Fråga	$x(t)\Leftrightarrow C_n,\, D_n$			$x(t)\Leftrightarrow X(\omega)$			$x(t)\Leftrightarrow X(s)$			$x[n]\Leftrightarrow X[z]$			$x[n]\Leftrightarrow X[\Omega]$		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Svar	**c**	**b**	**b**	**b**	**c**	**b**	**b**	**a**	**d**	**c**	**b**	**a**	**c**	**b**	**b**

1 c) $x(t)$ är en reellvärd signal

2 $x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$ $\quad (x(t) \Leftrightarrow D_n)$

$\Rightarrow x(-t) = \sum_{k=-\infty}^{\infty} D_k e^{jk\omega_0(-t)} = \Big/ \begin{matrix} n=-k \\ \Rightarrow k=-n \end{matrix} \Big/ = \sum_{n=-\infty}^{\infty} D_{-n} e^{jn\omega_0 t}$

$\Rightarrow x(-t) \Leftrightarrow D_{-n} = \hat{D}_n$ dvs. samband b) är korrekt

3 b) $\tau = 1$

$D_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}\,dt \overset{\omega_0 = \frac{2\pi}{T_0}}{=} \frac{1}{4}\int_{-\tau}^{\tau} 4\pi e^{-jn\frac{2\pi}{4}\cdot t}\,dt$

$= \pi\left[\frac{e^{-jn\frac{\pi}{2}t}}{-jn\pi/2}\right]_{-\tau}^{\tau} = \frac{2}{jn}\left(e^{jn\frac{\pi}{2}\tau} - e^{-jn\frac{\pi}{2}\tau}\right)$

$\Rightarrow D_1 = 2\cdot 2\cdot \frac{e^{j\frac{\pi}{2}\tau} - e^{-j\frac{\pi}{2}\tau}}{2j} = 4\cdot \sin\left(\frac{\pi}{2}\tau\right) = 4$ Enl. uppg.

$\Rightarrow \sin\left(\frac{\pi}{2}\tau\right) = 1 \Rightarrow \frac{\pi}{2}\tau = \frac{\pi}{2}(+k\cdot 2\pi) \Rightarrow \tau = 1$ (Enda rimliga lösn: $k=0$)

4 b) Om man ökar en signals tidsutbredning, så minskar dess frekvensutbredning (och tvärtom)

5 c) $\tilde{X}(\omega) = -j\omega^3 X(\omega)$

$\underline{x(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underline{X(\omega)} e^{j\omega t}\,d\omega \quad (x(t) \Leftrightarrow X(\omega))$

$\Rightarrow \underline{\tilde{x}(t)} = \frac{d^3 x(t)}{dt^3} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\frac{d^3}{dt^3}\left(e^{j\omega t}\right)d\omega$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{X(\omega)(j\omega)^3}_{= \tilde{X}(\omega) = j^2\cdot j\cdot \omega^3 X(\omega) = -j\omega^3 X(\omega)} e^{j\omega t}\,d\omega$

6 b) $x(t) = e^{-2|t|} \quad \left(= e^{2t}u_0(-t) + e^{-2t}u(t) \right)$

$$\Rightarrow \underline{X(\omega)} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{0-} e^{2t} e^{-j\omega t} dt + \int_{0-}^{\infty} e^{-2t} e^{-j\omega t} dt$$

$$= \left[\frac{e^{(2-j\omega)t}}{2-j\omega} \right]_{-\infty}^{0-} + \left[\frac{e^{(-2-j\omega)t}}{-2-j\omega} \right]_{0-}^{\infty}$$

$$= \Big/ \lim_{t \to -\infty} e^{2t} = 0 , \lim_{t \to \infty} e^{-2t} = 0 \Big/$$

$$= \frac{e^0 - 0}{2 - j\omega} + \underbrace{\frac{0 - e^0}{-2-j\omega}}_{= \frac{1}{2+j\omega}} = \frac{(2+j\omega) + (2-j\omega)}{(2-j\omega)(2+j\omega)}$$

$$= \underline{\underline{\frac{4}{4+\omega^2}}}$$

7 b) $X(\omega) = X(s)$ längs $j\omega$-axeln, dvs. $X(\omega) = X(s)\big|_{s=j\omega}$

8 a) $\tilde{x}(t) = x(t) - x(t+7)$

$$\tilde{X}(s) = X(s)(1 - e^{7s}) = X(s) - X(s)e^{7s}$$

$$\Rightarrow \tilde{x}(t) = x(t) - \mathcal{L}^{-1}\{X(s)e^{7s}\}$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \quad \Rightarrow$$

$$X(s)e^{7s} = \int_{-\infty}^{\infty} x(\lambda) \underbrace{e^{-s\lambda} \cdot e^{7s}}_{= e^{-s(\lambda-7)}} d\lambda = \Big/ \begin{matrix} t = \lambda - 7 \Rightarrow \\ \lambda = t+7,\ d\lambda = dt \end{matrix} \Big/$$

$$= \int_{-\infty}^{\infty} \underline{\underline{x(t+7)}} e^{-st} dt \quad , \text{ dvs } x(t+7) \Leftrightarrow X(s)e^{7s}$$

$$\Rightarrow \underline{\underline{\tilde{x}(t) = x(t) - x(t+7)}}$$

(9) d) $x(t) = e^{-3t}u(t) - e^{-2t}u_0(-t)$

$$X(s) = \frac{2s+5}{s^2+5s+6} = \frac{2s+5}{(s+3)(s+2)} = /\text{Part. br. uppd.}/$$

$$= \frac{1}{s+3} + \frac{1}{s+2}, \quad \text{Konv.omr. } -3 < \mathrm{Re}\{s\} < -2$$

$\Rightarrow X_1(s) = \frac{1}{s+3}$ har konv.omr. $\mathrm{Re}\{s\} > -3$

(till höger om den singulära punkten $s=-3$ i s-planet)

& $X_2(s) = \frac{1}{s+2}$ har konv.omr. $\mathrm{Re}\{s\} < -2$

(till vänster om den singulära punkten $s=-2$ i s-planet)

Uppgiftshjälpen $\Rightarrow x_1(t) = e^{-3t}u(t)$, $x_2(t) = -e^{-2t}u_0(-t)$

$$\Rightarrow \underline{x(t)} = x_1(t) + x_2(t) = \underline{e^{-3t}u(t) - e^{-2t}u_0(-t)}$$

(10) c) $(-2)^n u[n] \Leftrightarrow \frac{z}{z+2}, |z| < 2$

Konvergensområdet $|z|<2$ innebär att enhetscirkeln ligger i konvergensområdet, vilket innebär att tidssignalen är absolutsummerbar, men $(-2)^n u[n]$ är inte absolutsummerbar.

(11) b) $n \cdot x[n] \Leftrightarrow -z\frac{dX[z]}{dz}$

$$\underline{X[z]} = \sum_{n=-\infty}^{\infty} \underline{x[n]} \cdot z^{-n} \qquad (x[n] \Leftrightarrow X[z])$$

$$\Rightarrow \frac{dX[z]}{dz} = \sum_{n=-\infty}^{\infty} x[n] \cdot \frac{d}{dz}(z^{-n}) = \sum_{n=-\infty}^{\infty} x[n] \cdot (-n) \cdot z^{-n-1}$$

$$= -\frac{1}{z}\sum_{n=-\infty}^{\infty} n \cdot x[n] z^{-n}$$

$$\Rightarrow \underline{-z\frac{dX[z]}{dz}} = \sum_{n=-\infty}^{\infty} \underline{n \cdot x[n]} z^{-n}, \text{ dvs. } \underline{n \cdot x[n] \Leftrightarrow -z\frac{dX[z]}{dz}}$$

(12) a) $X[z] = \dfrac{z^2}{z^2-4}, \quad |z|>2$

$$x[n] = \sum_{k=0}^{\infty} 2^{2k}\delta[n-2k] \quad \Rightarrow$$

$$\underline{\underline{X[z]}} = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n} = \sum_{n=-\infty}^{\infty}\left(\sum_{k=0}^{\infty} 2^{2k}\delta[n-2k]\right) z^{-n}$$

$$= \sum_{k=0}^{\infty} 2^{2k} \underbrace{\sum_{n=-\infty}^{\infty} z^{-n}\cdot\delta[n-2k]}_{= z^{-n}\big|_{n=2k} = z^{-2k}} = \sum_{k=0}^{\infty} 4^k\cdot z^{-2k}$$

$$= \sum_{k=0}^{\infty}\left(\frac{4}{z^2}\right)^k = \Big/ \text{Om } \left|\frac{4}{z^2}\right|<1 \text{ dvs. } \underline{|z|>2} \Big/ = \frac{1}{1-\frac{4}{z^2}} = \underline{\frac{z^2}{z^2-4}}$$

(13) c) $X[\Omega] = \sin\left(\frac{\Omega}{2}\right)$

Fouriertransformen till $x[n]$ är 2π-periodisk

$X[\Omega]$ i a), b) & d) har perioden 2π,

men $\sin\left(\frac{\Omega}{2}\right)$ har perioden 4π.

(14) b) $x[-n] \Leftrightarrow X[-\Omega]$

$$\underline{X[\Omega]} = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} \Rightarrow \underline{\underline{X[-\Omega]}} = \sum_{k=-\infty}^{\infty} x[k]\underbrace{e^{-j(-\Omega)k}}_{= e^{j\Omega k}}$$

$$= \Big/ \begin{matrix} n=-k \\ \Rightarrow k=-n \end{matrix} \Big/ = \sum_{n=-\infty}^{\infty} \underline{x[-n]}\, e^{-j\Omega n}$$

(15) b) $X[\Omega] = \dfrac{1}{16e^{j\Omega}(e^{j\Omega}+1/4)}$

$$X[\Omega] = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} = \sum_{n=2}^{\infty}\left(-\frac{1}{4}\right)^n e^{-j\Omega n} = \sum_{n=2}^{\infty}\left(\frac{-1}{4e^{j\Omega}}\right)^n$$

$$= \Big/ \left|\frac{-1}{4e^{j\Omega}}\right| = \frac{1}{4} < 1 : \text{OK} \Big/ = \left(\frac{-1}{4e^{j\Omega}}\right)^2 \cdot \frac{1}{1-\frac{-1}{4e^{j\Omega}}} = \frac{1}{16e^{j\Omega}(e^{j\Omega}+\frac{1}{4})}$$