hjälpstorheterna *w*[*n*] och *q*[*n*], enligt figur S9.4-7 på nästa sida!

Hence $W(\Omega) = X(\Omega - \pi)$, as shown in Problem 9.4-6a.

$$Q(\Omega) = X(\Omega - \pi)H(\Omega)$$

and because

$$y[n] = (-1)^n q[n]$$

$$
\begin{aligned}
Y(\Omega) = Q(\Omega - \pi) &= X(\Omega - 2\pi)H(\Omega - \pi) \\
&= X(\Omega)H(\Omega - \pi) \\
&= X(\Omega)H_1(\Omega)
\end{aligned}
$$

Therefore

$$H_1\Omega = H(\Omega - \pi)$$

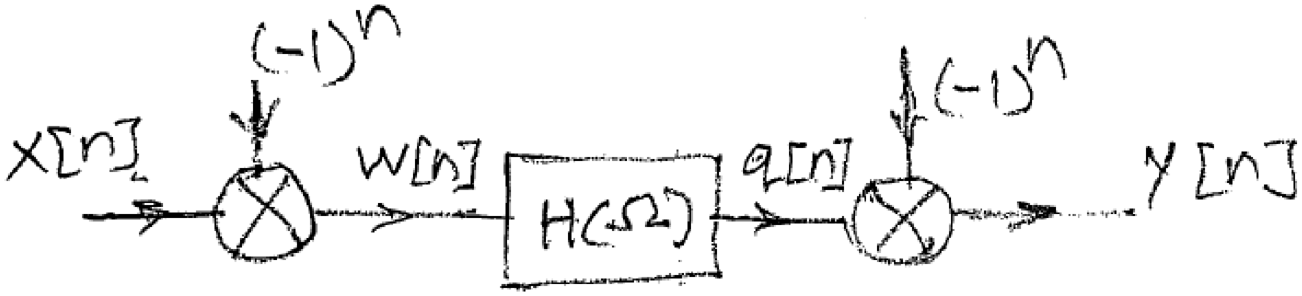

Figure S9.4-7