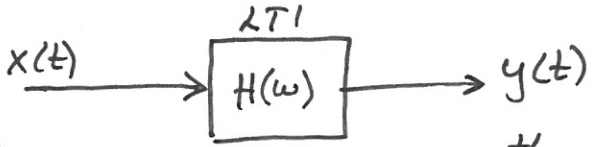
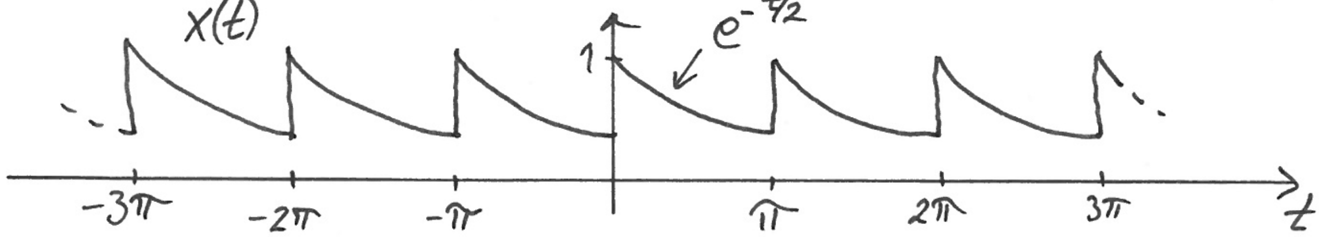
6.4-1



$$H(\omega) = \frac{j\omega}{3-\omega^2+j2\omega}$$



$x(t)$ är π-periodisk ($T_0 = \pi$ sek). LTI-system med frekvensfunktion $H(\omega) \Rightarrow y(t)$ är också π-periodisk,

$$y(t) = \sum_{n=-\infty}^{\infty} \hat{D}_n e^{jn\omega_0 t}, \quad \text{där } \omega_0 = \frac{2\pi}{T_0} = 2 \text{ rad/s och } \underline{\hat{D}_n = D_n \cdot H(n\cdot\omega_0)}$$

där D_n = insignalens komplexa fourierseriekoefficienter, dvs.

$$\underline{D_n} = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jn\omega_0 t}\,dt = \frac{1}{\pi}\int_0^{\pi} e^{-\frac{t}{2}} \cdot e^{-jn\cdot 2t}\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} e^{-(\frac{1}{2}+j2n)t}\,dt = \frac{1}{\pi}\left[\frac{e^{-(\frac{1}{2}+j2n)t}}{-(\frac{1}{2}+j2n)}\right]_0^{\pi}$$

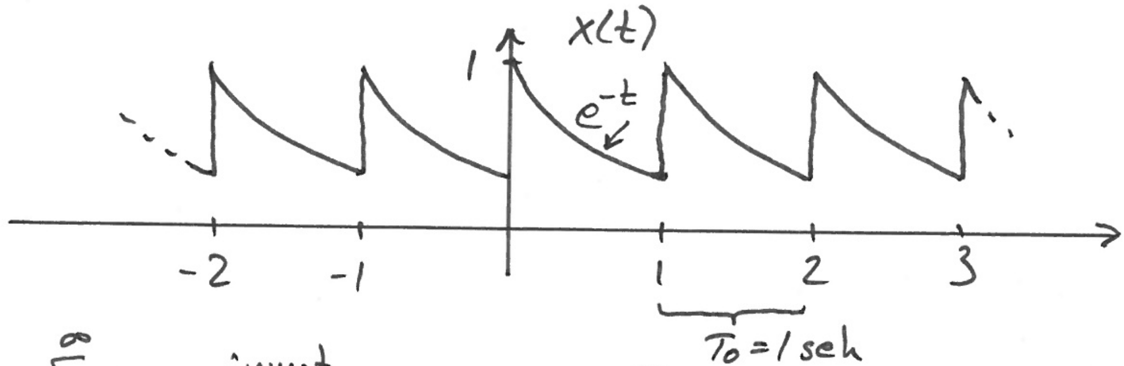
$$= \frac{-2}{\pi}\,\frac{e^{-(\frac{1}{2}+j2n)\pi} - e^{0}}{1+j4n} = /\, e^{j2n\pi} = (e^{j2\pi})^n = 1^n = 1\,/$$

$$= \frac{\frac{2}{\pi}(1-e^{-\pi/2})}{1+j4n} \approx \underline{\underline{\frac{0{,}504}{1+j4n}}}$$

$$\underline{H(n\cdot\omega_0)} = /\,\omega_0 = 2 \text{ rad/s}\,/ = \frac{j2n}{3-(2n)^2+j2\cdot 2n} = \underline{\frac{j2n}{3-4n^2+j4n}}$$

$$\underline{\underline{\text{Svar:}}} \quad \underline{\underline{y(t) = \sum_{n=-\infty}^{\infty} \hat{D}_n e^{j2nt}}}, \quad \text{där } \underline{\underline{\hat{D}_n = \frac{j1{,}008n}{(1+j4n)(3-4n^2+j4n)}}}$$

6.4-3 a)



$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}, \quad \text{där } \omega_0 = \frac{2\pi}{T_0} = 2\pi \text{ rad/s}, \text{ och}$$

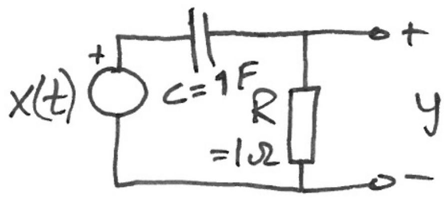
$$\underline{D_n} = \frac{1}{T_0}\int_{T_0} x(t) e^{-jn\omega_0 t} dt = \frac{1}{1}\int_0^1 e^{-t} \cdot e^{-j2\pi n t} dt$$

$$= \int_0^1 e^{-(1+j2\pi n)t} dt = \left[\frac{e^{-(1+j2\pi n)t}}{-(1+j2\pi n)}\right]_0^1$$

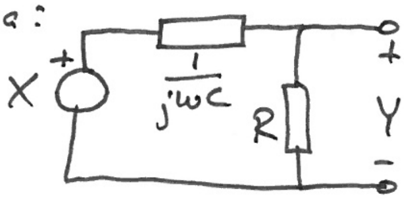
$$= \frac{e^{-1} \cdot e^{-j2\pi n} - e^0}{-(1+j2\pi n)} = \big/ e^{-j2\pi n} = (e^{-j2\pi})^n = 1^n = 1 \big/$$

$$= \underline{\underline{\frac{1 - e^{-1}}{1 + j2\pi n}}}$$

b)



Komplexschema:



Spänningsdelning: $Y = X \cdot \dfrac{R}{\frac{1}{j\omega C} + R} \Rightarrow$

$$\underline{H(\omega)} = \frac{Y}{X} = \frac{j\omega RC}{1 + j\omega RC} = \underline{\frac{j\omega}{1 + j\omega}}$$

Det elektriska nätet (den elektriska kretsen) utgör ett LTI-system

$$\Rightarrow \underline{\underline{y(t) = \sum_{n=-\infty}^{\infty} \hat{D}_n e^{jn\omega_0 t}}}, \quad \text{där } \hat{D}_n = D_n \cdot H(n \cdot \omega_0),$$

där $H(n \cdot \omega_0) = H(2\pi n) = \dfrac{j2\pi n}{1 + j2\pi n}$, dvs.

$$\underline{\underline{\hat{D}_n = \frac{(1 - e^{-1}) \cdot j2\pi n}{(1 + j2\pi n)^2}}}$$