9.2-3 Beräkna $X[\Omega] = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$

a) $x[n]=\delta[n] \Rightarrow \underline{\underline{X[\Omega]}} = \sum_{n=-\infty}^{\infty}\delta[n]e^{-j\Omega n} = e^{-j\Omega\cdot 0} = \underline{\underline{1}}$

b) $x[n]=\delta[n-k] \Rightarrow$

$\underline{\underline{X[\Omega]}} = \sum_{n=-\infty}^{\infty} \delta[n-k] e^{-j\Omega n} = \underline{\underline{e^{-j\Omega k}}}$ (Enhetsimpulsen finns vid $n=k$)

c) $x[n]=\gamma^n u[n-1]$

$\underline{\underline{X[\Omega]}} = \sum_{n=1}^{\infty} \gamma^n e^{-j\Omega n} = \sum_{n=1}^{\infty} (\gamma e^{-j\Omega})^n = \Big/ |\gamma e^{-j\Omega}| = |\gamma|<1 \text{ enl. uppg.} \Big/$

$= (\gamma e^{-j\Omega})^1 \cdot \frac{1}{1-\gamma e^{-j\Omega}} = \underline{\underline{\frac{\gamma}{e^{j\Omega}-\gamma}}}$

d) $x[n]=\gamma^n u[n+1]$

$\underline{\underline{X[\Omega]}} = \sum_{n=-1}^{\infty} \gamma^n e^{-j\Omega n} = \sum_{n=-1}^{\infty} (\gamma e^{-j\Omega})^n = \Big/ |\gamma|<1 \Big/$

$= (\gamma e^{-j\Omega})^{-1} \frac{1}{1-\gamma e^{-j\Omega}} = \underline{\underline{\frac{e^{j2\Omega}}{\gamma(e^{j\Omega}-\gamma)}}}$

e) $x[n]=(-\gamma)^n u[n]$

$\underline{\underline{X[\Omega]}} = \sum_{n=0}^{\infty} (-\gamma e^{-j\Omega})^n = \Big/ |\gamma|<1 \Big/ = \frac{1}{1-(-\gamma e^{-j\Omega})} = \underline{\underline{\frac{e^{j\Omega}}{e^{j\Omega}+\gamma}}}$

f) $x[n] = \gamma^{|n|} = \begin{cases} \gamma^{-n}; & n<0 \\ \gamma^{n}; & n\geq 0 \end{cases} = \gamma^n u[n] + \gamma^{-n} u[-n-1]$

$\underline{\underline{X[\Omega]}} = \sum_{n=0}^{\infty} \gamma^n e^{-j\Omega n} + \sum_{n=-\infty}^{-1} \gamma^{-n} e^{-j\Omega n} = \sum_{n=0}^{\infty} (\gamma e^{-j\Omega})^n + \sum_{n=1}^{\infty} (\gamma e^{j\Omega})^n$

$= \Big/ |\gamma|<1 \Big/ = \frac{1}{1-\gamma\cdot e^{-j\Omega}} + \gamma\cdot e^{j\Omega} \frac{1}{1-\gamma\cdot e^{j\Omega}}$

$= \frac{e^{j\Omega}}{e^{j\Omega}-\gamma} + \frac{\gamma e^{j\Omega}}{1-\gamma e^{j\Omega}} = \underline{\underline{\frac{e^{j\Omega}(1-\gamma^2)}{(e^{j\Omega}-\gamma)(1-\gamma e^{j\Omega})}}}$

9.2-4 $\quad x[n] = \frac{1}{2\pi}\int_{2\pi} X[\Omega] e^{j\Omega n} d\Omega$

a) $X[\Omega] = e^{jk\Omega}, \quad k \in \mathbb{Z}$

$$\underline{x[n]} = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{jk\Omega} e^{j\Omega n} d\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(k+n)\Omega} d\Omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j(k+n)\Omega}}{j(k+n)}\right]_{-\pi}^{\pi} = \frac{1}{(k+n)\pi}\,\frac{e^{j(k+n)\pi} - e^{-j(k+n)\pi}}{2j}$$

$$= \frac{\sin((k+n)\pi)}{(k+n)\pi} = \begin{cases} 0; & k+n \neq 0 \\ 1; & k+n = 0 \end{cases} = \underline{\underline{\delta[n+k]}}$$

$$\left(= \operatorname{sinc}((k+n)\pi) = \operatorname{sinc}_N(k+n)\right)$$

b) $X[\Omega] = \cos(k\Omega) = \frac{1}{2}e^{jk\Omega} + \frac{1}{2}e^{-jk\Omega}; \quad k \in \mathbb{Z}$

Uppg. a) $\Rightarrow$ $\underline{\underline{x[n] = \frac{1}{2}\left(\delta[n+k] + \delta[n-k]\right)}}$

c) $X[\Omega] = \cos^2\left(\frac{\Omega}{2}\right) = \frac{1}{2}\left(1 + \cos(\Omega)\right) = \underbrace{\frac{1}{2}}_{=X_1[\Omega]} + \underbrace{\frac{1}{2}\cos(\Omega)}_{=X_2[\Omega]}$

$X_1[\Omega] = \frac{1}{2}e^{j0\Omega} \Rightarrow /k=0$ i a$/ \Rightarrow x_1[n] = \frac{1}{2}\delta[n]$

$\Longrightarrow /k=1$ i b)$/$ $x_2[n] = \frac{1}{2}\cdot\frac{1}{2}\left(\delta[n+1] + \delta[n-1]\right)$

$$\Longrightarrow \underline{\underline{x[n] = \frac{1}{2}\delta[n] + \frac{1}{4}\left(\delta[n+1] + \delta[n-1]\right)}}$$

e) $X[\Omega] = 2\pi\delta(\Omega-\Omega_0), \; |\Omega| \le \pi$

$$\underline{x[n]} = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\delta(\Omega-\Omega_0)e^{j\Omega n}\,d\Omega \overset{\text{Dirac vid } \Omega=\Omega_0}{=} \underline{e^{j\Omega_0 n}}$$

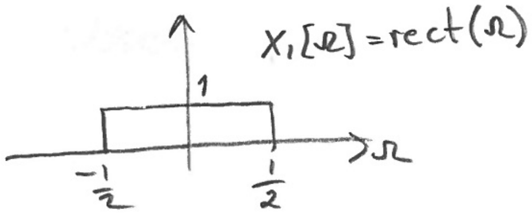
f) $X[\Omega] = \pi\left(\delta(\Omega-\Omega_0) + \delta(\Omega+\Omega_0)\right); \; |\Omega| \le \pi$

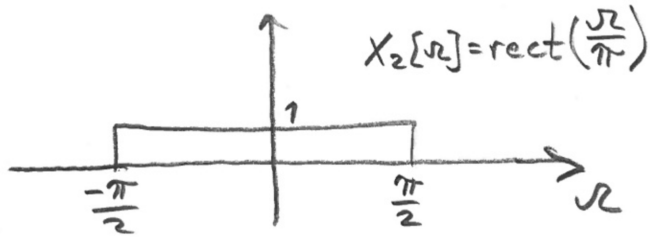
$$\underline{x[n]} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \pi\delta(\Omega-\Omega_0)e^{j\Omega n}\,d\Omega + \frac{1}{2\pi}\int_{-\pi}^{\pi} \pi\delta(\Omega+\Omega_0)e^{j\Omega n}\,d\Omega$$

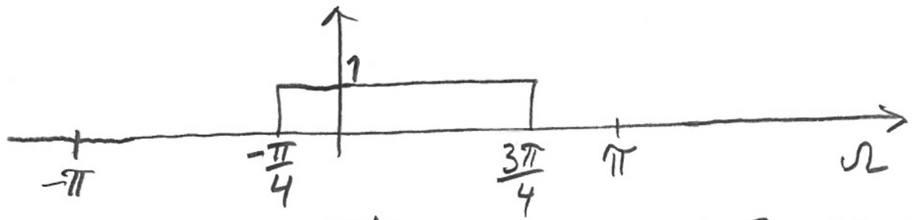
$$= \frac{1}{2}e^{j\Omega_0 n} + \frac{1}{2}e^{-j\Omega_0 n} = \underline{\cos(\Omega_0 n)}$$

9.2-5 Ekv. 9.29: $x[n] = \frac{1}{2\pi}\int_{2\pi} X[\Omega]e^{j\Omega n}\,d\Omega$

$$X[\Omega] = \text{rect}\left(\frac{\Omega-\frac{\pi}{4}}{\pi}\right); \; |\Omega| \le \pi$$







$X[\Omega] = X_2\left[\Omega-\frac{\pi}{4}\right]$
$= \text{rect}\left(\frac{\Omega-\pi/4}{\pi}\right)$

$$\underline{x[n]} = \frac{1}{2\pi}\int_{-\pi/4}^{3\pi/4} 1e^{j\Omega n}\,d\Omega = \frac{1}{2\pi}\left[\frac{e^{j\Omega n}}{jn}\right]_{-\pi/4}^{3\pi/4} =$$

$$= \frac{e^{j\frac{3\pi n}{4}} - e^{-j\frac{\pi n}{4}}}{j2\pi n} = \frac{e^{j\frac{\pi n}{4}}\left(e^{j\frac{\pi n}{2}} - e^{-j\frac{\pi n}{2}}\right)}{\pi n \cdot 2j} =$$

$$= \frac{e^{j\frac{\pi n}{4}} \cdot \sin\left(\frac{\pi}{2}n\right)}{2 \cdot \frac{\pi}{2}n} = \underline{\frac{e^{j\frac{\pi n}{2}}}{2}\text{sinc}\left(\frac{\pi}{2}n\right)} = \underline{\frac{e^{j\frac{\pi n}{2}}}{2}\text{sinc}_N\left(\frac{n}{2}\right)}$$

$$\left(|X[-\Omega]| \ne |X[\Omega]| \Rightarrow x[n] \in \mathbb{C}\,!\right)$$

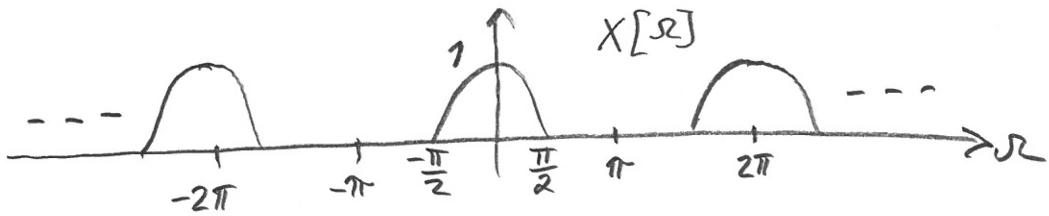
9.2-6

a)&b): $\underline{X[\Omega]} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=0}^{N_0} \underbrace{a^n e^{-j\Omega n}}_{=(ae^{-j\Omega})^n}$

$$= \frac{1-(ae^{-j\Omega})^{N_0+1}}{1-ae^{-j\Omega}} = \underline{\underline{\frac{a^{N_0+1}e^{-j\Omega N_0} - e^{j\Omega}}{a - e^{j\Omega}}}}$$

9.2-9

a)



$$X[\Omega] = \begin{cases} \cos(\Omega) \; ; & |\Omega| \le \frac{\pi}{2} \\ X[\Omega + 2\pi] & \forall \Omega \end{cases}$$

$$\underline{\underline{x[n]}} = \frac{1}{2\pi}\int_{2\pi} X[\Omega] e^{j\Omega n} d\Omega = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \cos(\Omega) e^{j\Omega n} d\Omega$$

$$= \Big/ \cos(\Omega) = \frac{e^{j\Omega}+e^{-j\Omega}}{2} \Big/ = \frac{1}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(e^{j\Omega(n+1)} + e^{j\Omega(n-1)}\right) d\Omega$$

$$= \frac{1}{4\pi}\left[\frac{e^{j\Omega(n+1)}}{j(n+1)} + \frac{e^{j\Omega(n-1)}}{j(n-1)}\right]_{-\pi/2}^{\pi/2} =$$

$$= \frac{1}{2\pi}\left(\frac{e^{j\frac{\pi}{2}(n+1)} - e^{-j\frac{\pi}{2}(n+1)}}{(n+1)\cdot 2j} + \frac{e^{j\frac{\pi}{2}(n-1)} - e^{-j\frac{\pi}{2}(n-1)}}{(n-1)\cdot 2j}\right)$$

$$= \frac{1}{2\pi}\left(\frac{\sin(\frac{\pi}{2}n + \frac{\pi}{2})}{n+1} + \frac{\sin(\frac{\pi}{2}n - \frac{\pi}{2})}{n-1}\right) = \Big/ \sin(\alpha \pm \tfrac{\pi}{2}) = \pm\cos(\alpha) \Big/$$

$$= \frac{1}{2\pi}\left(\frac{1}{n+1} - \frac{1}{n-1}\right)\cos\left(\frac{\pi}{2}n\right) = \underline{\underline{\frac{\cos(\frac{\pi}{2}n)}{\pi(1-n^2)}}}$$

9.2-10

a) $\underline{X[\Omega]} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \Big/ x[n] = \delta[n+2] + 2\delta[n-1] + 3\delta[n] + 2\delta[n-1] + \delta[n-2] \Big/$

$$= \sum_{n=-\infty}^{\infty} \left(\delta[n+2] e^{-j\Omega n} + 2\delta[n+1] e^{-j\Omega n} + 3\delta[n] e^{-j\Omega n} + 2\delta[n-1] e^{-j\Omega n} + \delta[n-2] e^{-j\Omega n} \right)$$

$$= e^{j2\Omega} + 2e^{j\Omega} + 3e^{0} + 2e^{-j\Omega} + e^{-j2\Omega} =$$

$$= 3 + 4 \cdot \frac{e^{j\Omega} + e^{-j\Omega}}{2} + 2 \cdot \frac{e^{j2\Omega} + e^{-j2\Omega}}{2}$$

$$\underline{\underline{= 3 + 4\cos(\Omega) + 2\cos(2\Omega)}}$$

b) $\underline{X[\Omega]} = \sum_{n=1}^{5} x[n] e^{-j\Omega n} = /\text{som i a)}/$

$$= e^{-j\Omega} + 2e^{-j2\Omega} + 3e^{-j3\Omega} + 2e^{-j4\Omega} + e^{-j5\Omega}$$

$$= e^{-j3\Omega}\left(3 + 2\left(e^{j\Omega} + e^{-j\Omega}\right) + \left(e^{j2\Omega} + e^{-j2\Omega}\right)\right)$$

$$\underline{\underline{= e^{-j3\Omega}\left(3 + 4\cos(\Omega) + 2\cos(2\Omega)\right)}}$$

Anm: Låt $x_a[n] = x[n]$ i uppg a) & $x_b[n] = x[n]$ i uppg. b)

$$\Rightarrow \quad x_b[n] = x_a[n-3]$$

$$= \underline{\underline{X_b[\Omega]}} = \sum_{n=-\infty}^{\infty} x_b[n] e^{-j\Omega n} = \Big/ x_b[n] = x_a[n-3] \text{ låt } m = n-3 \Big/$$

$$= \sum_{m=-\infty}^{\infty} x_a[m] e^{-j\Omega(m+3)}$$

$$= e^{-j3\Omega} \sum_{m=-\infty}^{\infty} x_a[m] e^{-j\Omega m} = \underline{\underline{e^{-j3\Omega} \cdot X_a[\Omega]}}$$

(Ges även direkt av egenskapen $x[n-k] \Leftrightarrow e^{-jk\Omega} X[\Omega]$)

c) $\underline{X[\Omega]} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=-3}^{3} 3n \cdot e^{-j\Omega n}$

$= -9e^{j3\Omega} - 6e^{j2\Omega} - 3e^{j\Omega} + 3e^{-j\Omega} + 6e^{-j2\Omega} + 9e^{-j3\Omega}$

$= 6j\left(\frac{e^{j\Omega} - e^{-j3\Omega}}{2j} + 2 \cdot \frac{e^{j2\Omega} - e^{-j2\Omega}}{2j} + 3\,\frac{e^{j3\Omega} - e^{-j3\Omega}}{2j}\right)$

$= \underline{6j\left(\sin(\Omega) + 2\sin(2\Omega) + 3\sin(3\Omega)\right)}$

d) $\underline{X[\Omega]} = \sum_{n=-2}^{2} x[n] e^{-j\Omega n} =$

$= 4e^{j2\Omega} + 2e^{j\Omega} + 2e^{-j\Omega} + 4e^{-j2\Omega}$

$= \underline{4\cos(\Omega) + 8\cos(2\Omega)}$

9.2-11

a) $\underline{x[n]} = \frac{1}{2\pi} \int_{2\pi} X[\Omega] e^{j\Omega n} d\Omega$

$= \frac{1}{2\pi} \int_{-\Omega_0}^{\Omega_0} 1 \cdot \underbrace{e^{-j\Omega n_0} \cdot e^{j\Omega n}}_{= e^{j\Omega(n-n_0)}} d\Omega$

$= \frac{1}{2\pi} \left[\frac{e^{j\Omega(n-n_0)}}{j(n-n_0)}\right]_{-\Omega_0}^{\Omega_0} = \frac{\sin(\Omega_0(n-n_0))}{\pi(n-n_0)}$

$= \underline{\frac{\Omega_0}{\pi} \operatorname{sinc}(\Omega_0(n-n_0))} = \underline{\frac{\Omega_0}{\pi} \operatorname{sinc}_N\left(\frac{\Omega_0}{\pi}(n-n_0)\right)}$

b) $\underline{\underline{x[n]}} = \frac{1}{2\pi}\int_{2\pi} X[\Omega]e^{j\Omega n}d\Omega$

$$= \frac{1}{2\pi}\left(\int_{-\Omega_0}^{0} 1\cdot e^{j\frac{\pi}{2}}\cdot e^{j\Omega n}d\Omega + \int_{0}^{\Omega_0} 1\cdot e^{-j\frac{\pi}{2}}\cdot e^{j\Omega n}d\Omega\right)$$

$$= \frac{1}{2\pi}\left(\left[\frac{j\cdot e^{j\Omega n}}{jn}\right]_{-\Omega_0}^{0} + \left[\frac{-je^{j\Omega n}}{jn}\right]_{0}^{\Omega_0}\right)$$

$$= \frac{1}{2\pi n}\left(e^0 - e^{-j\Omega_0 n} - (e^{j\Omega_0 n} - e^0)\right)$$

$$= \frac{1}{\pi n}\left(1 - \frac{e^{j\Omega_0 n} + e^{-j\Omega_0 n}}{2}\right) = \underline{\underline{\frac{1-\cos(\Omega_0 n)}{\pi n}}}$$

Kommentar, a) & b): Här ser vi tydligt hur de två signalerna skiljer sig åt, p.g.a. att deras respektive fasspektrum är olika — trots att de har samma amplitudspektrum.

9.2-14

a) $X[\Omega] = \Omega + \pi$: $\underline{\text{Nej}}$, är ingen fouriertransform, ty $X[\Omega]$ är inte 2π-periodisk.

b) $X[\Omega] = j + \pi$: Ja, $X[\Omega]$ = konstant, kan vara en f.-transform.

c) $X[\Omega] = \sin(10\Omega)$: Har period $\frac{2\pi}{10}$, dvs. $X[\Omega] = X[\Omega + 2\pi]$
$\Rightarrow X[\Omega]$ kan vara en fouriertransform

d) $X[\Omega] = \sin\left(\frac{\Omega}{10}\right)$: Har period $\frac{2\pi}{1/10} = 20\pi$, dvs. den är <u>inte</u> 2π-periodisk $\Rightarrow X[\Omega]$ kan <u>inte</u> vara en fouriertransform

e) $X[\Omega] = \delta(\Omega)$ är <u>inte</u> 2π-periodisk $\Rightarrow$ den kan <u>inte</u> vara en fouriertransform.

9.3-1 Använd sambanden $\gamma^n u[n] \Leftrightarrow \dfrac{e^{j\Omega}}{e^{j\Omega}-\gamma}$; $|\gamma|<1$,

$n\gamma^n u[n] \Leftrightarrow \dfrac{\gamma e^{j\Omega}}{(e^{j\Omega}-\gamma)^2}$; $|\gamma|<1$ & $x[n-k] \Leftrightarrow X[\Omega]e^{-jk\Omega}$ $k \in \mathbb{Z}$

a) Här behövs transformparet $u[n] = \dfrac{e^{j\Omega}}{e^{j\Omega}-1} + \pi\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k)$

$x[n] = u[n] - u[n-9] \Rightarrow$

$$X[\Omega] = \frac{e^{j\Omega}}{e^{j\Omega}-1} + \pi\delta(\Omega) - \left(\frac{e^{j\Omega}}{e^{j\Omega}-1} + \underbrace{\pi\delta(\Omega)\Big)e^{-j9\Omega}}_{=\pi\cdot\delta(\Omega)\cdot e^0}$$

$(|\Omega| \leq \pi)$

$$= \frac{e^{j\Omega}}{e^{j\Omega}-1}\left(1-e^{-j9\Omega}\right)$$

$$= \frac{e^{j\Omega}\cdot e^{-j\frac{9}{2}\Omega}\left(e^{j\frac{9}{2}\Omega}-e^{-j\frac{9}{2}\Omega}\right)\cdot 2j}{e^{j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}}-e^{-j\frac{\Omega}{2}}\right)\cdot 2j} =$$

$$= e^{-j4\Omega}\cdot\frac{\sin(4{,}5\Omega)}{\sin(0{,}5\Omega)} \qquad \forall\Omega$$

b) $x[n] = a^{n-m}u[n-m] = x_1[n-m]$ där $x_1[n] = a^n u[n]$

$$\Rightarrow X_1[\Omega] = \frac{e^{j\Omega}}{e^{j\Omega}-a}$$

$$\Rightarrow X[\Omega] = X_1[\Omega]e^{-jm\Omega} = \frac{e^{j\Omega(1-m)}}{e^{j\Omega}-a}$$

c) $x[n] = a^{n-3}(u[n] - u[n-10])$

$= a^{-3} \cdot a^n u[n] - a^7 \cdot a^{n-10} u[n-10]$

$$\Rightarrow X[\Omega] = a^{-3} \cdot \frac{e^{j\Omega}}{e^{j\Omega}-a} - a^7 \frac{e^{j\Omega}}{e^{j\Omega}-a} \cdot e^{-j10\Omega}$$

$$= \frac{a^{-3} e^{j\Omega}(1 - a^{10} e^{-j10\Omega})}{e^{j\Omega} - a}$$

d) $x[n] = a^{n-m} u[n] = a^{-m} \cdot a^n u[n]$

$$\Rightarrow X[\Omega] = \frac{a^{-m} \cdot e^{j\Omega}}{e^{j\Omega} - a}$$

e) $x[n] = a^n \cdot u[n-m] = a^m \cdot a^{n-m} \cdot u[n-m]$

$$\Rightarrow X[\Omega] = a^m \cdot \frac{e^{j\Omega}}{e^{j\Omega}-a} \cdot e^{-j\Omega m} = \frac{a^m e^{j\Omega(1-m)}}{e^{j\Omega} - a}$$

f) $X[\Omega] = \dfrac{a\, e^{j\Omega(1-m)}}{(e^{j\Omega}-a)^2}$

g) $X[\Omega] = \dfrac{e^{j\Omega}(a - m e^{j\Omega} + ma)}{(e^{j\Omega} - a)^2}$

h) $X[\Omega] = \dfrac{a + m(e^{j\Omega} - a)}{(e^{j\Omega} - a)^2} \cdot e^{j\Omega(1-m)}$

9.3-4

a) $x[n] = a^n \cos(\Omega_0 n) u[n] = \frac{1}{2}\left(a e^{j\Omega_0}\right)^n u[n] + \frac{1}{2}\left(a e^{-j\Omega_0}\right)^n u[n]$

Tab. 9.1:2 $\Rightarrow$

$$X[\Omega] = \frac{1}{2}\left(\frac{e^{j\Omega}}{e^{j\Omega} - a e^{j\Omega_0}} + \frac{e^{j\Omega}}{e^{j\Omega} - a e^{-j\Omega_0}}\right)$$

$$= \frac{e^{j\Omega}}{2}\left(\frac{e^{-j\Omega_0}}{e^{j(\Omega-\Omega_0)} - a} + \frac{e^{j\Omega_0}}{e^{j(\Omega+\Omega_0)} - a}\right)$$

$$= \frac{e^{j\Omega}\left(e^{j\Omega} - a e^{-j\Omega_0} + e^{j\Omega} - a e^{j\Omega_0}\right)}{2\left(e^{j(\Omega-\Omega_0)} - a\right)\left(e^{j(\Omega+\Omega_0)} - a\right)}$$

$$= \frac{e^{j\Omega}\left(e^{j\Omega} - a\cdot\cos(\Omega_0)\right)}{e^{j2\Omega} - 2a e^{j\Omega}\cos(\Omega_0) + a^2}$$

$$\left(\text{Jämför med } X[\Omega] = X[z]\Big|_{z=e^{j\Omega}} = \frac{z\left(z - a\cdot\cos(\Omega_0)\right)}{z^2 - 2a\cos(\Omega_0) z + a^2}\Bigg|_{z=e^{j\Omega}}\right)$$

Tabell 5.1: 11a

b) $x[n] = n^2 a^n u[n] = n^2 \cdot x_1[n]$ där $x_1[n] = a^n u[n]$

Tab. 9.2 $\Rightarrow$ $X[\Omega] = \underbrace{j\frac{d}{d\Omega}\left(\underbrace{j\frac{dX_1[\Omega]}{d\Omega}}_{\text{Ty Mult. med } n}\right)}_{\text{Ty Mult. med } n}$

där $X_1[\Omega] = \dfrac{e^{j\Omega}}{e^{j\Omega} - a}$

$$\Rightarrow X[\Omega] = j\frac{d}{d\Omega}\left(\frac{a e^{j\Omega}}{\left(e^{j\Omega} - a\right)^2}\right) = \frac{a e^{j\Omega}\left(e^{j\Omega} + a\right)}{\left(e^{j\Omega} - a\right)^3}$$

$$\left(\text{Här används kvotregeln } \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}\right)$$