Lösningar till uppgift 6.4-1, 6.4-2 och 6.4-3:

Se de två sista sidorna i dokumentet med lösningar relaterade till lektion 1 & 2!

7.2-5. (a) When $a > 0$, we cannot find the Fourier transform of $e^{at}u(t)$ by setting $s = j\omega$ in the Laplace transform of $e^{at}u(t)$ because the ROC is $Re s > a$, which does not include the $j\omega$-axis.

(b) The Laplace transform of $x(t)$ is

$$X(s) = \int_0^T e^{at}e^{-st}dt = \int_0^T e^{-(s-a)t}dt = \frac{1}{s-a}\left[1 - e^{-(s-a)T}\right]$$

Interestingly, because $x(t)$ has a finite width, the ROC of its $X(s)$ is the entire s-plane, which includes $j\omega$-axis. Hence, the Fourier transform

$$X(\omega) = X(s)\,|_{s=j\omega} = \frac{1}{s-a}\left[1 - e^{-(j\omega-a)T}\right]$$

To verify this, we find the Fourier transform of $x(t)$

$$X(\omega) = \int_0^T e^{at}e^{-j\omega t}dt = \int_0^T e^{-(j\omega-a)t}dt = \frac{1}{j\omega-a}\left[1 - e^{-(j\omega-a)T}\right]$$

Which agrees with $X(j\omega)$

7.4-1.

$$H(\omega) = \frac{1}{j\omega+1}$$

(a)

$$\begin{aligned} X(\omega) &= \frac{1}{j\omega+2} \\ Y(\omega) &= \frac{1}{(j\omega+1)(j\omega+2)} = \frac{1}{j\omega+1} - \frac{1}{j\omega+2} \\ y(t) &= (e^{-t} - e^{-2t})u(t) \end{aligned}$$

(b)

$$\begin{aligned} X(\omega) &= \frac{1}{j\omega+1} \\ Y(\omega) &= \frac{1}{(j\omega+1)^2} \\ y(t) &= te^{-at}u(t) \end{aligned}$$

(c)

$$
\begin{aligned}
X(\omega) &= -\frac{1}{j\omega - 1} \\
Y(\omega) &= \frac{-1}{(j\omega+1)(j\omega-1)} = \frac{1/2}{j\omega+1} - \frac{1/2}{j\omega-1} \\
y(t) &= \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{t}u(-t)
\end{aligned}
$$

(d)

$$
\begin{aligned}
X(\omega) &= \pi\delta(\omega) + \frac{1}{j\omega} \\
Y(\omega) &= \frac{1}{j\omega+1}\left[\pi\delta(\omega) + \frac{1}{j\omega}\right] \\
&= \pi\delta(\omega) + \frac{1}{j\omega(j\omega+1)} \qquad [\text{because} g(x)\delta(x) = g(0)\delta(x)] \\
&= \pi\delta(\omega) + \frac{1}{j\omega} - \frac{1}{j\omega+1} \\
y(t) &= (1 - e^{-t})u(t)
\end{aligned}
$$

Kommentar till 7.4-1: Den här uppgiften löses lika gärna (eller t.o.m. *hellre*) m.h.a. *laplacetransformen*. Ovan används dock fouriertransformen för att visa att även den kan användas. Notera att fouriertransformen bara kan användas när *båda* fouriertransformerna $X(\omega)$ och $H(\omega)$ existerar (vilket de gör här)!

7.4-2. (a)

$$X(\omega) = \frac{1}{j\omega + 1} \quad \text{and} \quad H(\omega) = \frac{-1}{j\omega - 2}$$

and

$$Y(\omega) = \frac{-1}{(j\omega - 2)(j\omega + 1)} = \frac{1}{3}\left[\frac{1}{j\omega + 1} - \frac{1}{j\omega - 2}\right]$$

Therefore

$$y(t) = \frac{1}{3}\left[e^{-t}u(t) + e^{2t}u(-t)\right]$$

(b)

$$X(\omega) = \frac{-1}{j\omega - 1} \quad \text{and} \quad H(\omega) = \frac{-1}{j\omega - 2}$$

and

$$Y(\omega) = \frac{1}{(j\omega - 1)(j\omega - 2)} = \frac{-1}{j\omega - 1} - \frac{-1}{j\omega - 2}$$

Therefore

$$y(t) = [e^{t} - e^{2t}]u(-t)]$$

$H(\omega) = \frac{1}{2 - j\omega} \Rightarrow$ (Tab. 7.1:2) $\Rightarrow h(t) = e^{2t} \cdot u(-t)$

$h(t<0) \neq 0 \Rightarrow$ Systemet är icke-kausalt.

7.4-3.

$$X_1(\omega) = \text{sinc}\left(\tfrac{\omega}{20000}\right) \quad \text{and} \quad X_2(\omega) = 1$$

Figure S7.4-3 shows $X_1(\omega)$, $X_2(\omega)$, $H_1(\omega)$ and $H_2(\omega)$. Now

$$\begin{aligned} Y_1(\omega) &= X_1(\omega)H_1(\omega) \\ Y_2(\omega) &= X_2(\omega)H_2(\omega) \end{aligned}$$

The spectra $Y_1(\omega)$ and $Y_2(\omega)$ are also shown in Figure S7.4-3. Because $y(t) = y_1(t)y_2(t)$, the frequency convolution property yields $Y(\omega) = Y_1(\omega) * Y_2(\omega)$. From the width property of convolution, it follows that the bandwidth of $Y(\omega)$ is the sum of bandwidths of $Y_1(\omega)$ and $Y_2(\omega)$. Because the bandwidths of $Y_1(\omega)$ and $Y_2(\omega)$ are 10 kHz, 5 kHz, respectively, the bandwidth of $Y(\omega)$ is 15 kHz.

7.4-4.

$$H(\omega) = 10^{-3}\,\text{sinc}\left(\tfrac{\omega}{2000}\right) \quad \text{and} \quad P(\omega) = 0.5 \times 10^{-6}\,\text{sinc}^2\left(\tfrac{\omega}{4\times 10^6}\right)$$

The two spectra are sketched in Figure S7.4-4. It is clear that $H(\omega)$ is much narrower than $P(\omega)$, and we may consider $P(\omega)$ to be nearly constant of value $P(0) = 10^{-6}/2$ over the entire band of $H(\omega)$. Hence,

$$Y(\omega) = P(\omega)H(\omega) \approx P(0)H(\omega) = 0.5 \times 10^{-6}H(\omega) \quad \Longrightarrow \quad y(t) = 0.5 \times 10^{-6}h(t)$$

Recall that $h(t)$ is the unit impulse response of the system. Hence, the output $y(t)$ is equal to the system response to an input $0.5 \times 10^{-6}\delta(\omega) = A\delta(\omega)$.

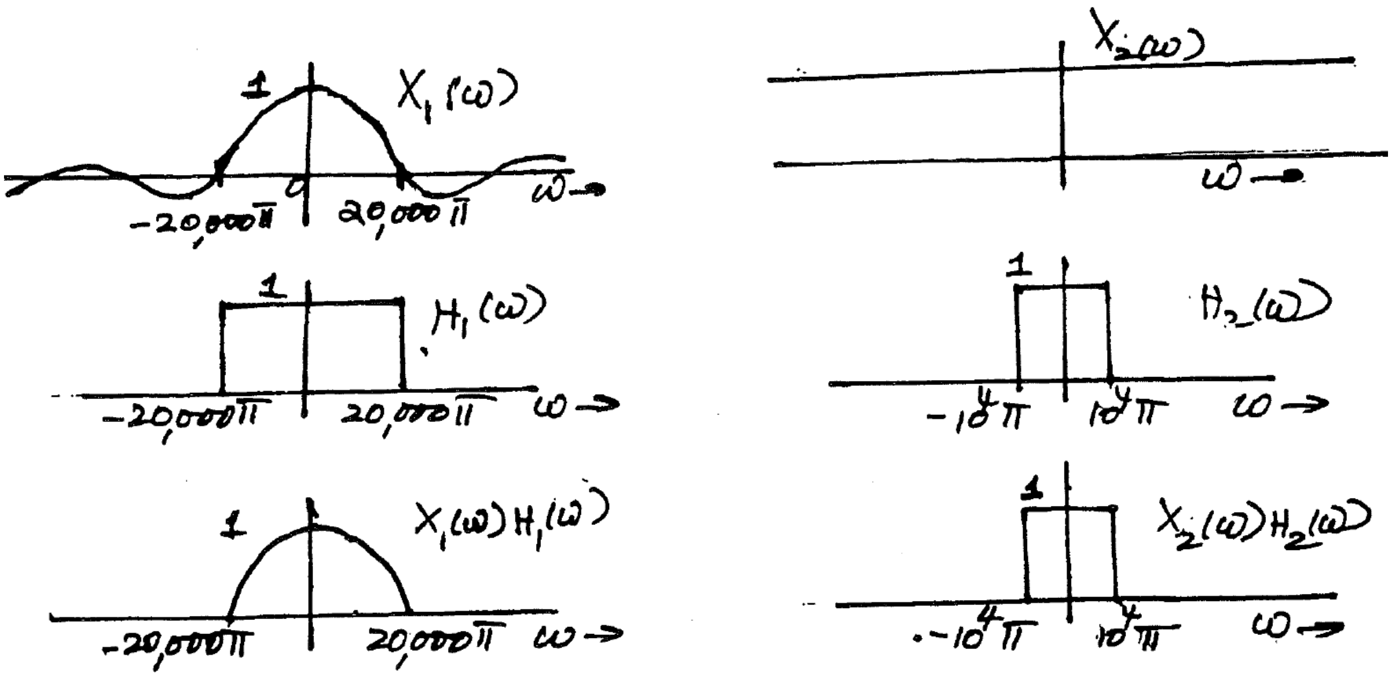


Figure S7.4-3

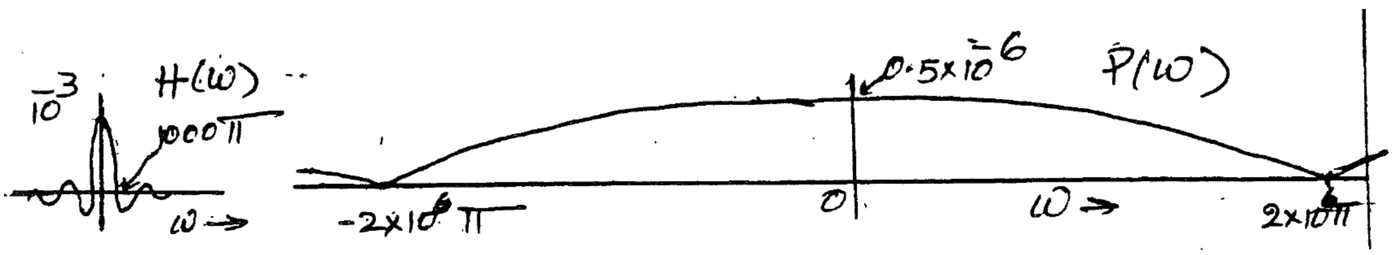


Figure S7.4-4

7.4-5. $$H(\omega) = 10^{-3}\,\text{sinc}\left(\tfrac{\omega}{2000}\right) \qquad \text{and} \qquad P(\omega) = 0.5\,\text{sinc}^2\left(\tfrac{\omega}{4}\right)$$

The two spectra are sketched in Figure S7.4-5. It is clear that $P(\omega)$ is much narrower than $H(\omega)$, and we may consider $H(\omega)$ to be nearly constant of value $H(0) = 10^{-3}$

over the entire band of $P(\omega)$. Hence,

$$Y(\omega) = P(\omega)H(\omega) \approx P(\omega)H(0) = 10^{-3}P(\omega) \quad \Longrightarrow \quad y(t) = 10^{-3}p(t)$$

Note that the dc gain of the system is $k = H(0) = 10^{-3}$. Hence, the output is nearly $kP(t)$.

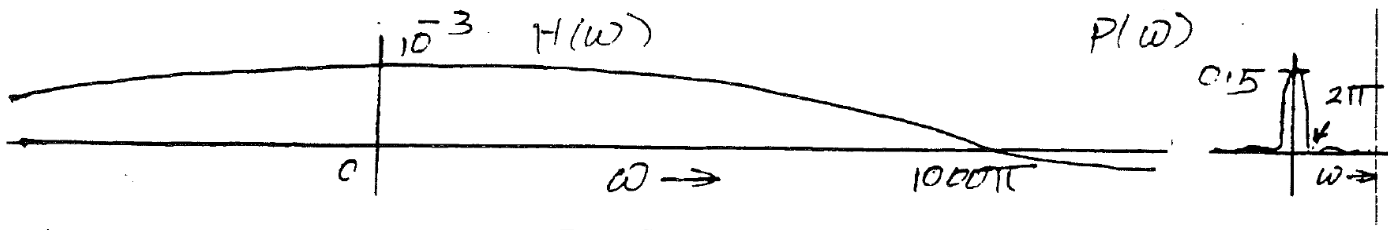


Figure S7.4-5

7.3-6. (a) The signal $x(t)$ in this case is a triangle pulse $\Delta(\frac{t}{2\pi})$ (Figure S7.3-6) multiplied by $\cos 10t$.

$$x(t) = \Delta\left(\frac{t}{2\pi}\right) \cos 10t$$

Also from Table 4.1 (pair 19) $\Delta(\frac{t}{2\pi}) \Longleftrightarrow \pi\, \text{sinc}^2(\frac{\pi\omega}{2})$ From the modulation property (4.41), it follows that

$$x(t) = \Delta\left(\frac{t}{2\pi}\right) \cos 10t \Longleftrightarrow \frac{\pi}{2}\left\{\text{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \text{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}$$

The Fourier transform in this case is a real function and we need only the amplitude spectrum in this case as shown in Figure S7.3-6a.

(b) The signal $x(t)$ here is the same as the signal in (a) delayed by 2π. From time shifting property, its Fourier transform is the same as in part (a) multiplied by $e^{-j\omega(2\pi)}$. Therefore

$$X(\omega) = \frac{\pi}{2}\left\{\text{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \text{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\} e^{-j2\pi\omega}$$

The Fourier transform in this case is the same as that in part (a) multiplied by $e^{-j2\pi\omega}$. This multiplying factor represents a linear phase spectrum $-2\pi\omega$. Thus we have an amplitude spectrum [same as in part (a)] as well as a linear phase spectrum $\angle X(\omega) = -2\pi\omega$ as shown in Figure S7.3-6b. the amplitude spectrum in this case as shown in Figure S7.3-6b.

Note: In the above solution, we first multiplied the triangle pulse $\Delta(\frac{t}{2\pi})$ by $\cos 10t$ and then delayed the result by 2π. This means the signal in (b) is expressed as $\Delta(\frac{t-2\pi}{2\pi}) \cos 10(t-2\pi)$.

We could have interchanged the operation in this particular case, that is, the triangle pulse $\Delta(\frac{t}{2\pi})$ is first delayed by 2π and then the result is multiplied by $\cos 10t$. In this alternate procedure, the signal in (b) is expressed as $\Delta(\frac{t-2\pi}{2\pi}) \cos 10t$.

This interchange of operation is permissible here only because the sinusoid $\cos 10t$ executes integral number of cycles in the interval 2π. Because of this both the expressions are equivalent since $\cos 10(t-2\pi) = \cos 10t$.

(c) In this case the signal is identical to that in (b), except that the basic pulse is $\text{rect}(\frac{t}{2\pi})$ instead of a triangle pulse $\Delta(\frac{t}{2\pi})$. Now

$$\text{rect}\left(\frac{t}{2\pi}\right) \Longleftrightarrow 2\pi\, \text{sinc}(\pi\omega)$$

Using the same argument as for part (b), we obtain

$$X(\omega) = \pi\{\text{sinc}[\pi(\omega+10)] + \text{sinc}[\pi(\omega-10)]\}e^{-j2\pi\omega}$$

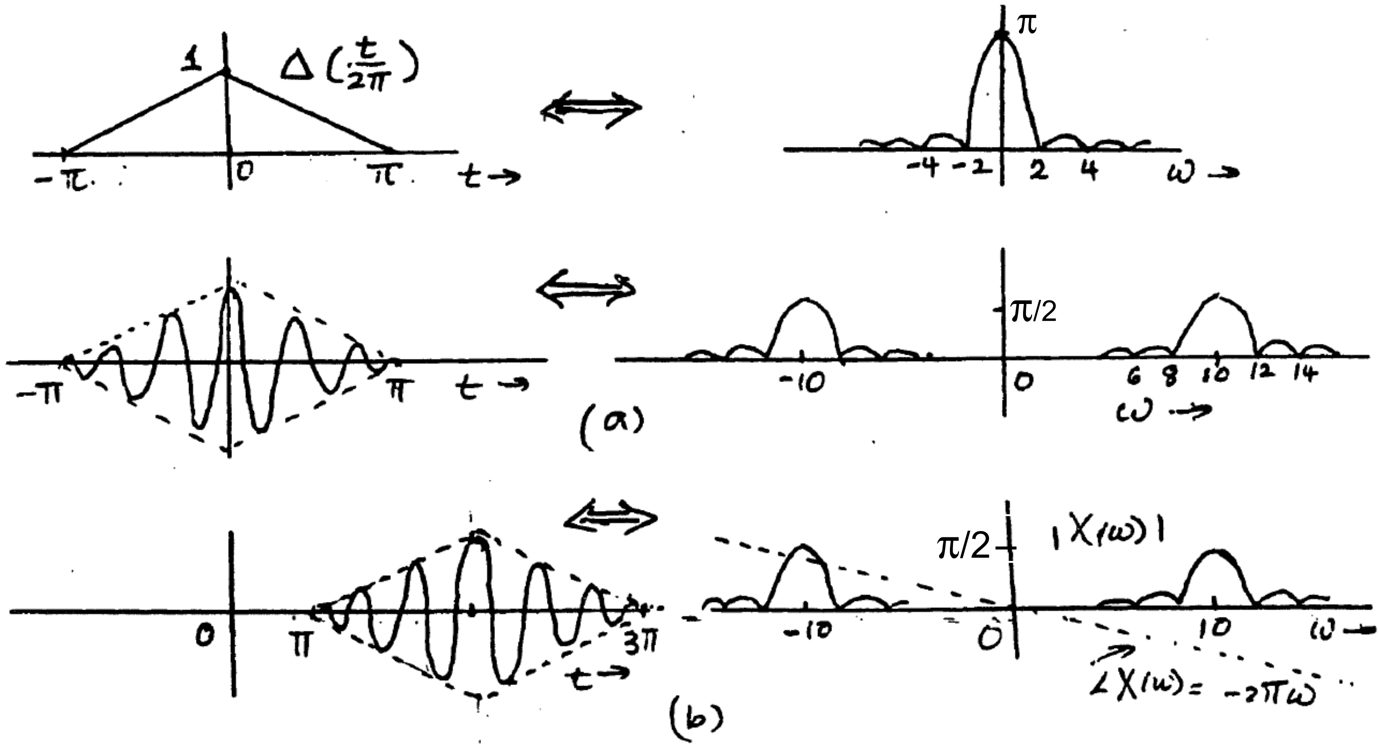


Figure S7.3-6

7.3-7. (a)

$$X(\omega) = \text{rect}\left(\frac{\omega-4}{2}\right) + \text{rect}\left(\frac{\omega+4}{2}\right)$$

Also

$$\frac{1}{\pi}\text{sinc}(t) \Longleftrightarrow \text{rect}\left(\frac{\omega}{2}\right)$$

Therefore

$$x(t) = \frac{2}{\pi}\text{sinc}(t)\cos 4t$$

(b)

$$X(\omega) = \Delta\left(\frac{\omega+4}{4}\right) + \Delta\left(\frac{\omega-4}{4}\right)$$

Also

$$\frac{1}{\pi}\text{sinc}^2(t) \Longleftrightarrow \Delta\left(\frac{\omega}{4}\right)$$

Therefore

$$x(t) = \frac{2}{\pi}\text{sinc}^2(t)\cos 4t$$

7.6-2. Consider a signal

$$x(t) = \text{sinc}(kt) \quad \text{and} \quad X(\omega) = \frac{\pi}{k}\text{rect}(\frac{\omega}{2k})$$

$$\begin{aligned} E_x = \int_{-\infty}^{\infty} \text{sinc}^2(kt)\,dt &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi^2}{k^2}\left[\text{rect}\left(\frac{\omega}{2k}\right)\right]^2 d\omega \\ &= \frac{\pi}{2k^2}\int_{-k}^{k} d\omega = \frac{\pi}{k} \end{aligned}$$

7.6-6. Application of duality property [Eq. (4.31)] to pair 3 (Table 4.1) yields

$$\frac{2a}{t^2 + a^2} \Longleftrightarrow 2\pi e^{-a|\omega|}$$

The signal energy is given by

$$E_x = \frac{1}{\pi}\int_0^{\infty} |2\pi e^{-a\omega}|^2\,d\omega = 4\pi\int_0^{\infty} e^{-2a\omega}\,d\omega = \frac{2\pi}{a}$$

The energy contained within the band (0 to W) is

$$E_W = 4\pi\int_0^{W} e^{-2a\omega}\,d\omega = \frac{2\pi}{a}[1 - e^{-2aW}]$$

If $E_W = 0.99E_x$, then

$$e^{-2aW} = 0.01 \quad \Longrightarrow W = \frac{2.3025}{a}\text{ rad/s}$$

$$\Rightarrow \quad B = 0.366/a \text{ Hz}$$

Kommentar till 7.7-1 nedan:

Utför hellre beräkningarna i frekvensdomänen, dvs. falta i de tre fallen $M(\omega)$ med bärvågens fouriertransform (två dirac:er), dvs. använd sambandet

$m(t)\cos(\omega_c t) \Leftrightarrow ½(M(\omega+\omega_c) + M(\omega+\omega_c))$.

Vid AM är man vanligen *mer* intresserad av den modulerade signalens frekvensspektrum än den modulerade signalen själv (dvs. $\varphi_{\text{DSB-SC}}(t)$ i författarens lösning nedan).

Komponenterna i det undre sidbandet (LSB) och det övre sidbandet (USB) kan lika gärna identifieras (och markeras i ditt svar) i den modulerade signalens *frekvensspektrum* i stället för i tidsuttrycket för $\varphi_{\text{DSB-SC}}(t)$.

7.7-1. (i) For $m(t) = \cos 1000t$

$$\begin{aligned}\varphi_{DSB-SC}(t) &= m(t)\cos 10,000t = \cos 1000t \cos 10,000t \\ &= \frac{1}{2}[\underbrace{\cos 9000t}_{\text{LSB}} + \underbrace{\cos 11,000t}_{\text{USB}}]\end{aligned}$$

(ii) For $m(t) = 2\cos 1000t + \cos 2000t$

$$\begin{aligned}\varphi_{DSB-SC}(t) &= m(t)\cos 10,000t = [2\cos 1000t + \cos 2000t]\cos 10,000t \\ &= \cos 9000t + \cos 11,000t + \frac{1}{2}[\cos 8000t + \cos 12,000t] \\ &= \underbrace{[\cos 9000t + \frac{1}{2}\cos 8000t]}_{\text{LSB}} + \underbrace{[\cos 11,000t + \frac{1}{2}\cos 12,000t]}_{\text{USB}}\end{aligned}$$

(iii) For $m(t) = \cos 1000t \cos 3000t$ $=$

$$\begin{aligned}\varphi_{DSB-SC}(t) &= m(t)\cos 10,000t = \frac{1}{2}[\cos 2000t + \cos 4000t]\cos 10,000t \\ &= \frac{1}{2}[\cos 8000t + \cos 12,000t] + \frac{1}{2}[\cos 6000t + \cos 14,000t] \\ &= \frac{1}{2}[\underbrace{\cos 8000t + \cos 6000t}_{\text{LSB}}] + \frac{1}{2}[\underbrace{\cos 12,000t + \cos 14,000t}_{\text{USB}}]\end{aligned}$$

This information is summarized in a table below. Figure S7.7-1 shows various spectra.

Anm: I fall (iii), där den modulerade signalen är $\varphi_{\text{DSB-SC}}(t) = \cos(1000t)\cos(3000t)\cos(10000t)$, är en lämplig övning att tolka den modulerade signalen som två efterföljande amplitudmoduleringar.
Gör detta i frekvensdomänen, genom att falta med dirac:er i två motsvarande steg!

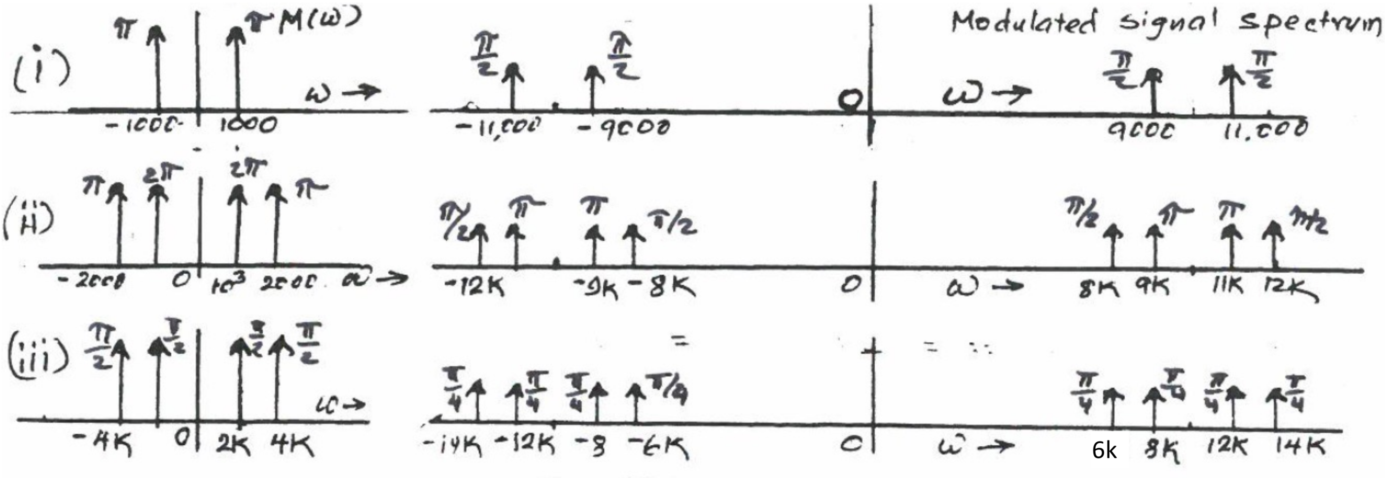


Figure S7.7-1

case	Baseband frequency	DSB frequency	LSB frequency	USB frequency
i	1000	9000 and 11,000	9000	11,000
ii	1000	9000 and 11,000	9000	11,000
	2000	8000 and 12,000	8000	12,000
iii	2000	8000 and 12,000	8000	12,000
	4000	6000 and 14,000	6000	14,000

7.7-2. (a) The signal at point b is

$$\begin{aligned} x_a(t) &= m(t)\cos^3\omega_c t \\ &= m(t)\left[\frac{3}{4}\cos\omega_c t + \frac{1}{4}\cos 3\omega_c t\right] \end{aligned}$$

The term $\frac{3}{4}m(t)\cos\omega_c t$ is the desired modulated signal, whose spectrum is centered at $\pm\omega_c$. The remaining term $\frac{1}{4}m(t)\cos 3\omega_c t$ is the unwanted term, which represents the modulated signal with carrier frequency $3\omega_c$ with spectrum centered at $\pm 3\omega_c$ as shown in Figure S7.7-2. The bandpass filter centered at $\pm\omega_c$ allows to pass the desired term $\frac{3}{4}m(t)\cos\omega_c t$, but suppresses the unwanted term $\frac{1}{4}m(t)\cos 3\omega_c t$. Hence, this system works as desired with the output $\frac{3}{4}m(t)\cos\omega_c t$.

(b) Figure S7.7-2 shows the spectra at points b and c.

(c) The minimum usable value of ω_c is $2\pi B$ in order to avoid spectral folding at dc.

(d)

$$\begin{aligned} m(t)\cos^2\omega_c t &= \frac{m(t)}{2}\left[1+\cos 2\omega_c t\right] \\ &= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t \end{aligned}$$

6k

The signal at point b consists of the baseband signal $\frac{1}{2}m(t)$ and a modulated signal $\frac{1}{2}m(t)\cos 2\omega_c t$, which has a carrier frequency $2\omega_c t$, not the desired value $\omega_c t$. Both the components will be suppressed by the filter, whose center center frequency is ω_c. Hence, this system will not do the desired job.

(e) The reader may verify that the identity for $\cos n\omega_c t$ contains a term $\cos\omega_c t$ when n is odd. This is not true when n is even. Hence, the system works for a carrier $\cos^n \omega_c t$ only when n is odd.

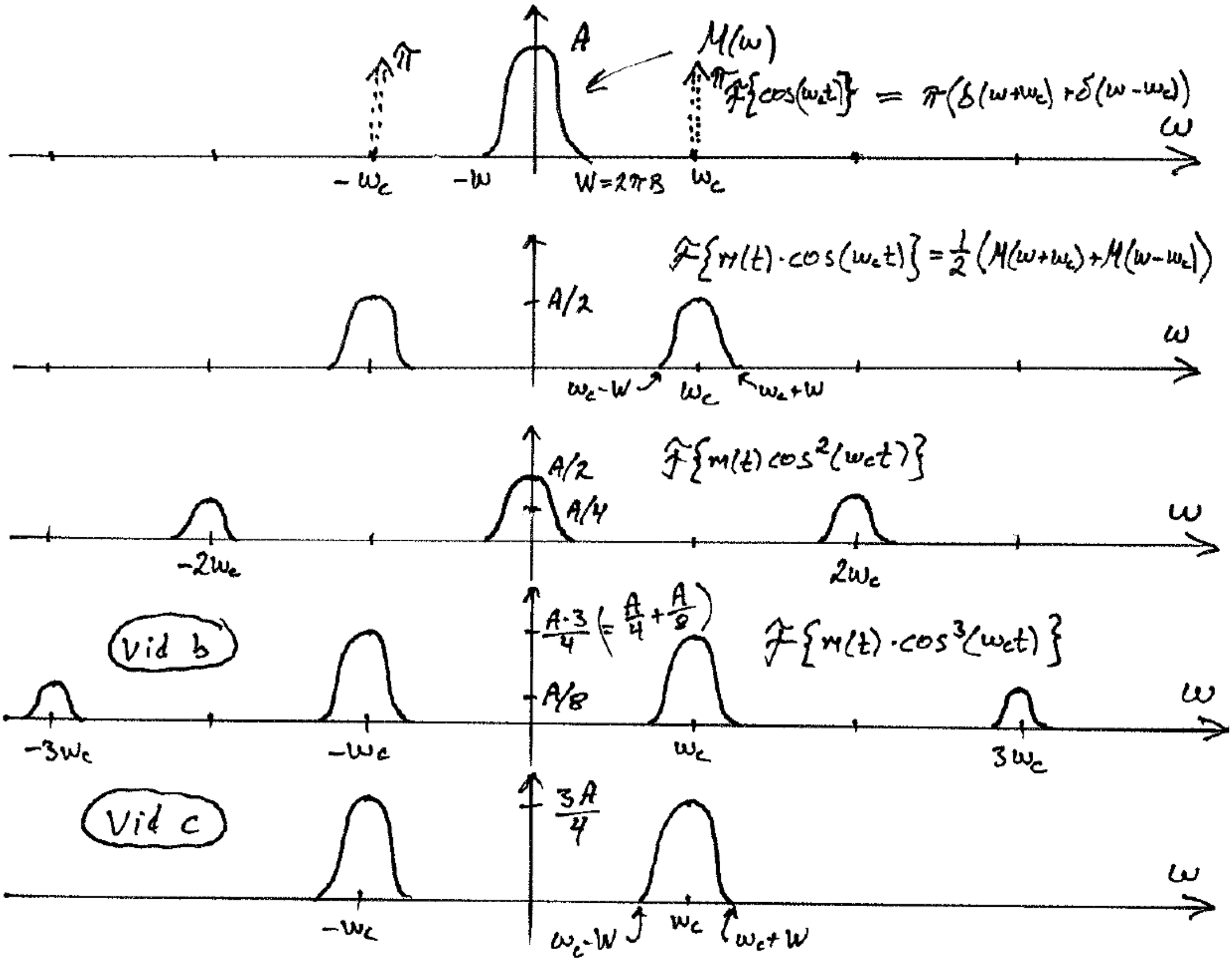


Figure S7.7-2

Det är lämpligare (rekommenderat!) att, på samma sätt som i uppgift 7.7-1, betrakta vad som händer med de olika ingående signalernas *spektrum*, genom att falta med fouriertransformen till cos(ω_ct), som består av två dirac:er med vikt π – se figur ovan. Se den inledande kommentaren till lösningen för 7.7-1.

7.7-3. This signal is identical to that in Figure 3.8a with period T_0 (instead of 2π). We find the Fourier series for this signal as

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos \omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t + \cdots\right]$$

Hence, $y(t)$, the output of the multiplier is

$$y(t) = m(t)x(t) = m(t)\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos \omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t + \cdots\right)\right]$$

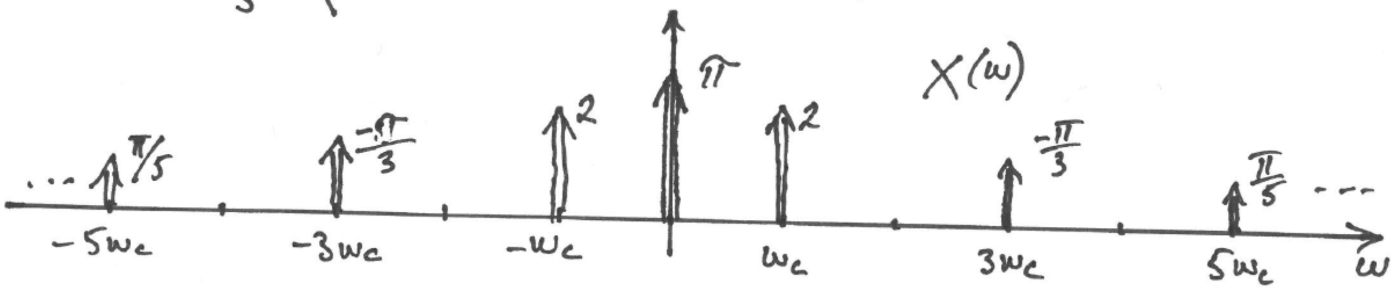
The bandpass filter suppresses the signals $m(t)$ and $m(t)\cos n\omega_c t$ for all $n \neq 1$. Hence, the bandpass filter output is

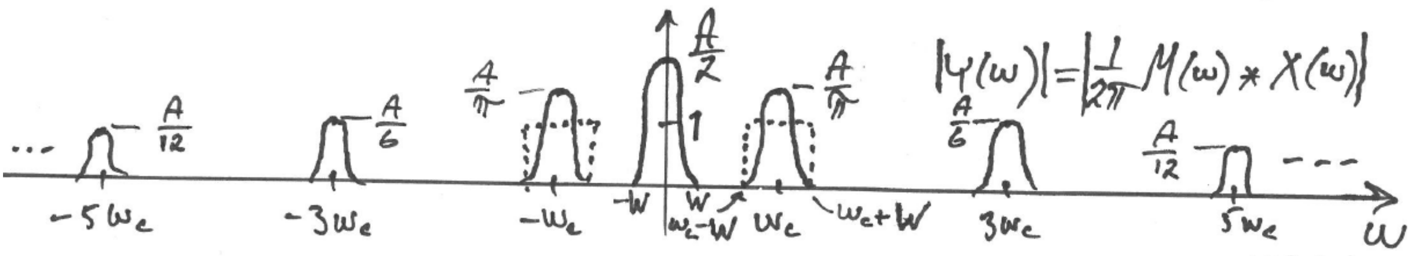
$$km(t)\cos \omega_c t = \frac{2}{\pi} m(t)\cos \omega_c t$$

Tilläggsuppgift:

$x(t)$ enligt ovan $\Rightarrow$

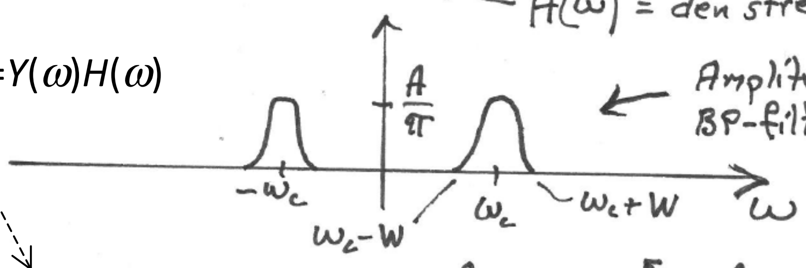
$$\Rightarrow X(\omega) = \frac{1}{2}\cdot 2\pi\delta(\omega) + \frac{2}{\pi}\cdot\pi\left(\delta(\omega+\omega_c)+\delta(\omega-\omega_c)\right) - \frac{1}{3}\cdot\pi\left(\delta(\omega+3\omega_c)+\delta(\omega-3\omega_c)\right) + \frac{1}{5}\cdot\pi\left(\delta(\omega+5\omega_c)+\delta(\omega-5\omega_c)\right) + \dots$$





$H(\omega)$ = den streckede ; $W = 2\pi B$

$Z(\omega)=Y(\omega)H(\omega)$



Amplitudspektrum för BP-filtrets utsignal.

Utsignalen $z(t) = k\cdot m(t)\cdot\cos(\omega_c t)$ $\Rightarrow k\cdot\frac{A}{2} = $ /figur/ $\frac{A}{\pi} \Rightarrow k = \frac{2}{\pi}$

7.7-4. (a) Figure S7.7-4 shows the signals at points a, b, and c.

(b) From the spectrum at point c, it is clear that the channel bandwidth must be at least 30,000 rad/s (from 5000 to 35,000 rad/s.).

(c) Figure S7.7-4 shows the receiver to recover $m_1(t)$ and $m_2(t)$ from the received modulated signal.

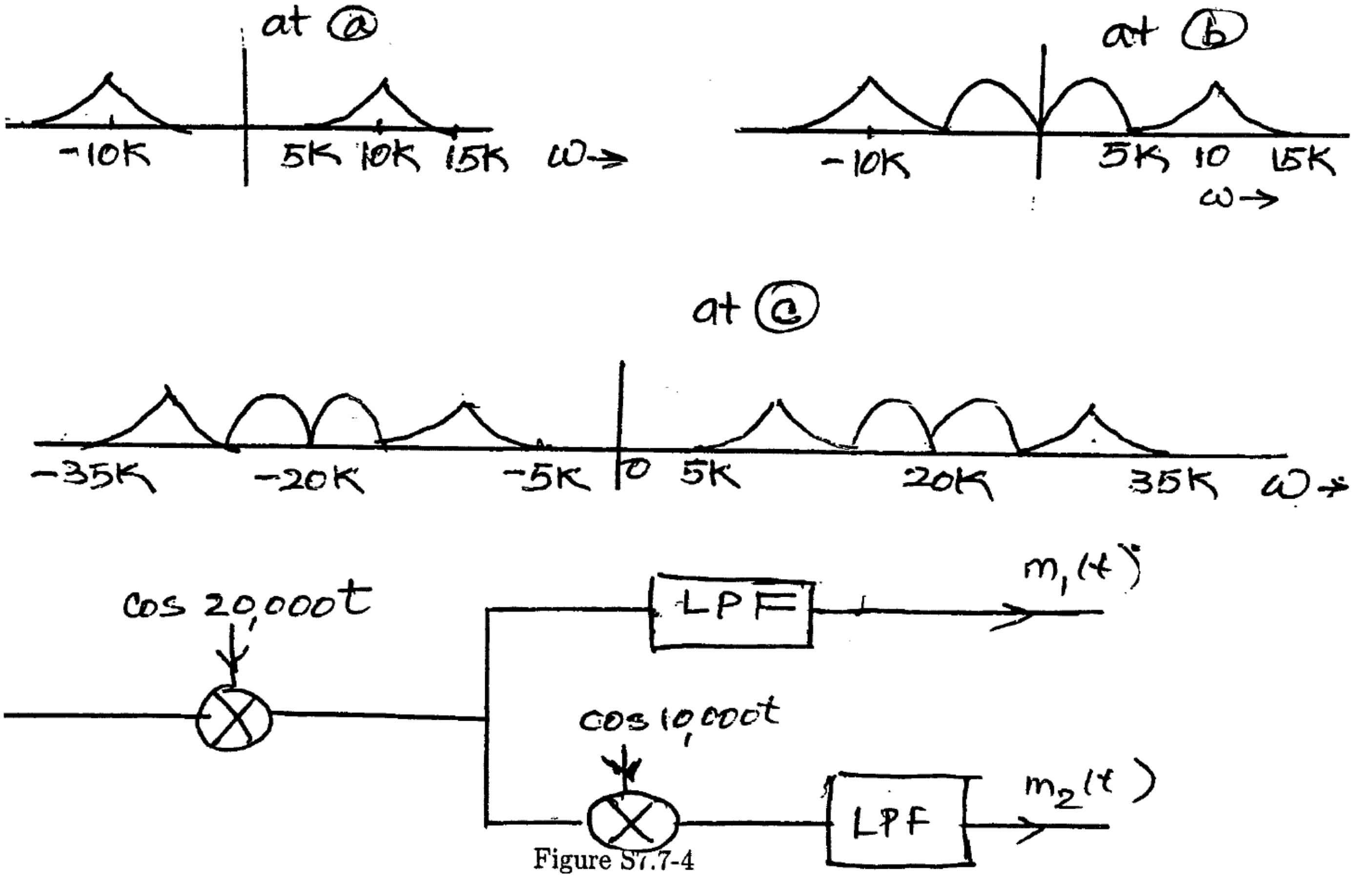


Figure S7.7-4

OBS: Ett förtydligande av lösningen ovan finns överst på nästa sida!

7.7-5. (a) Figure S7.7-5 shows the output signal spectrum $Y(\omega)$.

(b) Observe that $Y(\omega)$ is the same as $M(\omega)$ with the frequency spectrum inverted, that is, the high frequencies are shifted to lower frequencies and vice versa. Thus, the scrambler in Figure P7.7-5 inverts the frequency spectrum. To get back the original spectrum $M(\omega)$, we need to invert the spectrum $Y(\omega)$ once again. This can be done by passing the scrambled signal $y(t)$ through the same scrambler.

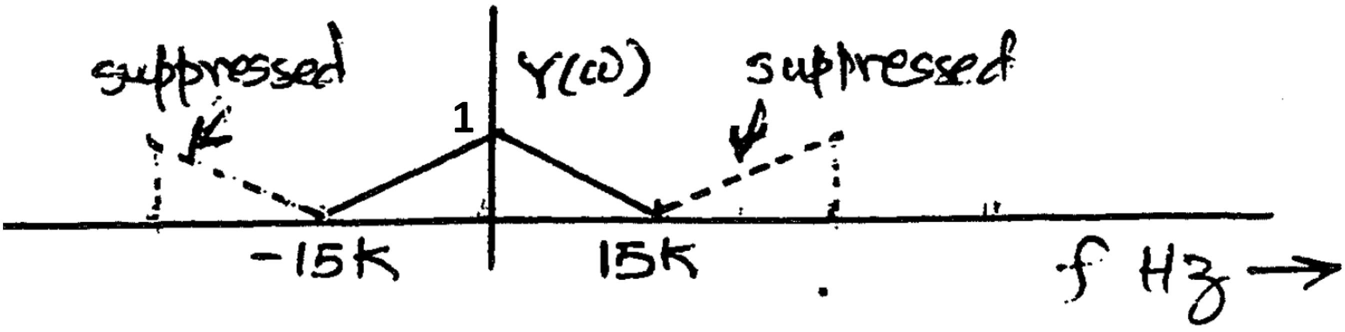
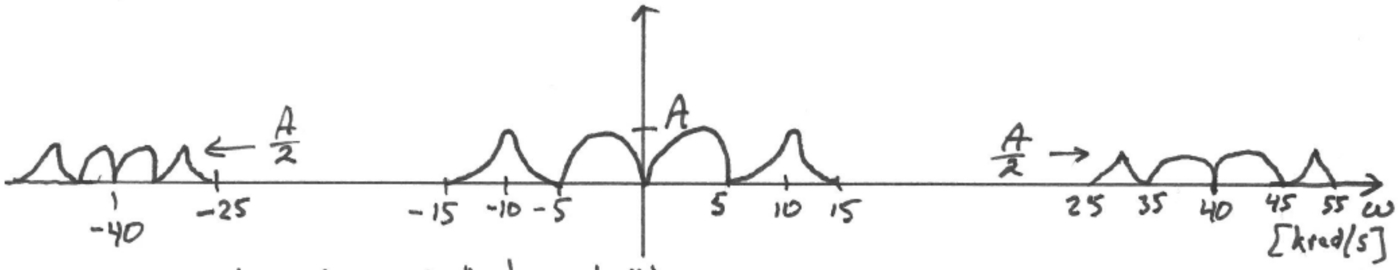


Figure S7.7-5

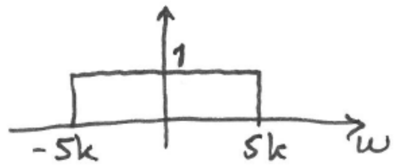
Förtydligande av lösningen till 7.7-4:

Spektrum efter första multiplikationen (vid demoduleringen):

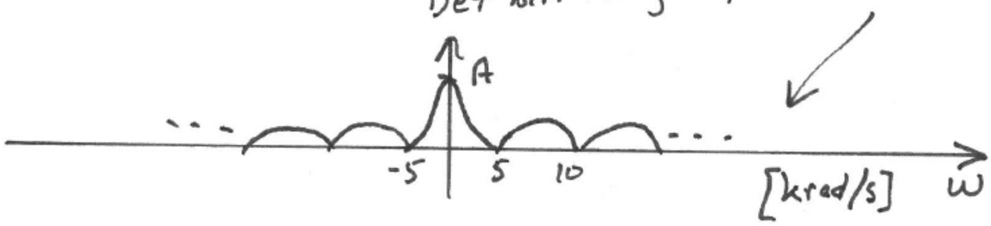


Här antas (för enhetlighetens skull) både $M_1(\omega)$ och $M_2(\omega)$ ha maxamplitud A. Då har de olika spektrumkomponenterna i spektrumen vid ⓐ, ⓑ och ⓒ också maxamplitud A (p.g.a. att bärvågsamplituderna vid moduleringen är 2)

$\Rightarrow$ Det övre LP-filtret (som har $m_1(t)$ som utsignal) är ett idealt amplitudnormerat filter med gränsvinkelfr. 5 krad/s:



Det nedre LP-filtret är ett likadant filter!
Rita gärna spektrumet för insignalen till det undre LP-filtret!
Det blir många spektrumkomponenter...



8.1-1. The bandwidths of $x_1(t)$ and $x_2(t)$ are 100 kHz and 150 kHz, respectively. Therefore the Nyquist sampling rates for $x_1(t)$ is 200 kHz and for $x_2(t)$ is 300 kHz. Also ${x_1}^2(t) \Longleftrightarrow \frac{1}{2\pi}F_1(\omega) * F_1(\omega)$, and from the width property of convolution the bandwidth of ${x_1}^2(t)$ is twice the bandwidth of $x_1(t)$ and that of ${x_2}^3(t)$ is three times the bandwidth of $x_2(t)$ (se also Prob. 4.3-10). Similarly the bandwidth of $x_1(t)x_2(t)$ is the sum of the bandwidth of $x_1(t)$ and $x_2(t)$. Therefore the Nyquist rate for ${x_1}^2(t)$ is 400 kHz, for ${x_2}^3(t)$ is 900 kHz, for $x_1(t)x_2(t)$ is 500 kHz.

8.1-2. (a)

$$\text{sinc}^2(100\pi t) \Longleftrightarrow 0.01\Delta(\frac{\omega}{400\pi})$$

The bandwidth of this signal is 200 π rad/s or 100 Hz. The Nyquist rate is 200 Hz (samples/sec).

(b) The Nyquist rate is 200 Hz, the same as in (a), because multiplication of a signal by a constant does not change its bandwidth.

(c)

$$\text{sinc}(100\pi t) + 3\,\text{sinc}^2(60\pi t) \Longleftrightarrow 0.01\,\text{rect}(\frac{\omega}{200\pi}) + \frac{1}{20}\Delta(\frac{\omega}{240\pi})$$

The bandwidth of $\text{rect}(\frac{\omega}{200\pi})$ is 50 Hz and that of $\Delta(\frac{\omega}{240\pi})$ is 60 Hz. The bandwidth of the sum is the higher of the two, that is, 60 Hz. The Nyquist sampling rate is 120 Hz.

(d)

$$\begin{aligned} \text{sinc}(50\pi t) &\Longleftrightarrow 0.02\,\text{rect}(\frac{\omega}{100\pi}) \\ \text{sinc}(100\pi t) &\Longleftrightarrow 0.01\,\text{rect}(\frac{\omega}{200\pi}) \end{aligned}$$

The two signals have bandwidths 25 Hz and 50 Hz respectively. The spectrum of the product of two signals is $1/2\pi$ times the convolution of their spectra. From width property of the convolution, the width of the convoluted signal is the sum of the widths of the signals convolved. Therefore, the bandwidth of $\text{sinc}(50\pi t)\text{sinc}(100\pi t)$ is $25 + 50 = 75$ Hz. The Nyquist rate is 150 Hz.

8.1-5. If the corrupted spectrum is not filtered out, we need the minimum sampling rate 22 Hz. This is clarified by Figure S8.1-5a. For $f_s = 22$, the uncorrupted spectrum remains intact and can be recovered by a lowpass filter of cutoff frequency 10 Hz.

When the corrupted spectrum is suppressed, the resulting signal spectrum band is only 10Hz. Hence it is adequate to use $f_s = 20$ Hz, as shown in Figure S8.1-5b.

OBS: I texten ovan skall det vara ”kHz”, inte ”Hz”!!

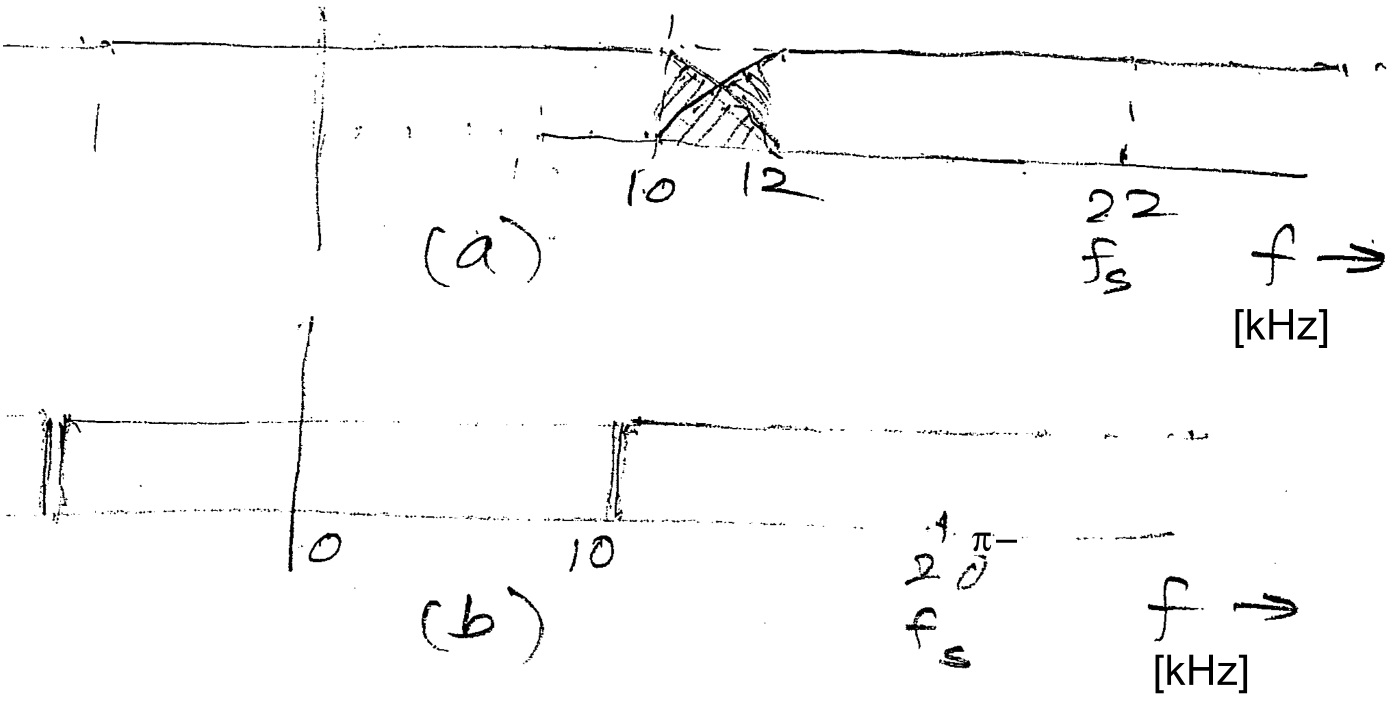


Figure S8.1-5

8.1-6.

$$\Delta\left(\frac{t-1}{2}\right) \Longleftrightarrow \text{sinc}^2\left(\frac{\omega}{2}\right)e^{-j\omega}$$

The spectrum $|X(\omega)|$ shows that most of the signal energy is concentrated within the band of 1 Hz. It can be shown that 90.28% energy is contained within the band of 1 Hz. If we use 90% energy criterion for bandwidth, sampling rate of 2 Hz is adequate. However, for a better approximation, (higher energy bandwidth criterion), we may go to $f_s = 4$ Hz. Theoretically, of course, $f_s = \infty$.

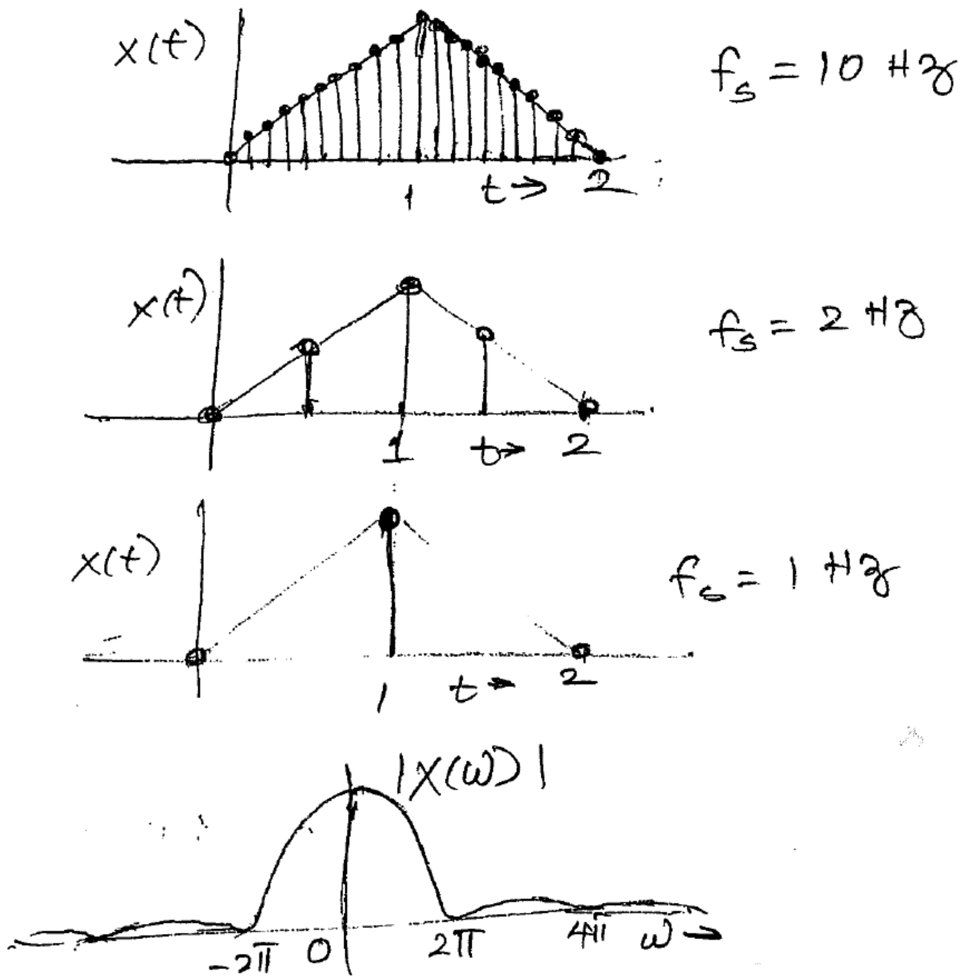
Se figur S8.1-6, nästa sida!

8.1-7. (a)

$$X(\omega) = \Delta\left(\frac{\omega}{20\pi}\right) + \pi[\delta(\omega + 20\pi) + \delta(\omega - 20\pi)]$$

The bandwidth is 10 Hz. There is an impulse at 10Hz, as seen from $X(\omega)$ shown in Figure S8.1-7a. The Nyquist rate is 20 Hz. Hence, $f_s = 10$Hz will not permit reconstruction of $x(t)$. This is verified from the sampled signal spectrum in Figure S8.1-7a, shown as a function of f in Hz.

(b) The Nyquist rate is 20 Hz. Hence the sampling rate $f_s = 20$ Hz is adequate despite the fact that $x(t)$ contains an impulse at the highest frequency 10 Hz. This is because, the impulse component is $\cos 20\pi t$.
To reconstruct $x(t)$ from the spectrum in Figure S8.1-7b, we need an ideal lowpass filter of cutoff frequency 10 Hz and gain $T = 1/20$. Because the rect function value is 0.5 at the edge (cutoff), the lowpass filter gain at the cutoff frequency 10 Hz is $0.5 \times 1/20 = 1/40$. Hence for the input of an impulse of strength 40π at



Figur S8.1-6

± 10 Hz, the output will be an impulse of strength 40π at $f = \pm 10$ Hz. Hence, the filter output is the spectrum

$$X(\omega) = \Delta\left(\frac{\omega}{20\pi}\right) + \pi[\delta(\omega + 20\pi) + \delta(\omega - 20\pi)]$$

and

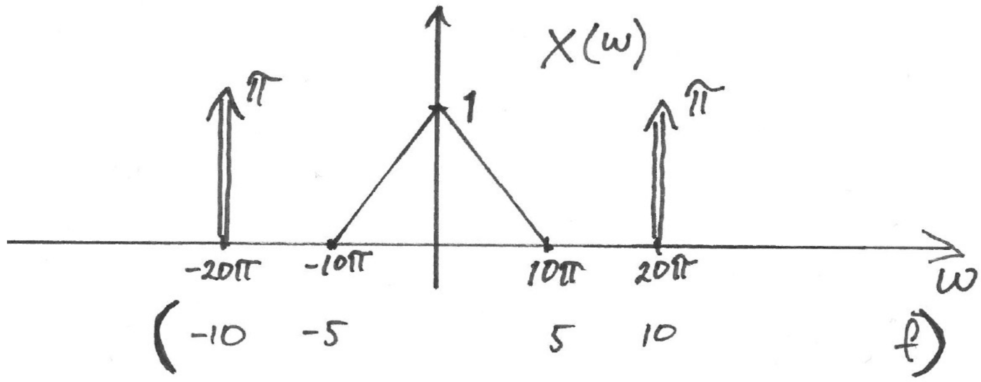
$$x(t) = 5\text{sinc}^2(5\pi t) + \cos 20\pi t$$

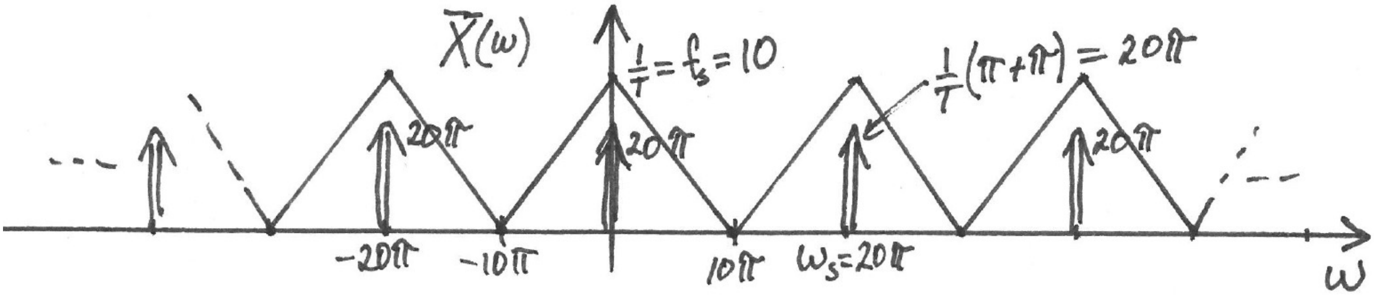
(c)

In this case the impulses are because of a sine term. Hence, f_s =20 Hz is inadequate, even in theory. It is easily verified that the samples of $\sin 20\pi t$ at a rate 20 Hz ($T = 1/20$) are $\sin 20\pi nT = \sin \pi n = 0$, In this case the impulse at $\pm 10, \pm 30, \pm 50, \cdots$ cancel out because the two impulses have opposite phases (Figure S8.1-7c).

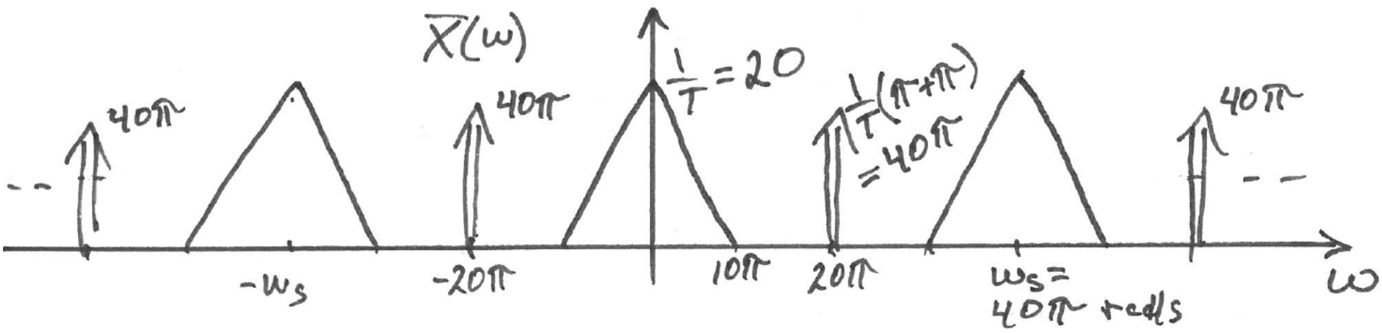
(d) Yes. In this case, the impulses do not overlap and there is no cancelation. Hence using a low pass filter of cutoff frequency 10.5 Hz, and gain $T = 1/21$, we can recover $x(t)$ from $\overline{x}(t)$ (see Figure S8.1-7d).

Figur S8.1-7a

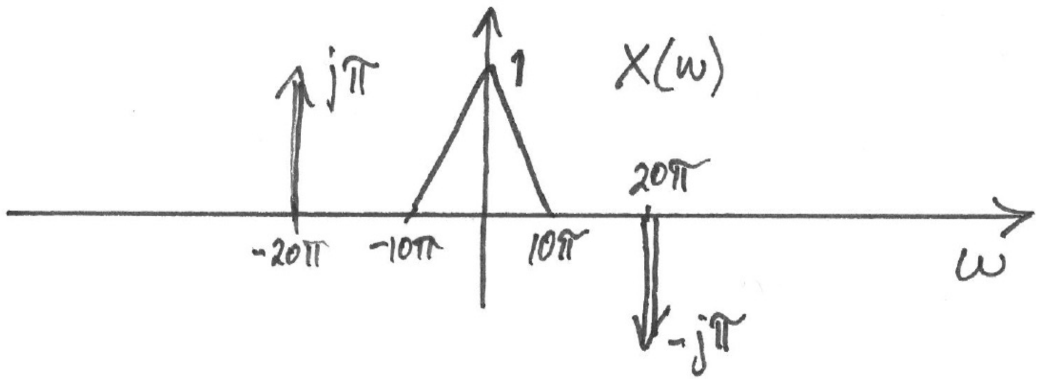


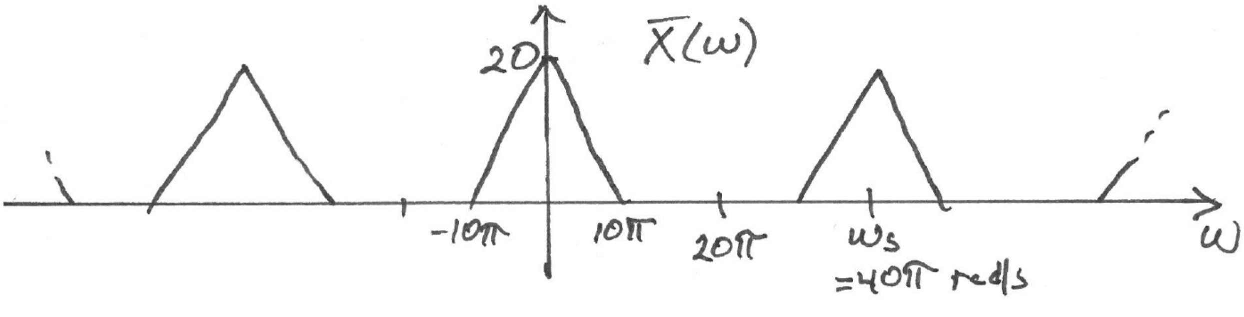


Figur S8.1-7b

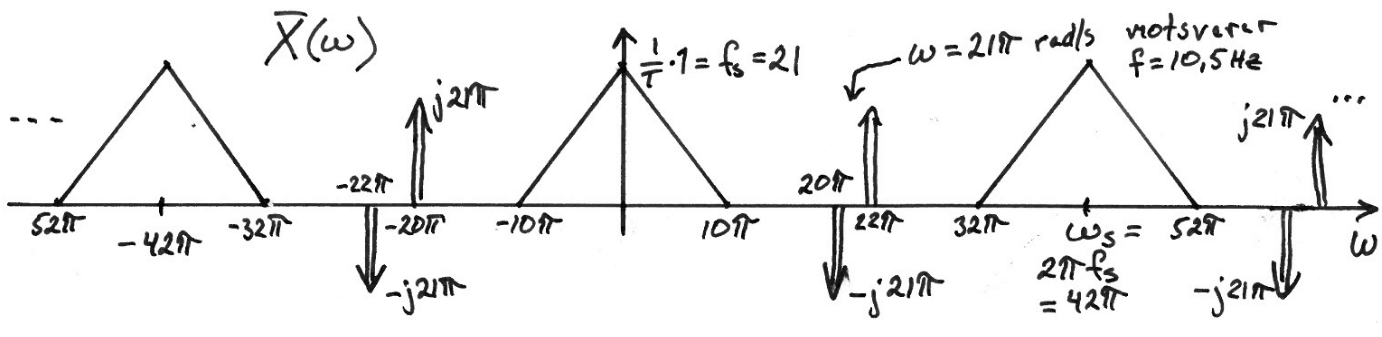


Figur S8.1-7c





Figur S8.1-7d



8.1-9. This problem is trivial when worked out in the frequency-domain. The sampled signal spectrum is given by

$$\overline{X}(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - 2\pi n f_s)$$

We repeat the spectrum periodically with period $(f_1 + f_2)$ Hz, as shown in Figure S8.1-9. The amplitude at the origin is $1/T = f_1 + f_2$. From Figure S8.1-8a, it is obvious that the resulting spectrum $\overline{X}(\omega)$ is constant for all ω and has a value $f_1 + f_2$. Hence

$$\overline{X}(\omega) = f_1 + f_2$$

and

$$\overline{x}(t) = (f_1 + f_2)\delta(t)$$

Hence all the samples of $x(t)$ at a rate $f_s = f_1 + f_2$ are zero except the sample at $t = 0$ or $n = 0$.

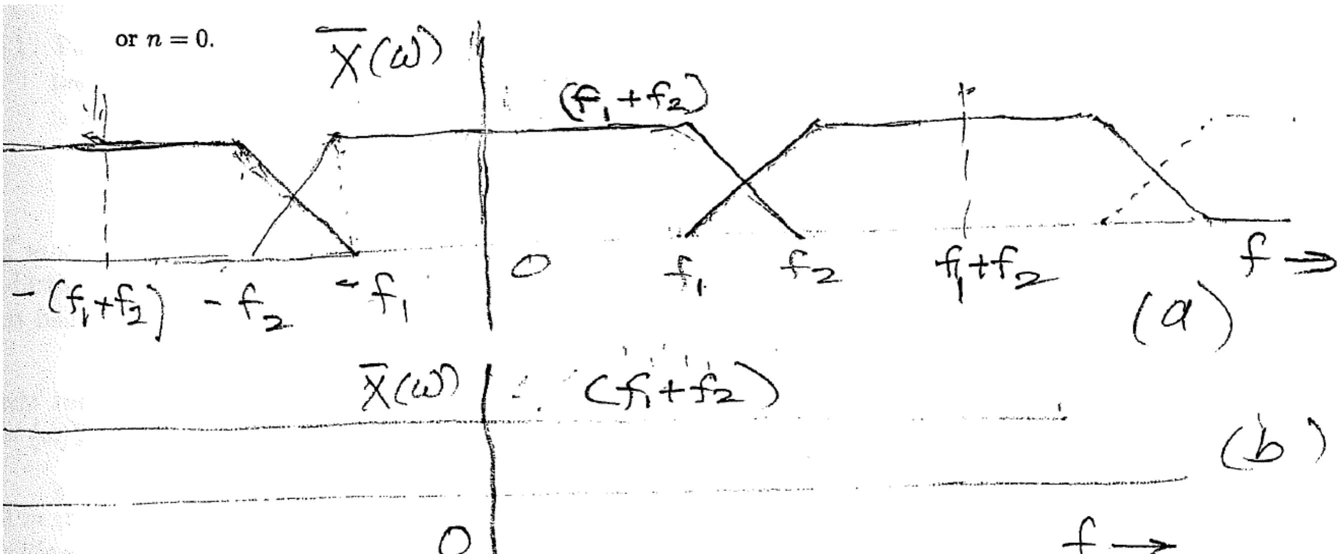


Fig S8.1-9

8.2-1. The signal $x(t) = \text{sinc}(200\pi t)$ is sampled by a rectangular pulse sequence $p_T(t)$ whose period is 4 ms so that the fundamental frequency (which is also the sampling frequency) is 250 Hz. Hence, $\omega_s = 500\pi$. The Fourier series for $p_T(t)$ is given by

$$p_T(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos n\omega_s t$$

Use of Eqs. (3.66) yields $C_0 = \frac{1}{5}$, $C_n = \frac{2}{n\pi} \sin\left(\frac{n\pi}{5}\right)$, that is,

$$C_0 = 0.2, \quad C_1 = 0.374, \quad C_2 = 0.303, \quad C_3 = 0.202, \quad C_4 = 0.093, \quad C_5 = 0, \cdots$$

Consequently

$$\overline{x}(t) = x(t)p_T(t) = 0.2\,x(t) + 0.374x(t)\,\cos 500\pi t + 0.303\,x(t)\,\cos 1000\pi t + 0.202\,x(t)\,\cos 1500\pi t + \cdot$$

and

$$\begin{aligned} \overline{X}(\omega) &= 0.2\,X(\omega) + 0.187\,[X(\omega - 500\pi) + X(\omega + 500\pi)] \\ &\quad +0.151\,[X(\omega - 1000\pi) + X(\omega + 1000\pi)] \\ &\quad +0.101\,[X(\omega - 1500\pi) + X(\omega + 1500\pi)] + \cdots \end{aligned}$$

In the present case $X(\omega) = 0.005\,\text{rect}(\frac{\omega}{400\pi})$. The spectrum $\overline{X}(\omega)$ is shown in Figure S8.2-1. Observe that the spectrum consists of $X(\omega)$ repeating periodically at the interval of 500π rad/s (250 Hz). Hence, there is no overlap between cycles, and $X(\omega)$ can be recovered by using an ideal lowpass filter of bandwidth 100 Hz. An ideal lowpass filter of unit gain (and bandwidth 100 Hz) will allow the first term on the he right-side of the above equation to pass fully and suppress all the other terms. Hence

the output $y(t)$ is

$$y(t) = 0.2\,x(t)$$

Because the spectrum $\overline{X}(\omega)$ has a zero value in the band from 100 to 150 Hz, we can use an ideal lowpass filter of bandwidth B Hz where $100 < B < 150$. But if $B > 150$ Hz, the filter will pick up the unwanted spectral components from the next cycle, and the output will be distorted.

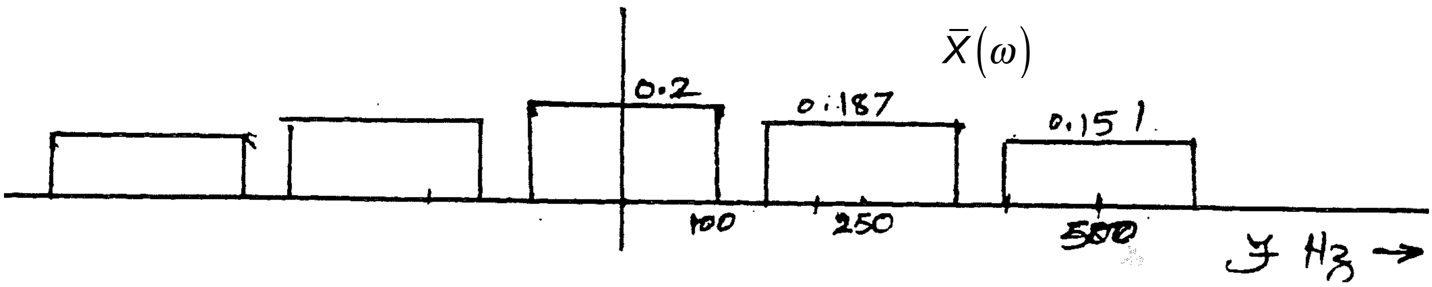


Figure S8.2-1

8.2-3. (a) Figure S8.2-3a shows the signal reconstruction from its samples using the first-order hold circuit. Each sample generates a triangle of width $2T$ and centered at the sampling instant. The height of the triangle is equal to the sample value. The resulting signal consists of straight line segments joining the sample tops.

(b) The frequency response of this circuit is:

$$H(\omega) = \mathcal{F}\{h(t)\} = \mathcal{F}\left\{\Delta\left(\frac{t}{2T}\right)\right\} = T\,\text{sinc}^2\left(\frac{\omega T}{2}\right)$$

Because $H(\omega)$ is positive for all ω, it also represents the amplitude response. Figure S8.2-3b shows this amplitude response and the ideal amplitude response (lowpass) required for signal reconstruction.

(c) A minimum of T secs delay is required to make $h(t)$ causal (realizable). Such a delay would cause the reconstructed signal in Figure S8.2-3a to be delayed by T secs.

(d) The impulse response and the frequency response of a ZOH circuit are

$$h(t) = \text{rect}\left(\frac{t - T/2}{T}\right)$$

and

$$H_{\text{zoh}}(\omega) = T\,\text{sinc}(\omega T/2)e^{-j\omega T/2}$$

The frequency response of the cascade two sections of the ZOH circuits is given by

$$H_{\text{cascade}}(\omega) = T^2\,\text{sinc}^2(\omega T/2)e^{-j\omega T}$$

In part (b), we found that the frequency response of an FOH circuit as

$$H_{\text{foh}}(\omega) = T\,\text{sinc}^2(\omega T/2)$$

This shows that the frequency response of the cascading two ZOH circuits is T times the frequency response of the FOH circuit with time delay T seconds. the time delay is a desirable feature as it makes the FOH circuit causal, and therefore, realizable. Thus, the cascade of two ZOH acts identical to an FOH circuit except for the amplification by factor T and delay by T seconds.

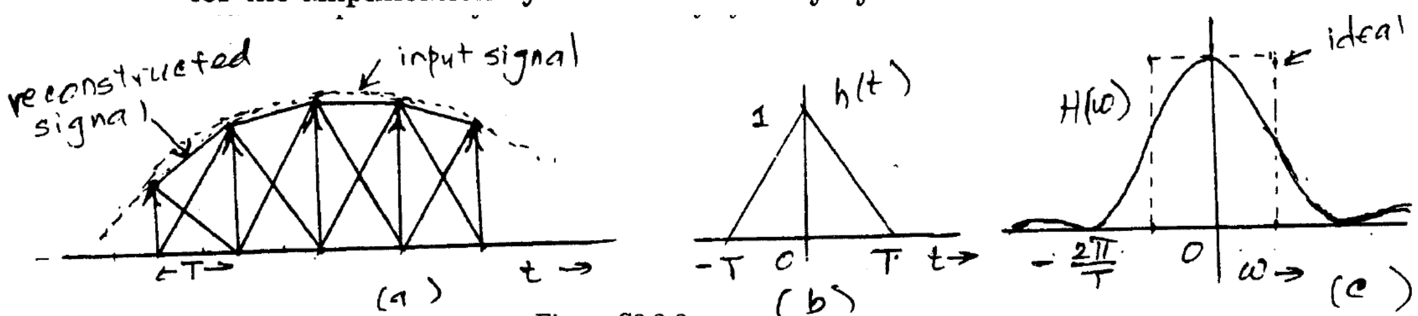


Figure S8.2-3

8.2-6.

$$f_s = 20\text{Hz}$$

$$|f_a| = |f - mf_s| \qquad |f_a| \leq f_s/2 = 10$$

(a) $f = 8$ Hz is less that $f_s/2 = 10$ Hz.
Hence, this frequency is not aliased and $|f_a| = f = 8$ Hz

(b) $f = 12$ Hz
$|f_a| = |12 - 20| = 8$ Hz

(c) $f = 20$ Hz
$|f_a| = |20 - 20| = 0$ Hz

(d) $f = 22$ Hz
$|f_a| = |22 - 20| = 2$ Hz

(e) $f = 32$ Hz
$|f_a| = |32 - 40| = 8$ Hz

8.2-8.

$$X(\omega) = \pi\left[3\delta(\omega \pm 6\pi)) + \delta(\omega \pm 16\pi)) + 2\delta(\omega \pm 20\pi))\right]$$

The highest frequency is 10 Hz. The Nyquist rate is 20 Hz. 25% above this rate is 25 Hz is the actual sampling rate. Hence $T = 0.04$. Therefore $\overline{X}(\omega)$ consists of $25X(\omega)$ repeating periodically with period 25 Hz (or 50π rad/sec), as shown in Figure S8.2-8a. To reconstruct $x(t)$ from the sampled signal $\overline{x}(t)$, we pass $\overline{X}(\omega)$ through a lowpass filter of gain 1/25 and having a cutoff frequency anywhere between $(10 + \epsilon)$ Hz to $(15 - \epsilon)$ Hz where ϵ is an arbitrarily small number.
If the sampling rate is 25% below the NYquist rate, that is $f_s = 15$ Hz, the components of frequencies 8 Hz and 10 Hz will be aliased to other frequencies. The 8 Hz will appear as $|f_a| = |8 - 15| = 7$ Hz and 10 Hz will be aliased as $|10 - 15| = 5$ Hz. Hence the output contains frequencies 3 Hz, 5 Hz and 7 Hz.

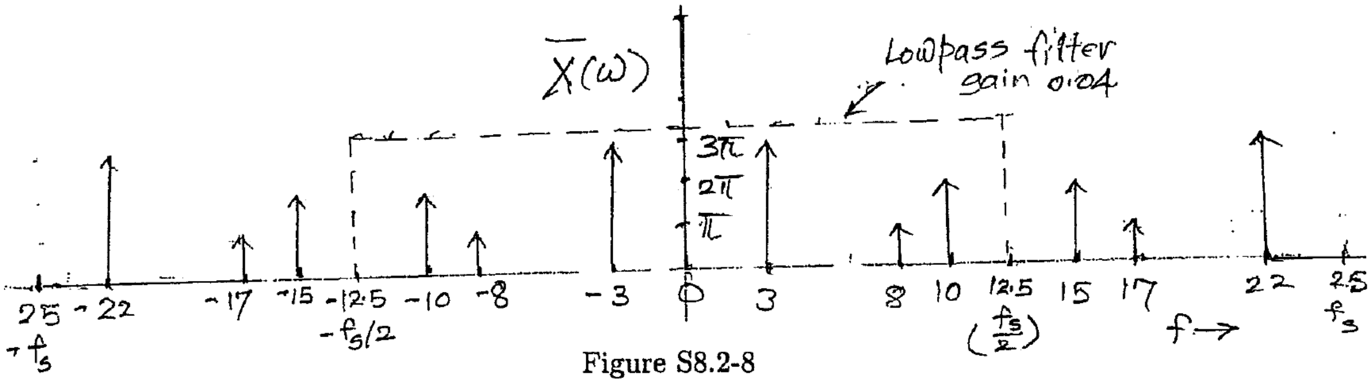


Figure S8.2-8

8.5-1.

$$T_0 = \frac{1}{f_o} = \frac{1}{50} = 20\text{ms}$$

$$B = 10000 \quad \text{Hence} \quad f_s \geq 2B = 20000$$

$$T = \frac{1}{f_s} = \frac{1}{20000} = 50\mu s$$

$$N_0 = \frac{T_0}{T} = \frac{20 \times 10^{-3}}{50 \times 10^{-6}} = 400$$

Since N_0 must be a power of 2, we choose $N_0 = 512$. Also $T = 50\mu$s, and $T_0 = N_0 T = 512 \times 50\mu s = 25.6$ms, $f_o = 1/T_0 = 39.0625$ Hz. Since $x(t)$ is of 10 ms duration, we need zero padding over 15.6 ms. Alternatively, we could also have used

$$T = \frac{20 \times 10^{-3}}{512} = 39.0625\ \mu\text{s}$$

This gives $T_0 = 20$ ms, $f_o = 50$ Hz. And

$$f_s = \frac{1}{T} = 25600\text{Hz}$$

There are also other possibilities of reducing T as well as increasing the frequency resolution.

8.5-2. For the signal $x(t)$,

$$T_0 \geq \frac{1}{0.25} = 4, \qquad T \leq \frac{1}{f_s} = \frac{1}{3 \times 2} = \frac{1}{6}$$

Let us choose $T = 1/8$. Also $T_0 = 4$. Therefore, $N_0 = T_0/T = 32$. The signal $x(t)$ repeats every 4 seconds with samples every 1/8 second. The samples are $Tx(kT) = (1/8)x(k/8)$. Thus, the first sample is (at $k = 0$) $1 \times (1/8) = 1/8$. The 32 samples are (starting at $k = 0$)

$$\frac{1}{8}, \frac{7}{64}, \frac{3}{32}, \frac{5}{64}, \frac{1}{16}, \frac{3}{64}, \frac{1}{32}, \frac{1}{64}, 0, 0, 0, 0, 0, 0, 0, 0,$$
$$0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{64}, \frac{1}{32}, \frac{3}{64}, \frac{1}{16}, \frac{5}{64}, \frac{3}{32}, \frac{7}{64}$$

The samples of $x(t)$ and $g(t)$ are shown in Figure S8.5-2.

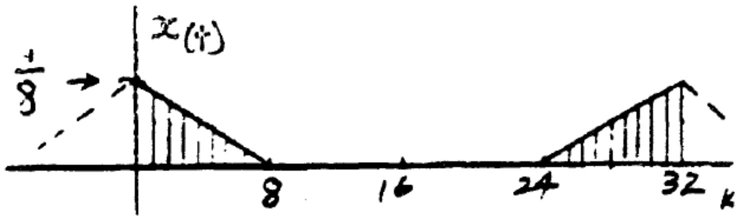
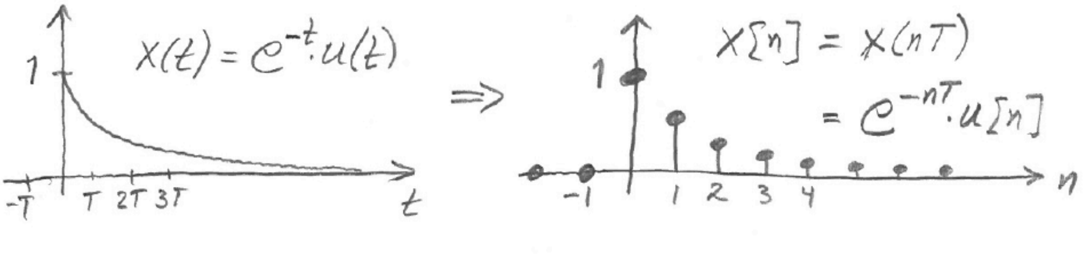


Figure S8.5-2

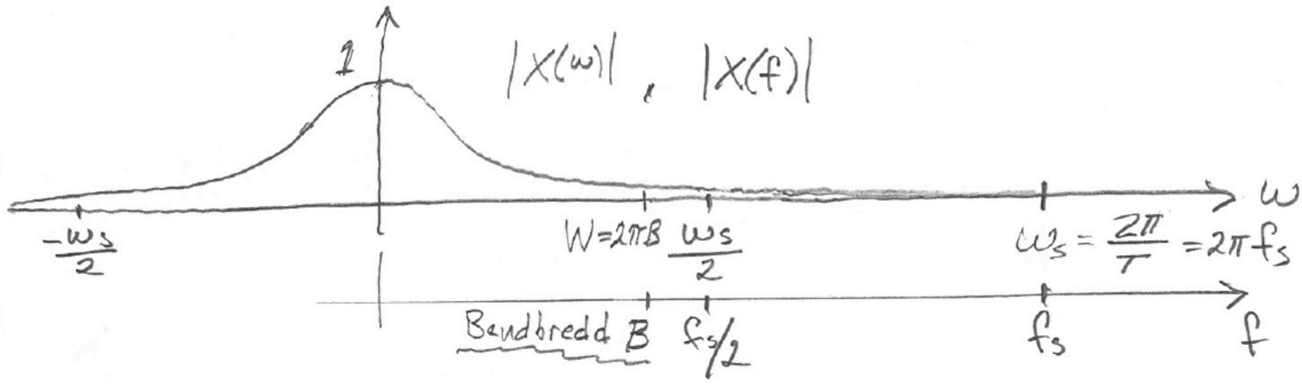
8.5 - 3

$x(t) = e^{-t} \cdot u(t) \Rightarrow x[n] = x(nT) = e^{-nT} \cdot u[n]$



$$\Rightarrow X(\omega) = \frac{1}{1+j\omega} \Rightarrow |X(\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

$|X(\omega)|$, $|X(f)|$



$x(t)$ samples med sampelfrekvens $\underline{f_s > 2B}$

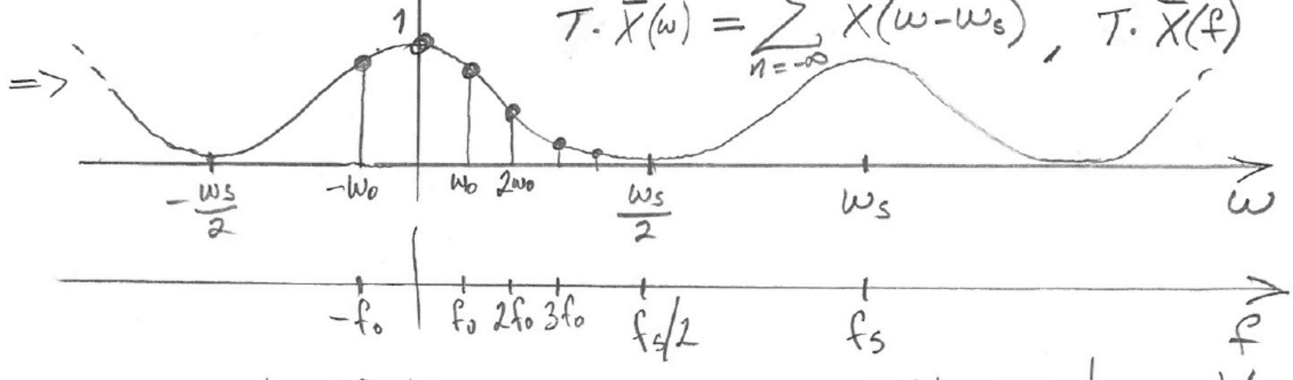
a) Låt B vara den frekvens där $|X(f=B)| = \frac{1}{100} \cdot |X(f)|_{max}$

dvs. $\frac{1}{\sqrt{1+(2\pi B)^2}} \overset{(2\pi B)^2 \gg 1}{\approx} \frac{1}{2\pi B} = \frac{1}{100} \cdot 1 \Rightarrow B = \frac{50}{\pi}$ Hz

$\Rightarrow f_s > 2B = \frac{100}{\pi} \approx 31{,}83$. Välj $f_s = 32$ Hz

(Enkelt att räkna om $f_s \in \mathbb{Q}$), dvs. $\underline{T = \frac{1}{32} \text{ sek}}$

$$\Rightarrow \quad T \cdot \bar{X}(\omega) = \sum_{n=-\infty}^{\infty} X(\omega - \omega_s), \quad T \cdot \bar{X}(f)$$

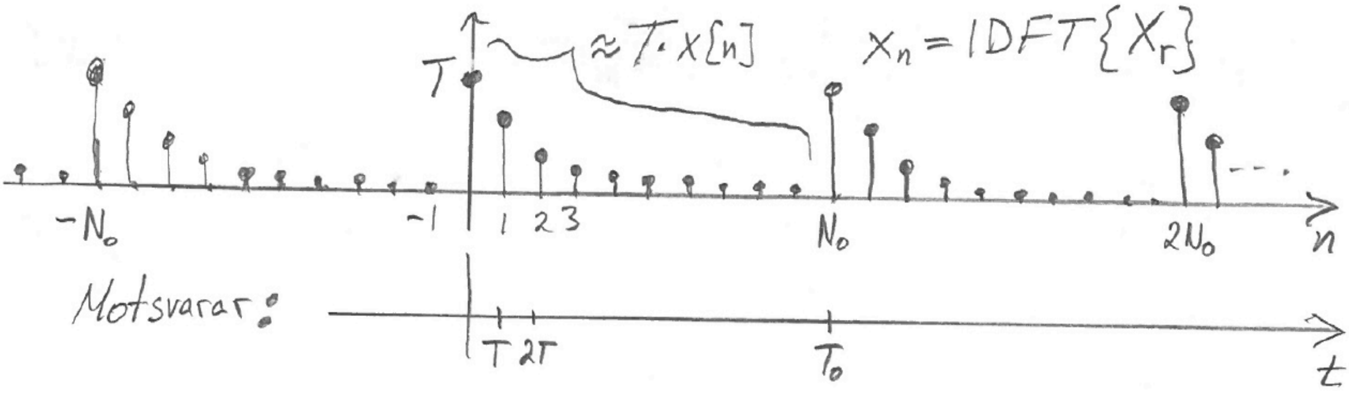


Spektrumet $\frac{1}{T}\bar{X}(f)$ samples nu med sampeltakten $T_0 = \frac{1}{f_0}$ sampel/Hz,

dvs. DFT:n $\underline{X_r = T \cdot \bar{X}(r \cdot f_0)}$, vilket medför att

IDFT:n x_n utgör en N_0-periodisk upprepning av $T \cdot x[n]$,

där $N_0 = \frac{T_0}{T}$, vilket motsvarar en T_0-periodisk upprepning av $T \cdot x(t)$.



f_0 väljs "tillräckligt" liten, så att $\bar{X}(f)$ kan återskapas "tillräckligt" väl från X_r $\Longleftrightarrow$ $N_0 = \frac{T_0}{T}$ (där $T_0 = \frac{1}{f_0}$) väljs "tillräckligt" stor, så att de N_0-periodiska upprepningarna av $T\cdot x[n]$ endast överlapper marinellt – dvs. så att $x_n \approx T\cdot x[n]$ (idealt: $x_n = T\cdot x[n]$) för $0 \leq n \leq N_0 - 1$

Med samma kriterium som vid valet av f_s ($< \frac{1}{100}$ av max-amplituden för $|X(f)|$ vid $f = f_s/2$), så får vi $e^{-T_0} < \frac{1}{100} \cdot e^{0} \Longrightarrow$

$$\left.\begin{array}{l} T_0 > \ln 100 \approx 4{,}6 \text{ sek.} \\ \hline N_0 = \frac{T_0}{T} = 32\,T_0 \in \mathbb{N} \Rightarrow T_0 \in \mathbb{N} \end{array}\right\} T_0 = 5, 6, 7, 8, \ldots$$

Om $N_0 = 2^b$, så kan DFT:n X_r beräknes med en effektiv FFT-algoritm $\Rightarrow$ Välj gärna $T_0 = 8$ sek

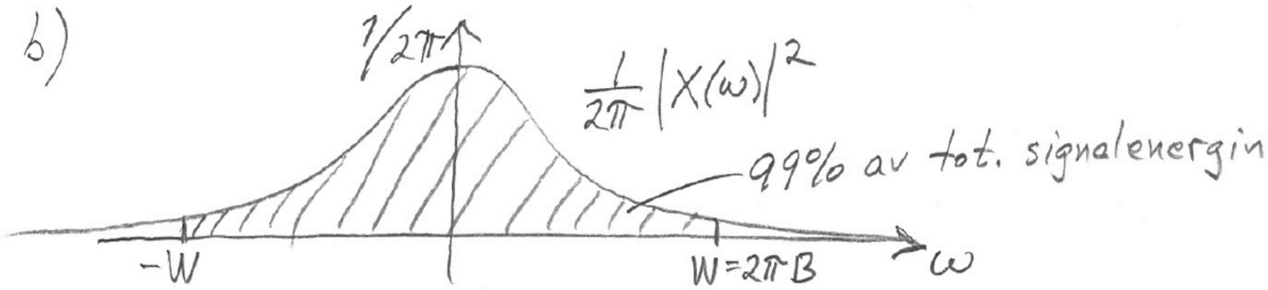
$$\Rightarrow \underline{\underline{N_0 = 32\cdot 8 = 256 = 2^8}}$$

<u>DFT:n</u>

$$\underline{\underline{X_r}} = \sum_{n=0}^{N_0-1} x_n e^{-jr\Omega_0 n} = \sum_{n=0}^{N_0-1} T\cdot x[n]\, e^{-jr\cdot\frac{2\pi}{N_0}n}$$

$$\underline{\underline{= \frac{1}{32}\sum_{n=0}^{255} e^{-\frac{n}{32}} e^{-j\frac{2\pi rn}{256}}}} \left(= \frac{1}{32}\sum_{n=0}^{255} e^{-(4+j\pi r)\frac{n}{128}}\right)$$

b)



Dvs $\underline{E_{99}} = \frac{1}{2\pi}\int_{-W}^{W} |X(\omega)|^2 d\omega = 2\cdot\frac{1}{2\pi}\int_{0}^{W}\frac{1}{1+\omega^2}d\omega = \frac{1}{\pi}\Big[\arctan\omega\Big]_0^W$

$= \underline{\frac{1}{\pi}\arctan W}$

$\underline{\underline{E_x}} = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_0^{\infty} e^{-2t}dt = \left[\frac{e^{-2t}}{-2}\right]_0^{\infty} = \underline{\frac{1}{2}}$ (= tot. signalenergin)

$E_{99} = 0{,}99\cdot E_x \Rightarrow \frac{1}{\pi}\arctan W = 0{,}99\cdot\frac{1}{2} \Rightarrow$

$\Rightarrow \underline{W} = \tan\left(\frac{0{,}99\pi}{2}\right) \approx \underline{63{,}66 \text{ rad/s}} \Rightarrow$

$\Rightarrow \underline{B} = \frac{W}{2\pi} \approx \underline{10{,}13 \text{ Hz}}$ $\Rightarrow$ Välj $\underline{f_s > 2B} = \underline{20{,}3 \text{ Hz}}$

$\Rightarrow T < \frac{1}{20{,}3}$. Lämpligen är $T \in \mathbb{Q} \Rightarrow$ t.ex $\underline{\underline{T = \frac{1}{21}}}$

I a-uppgiften erhölls $T_0 \in \{5,6,7,8,\ldots\}$

För $T_{0,min}$ erhölls $\underline{\underline{N_0}} = \frac{T_0}{T} = 5\cdot 21 = \underline{\underline{105}}$

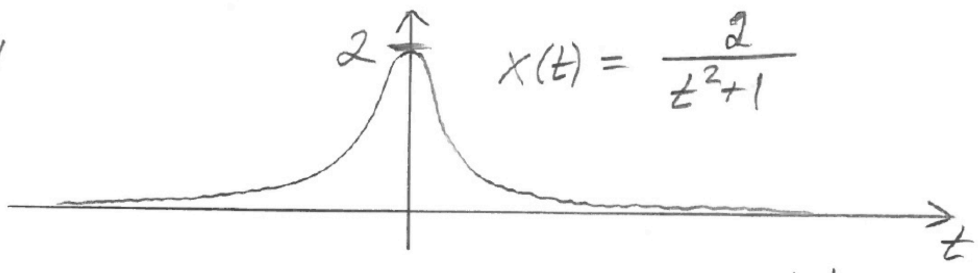
Doch, om man önskar $N_0 = 2^b$ (se uppg. a), så är det lämpligare att välja $\underline{\underline{T = \frac{1}{32} \text{ sek}}}$ & $\underline{\underline{T_0 = 8 \text{ sek}}}$

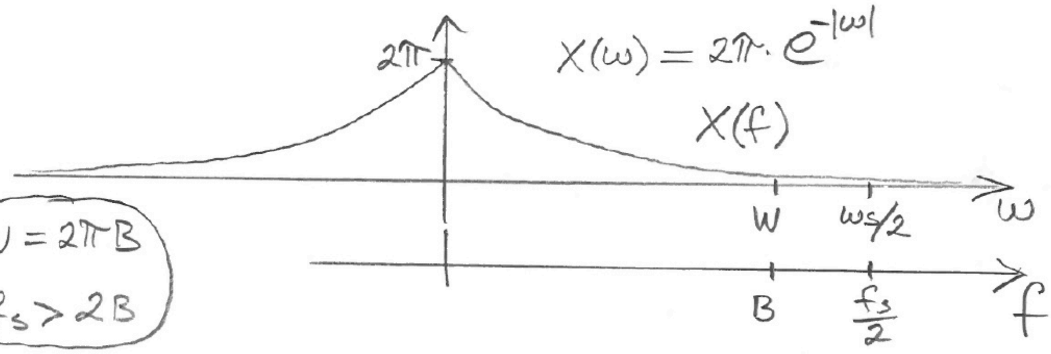
$\Rightarrow \underline{\underline{N_0 = 2^8 = 256}}$, dvs. samma som i uppg. a)

$\Rightarrow$ Samma DFT som i a)

-

8.5–4





$W = 2\pi B$

$f_s > 2B$

a) Se resonemanget i lösningen till 8.5–3 a):

Här: $|X(W)| = \frac{1}{100} \cdot |X(\omega)|_{max} \Rightarrow 2\pi e^{-2\pi B} = \frac{2\pi}{100}$

$\Rightarrow B = \frac{\ln 100}{2\pi} \approx 0{,}733 \text{ Hz}$

$f_s > 2B \Rightarrow T = \frac{1}{f_s} < \frac{1}{2 \cdot 0{,}733} = \frac{1}{1{,}466} \approx 0{,}682 \text{ sek}$

Val av $N_0 \Rightarrow$ val av $f_0 = \frac{1}{T_0}$:

Välj förslagsvis T_0 så att $x(T_0) = \frac{1}{100} x(t)|_{max}$,

dvs. $\frac{2}{T_0^2+1} = \frac{2}{100} \Rightarrow T_0 = \sqrt{99} \approx 10 \text{ sek}$

$N_0 = \frac{T_0}{T} \approx 10 \cdot 1{,}466 = 14{,}66$

Välj $N_0 = 16$ ($= 2^4$, en 2-potens: bra!)

$\Leftrightarrow$ välj $T = \frac{T_0}{N} = \frac{10}{16} = \frac{1}{1{,}6} = 0{,}625 \text{ sek}$

DFT:n $X_r = \sum_{n=0}^{N_0-1} x_n e^{-jr\Omega_0 n} =$ / $x_n = T \cdot x[n] = T\,x(nT)$

$= \frac{10}{16} \cdot \frac{2}{(\frac{10}{16}n)^2+1}$, $\Omega_0 = \frac{2\pi}{N_0}$ /

$$= \frac{10}{16} \sum_{n=0}^{15} \frac{2^9}{100n^2 + 2^8} \cdot e^{-j\frac{\pi r n}{8}}$$

b) $$E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 2\cdot\frac{1}{2\pi}\int_0^{\infty} 4\pi^2 e^{-2\omega} d\omega = 2\pi$$

$$E_{99} = 2\cdot\frac{1}{2\pi}\int_0^{W} 4\pi^2 e^{-2\omega} d\omega = 2\pi\left(1 - e^{-2W}\right)$$

$$E_{99} = 0{,}99\, E_x \Rightarrow 1 - e^{-2W} = 0{,}99 \Rightarrow W \approx 2{,}303 \text{ rad/s}$$

$$\Rightarrow B = \frac{W}{2\pi} \approx 0{,}366 \text{ Hz} \Rightarrow f_s > 2B \approx 0{,}733 \text{ Hz}$$

$$\Rightarrow \underline{T} < \frac{1}{f_s} \approx \underline{1{,}36 \text{ sek}}$$

Uppg. a) $\Rightarrow \underline{T_0 = 10 \text{ sek}}$

$$N_0 = \frac{T_0}{T} > 10\cdot 0{,}733 = 7{,}33$$

Välj gärna $N_0 = 2^b \Rightarrow$ Välj $\underline{\underline{N_0 = 8}}$

$\Leftrightarrow$ Välj $\underline{\underline{T}} = \dfrac{T_0}{N_0} = \dfrac{10}{8} = \underline{\underline{1{,}25 \text{ sek}}}$

<u>DFT:n</u>

$$x_n = T\,x[n] = T\,x(nT) = \frac{10}{8}\,\frac{2}{\left(\frac{10}{8}n\right)^2 + 1} = \frac{10}{8}\cdot\frac{2^7}{100n^2 + 2^6}$$

$$\Omega_0 = \frac{2\pi}{N_0} = \frac{\pi}{4} \text{ rad}$$

$$\Rightarrow \underline{\underline{X_r}} = \sum_{n=0}^{N_0-1} x_n e^{-jr\Omega_0 n} = \underline{\underline{\frac{10}{8}\sum_{n=0}^{7}\frac{2^7}{100n^2+2^6}e^{-j\frac{\pi r n}{4}}}}$$

8.5-5. The widths of $x(t)$ and $g(t)$ are 1 and 2 respectively. Hence the width of the convolved signal is $1+2=3$. This means we need to zero-pad $x(t)$ for 2 secs. and $g(t)$ for 1 sec., making $T_0 = 3$ for both signals. Since $T = 0.125$

$$N_0 = \frac{3}{0.125} = 24$$

N_0 must be a power of 2. Choose $N_0 = 32$. This permits us to adjust T_0 to 4. Hence the final values are $T = 0.125$ and $T_0 = 4$. The samples of $x(t)$ and $g(t)$ are shown in Figure S8.5-5.

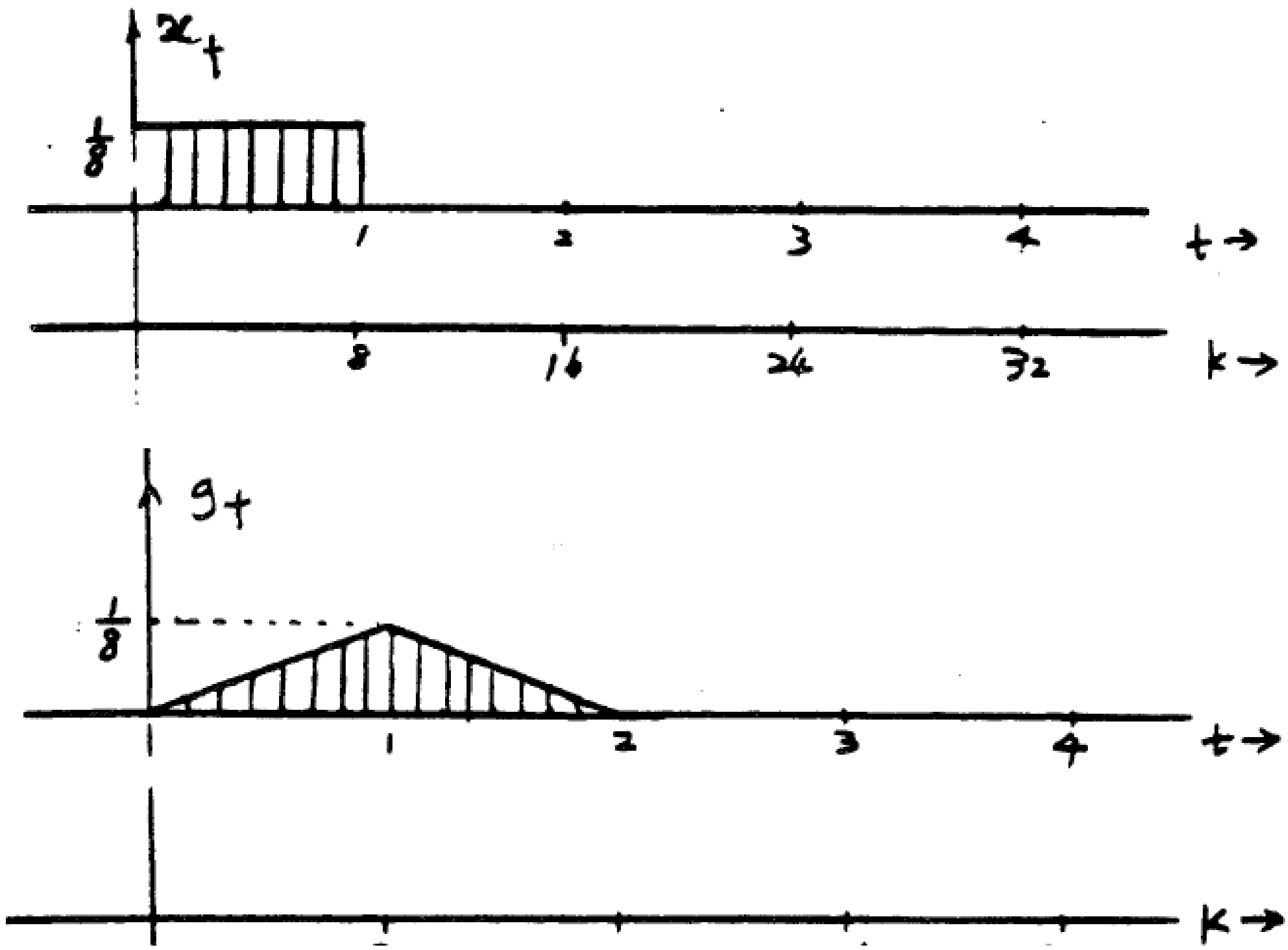


Figure S8.5-5