6.1-7

b) Förekommande vinkelfrekvenser: $\omega_1 = 4$ rad/s $\omega_2 = 7$ rad/s

$\frac{\omega_1}{\omega_2} = \frac{4}{7} \in \mathbb{Q} \Rightarrow$ Periodisk signal, periodtid $T_0 = \frac{2\pi}{\omega_0}$

där $\omega_0 = SGD(\omega_1, \omega_2) = SGD(2^2, 7) = 1$ rad/s $\Rightarrow T_0 = 2\pi$ sek

d) Vinkelfrekvenser: $\omega_1 = \pi$ rad/s, $\omega_2 = 2\pi$ rad/s

$\frac{\omega_1}{\omega_2} = \frac{\pi}{2\pi} = \frac{1}{2} \in \mathbb{Q} \Rightarrow$ Periodisk med periodtid $T_0 = \frac{2\pi}{\omega_0}$

där $\omega_0 = SGD(\omega_1, \omega_2) = SGD(\pi, 2\pi) = \pi$ rad/s $\Rightarrow T_0 = 2$ sek

e) Vinkelfrekvenser $\omega_1 = \sqrt{2}$ rad/s, $\omega_2 = 2$ rad/s

$\frac{\omega_1}{\omega_2} = \frac{\sqrt{2}}{2} \notin \mathbb{Q} \Rightarrow$ Signalen är ej periodisk

g) Vinkelfrekvenser $\omega_1 = 3$ rad/s, $\omega_2 = \frac{15}{4}$ rad/s

$\frac{\omega_1}{\omega_2} = \frac{3 \cdot 4}{15} \in \mathbb{Q} \Rightarrow$ Periodisk med periodtid $T_0 = \frac{2\pi}{\omega_0}$,

där $\omega_0 = SGD(\omega_1, \omega_2) = SGD\left(\frac{1}{4} \cdot 4 \cdot 3, \frac{1}{4} \cdot 3 \cdot 5\right) = \frac{3}{4}$ rad/s $\Rightarrow T_0 = \frac{8\pi}{3}$ sek

a) Periodisk, $\omega_0 = 1$ rad/s, $T_0 = 2\pi$ sek

c) Ej periodisk

f) Vinkelfrekvenser $\omega_1 = \frac{5}{2}$ rad/s, $\omega_2 = \frac{6}{5}$ rad/s, $\omega_3 = \frac{1}{7}$ rad/s

$\frac{\omega_1}{\omega_2} = \frac{5 \cdot 5}{2 \cdot 6} \in \mathbb{Q}$, $\frac{\omega_2}{\omega_3} = \frac{6 \cdot 7}{5} \in \mathbb{Q} \Rightarrow T_0$-periodisk,

$T_0 = \frac{2\pi}{\omega_0}$ där $\omega_0 = SGD(\omega_1, \omega_2, \omega_3) =$

$= SGD\left(\frac{1}{2} \cdot 5 \cdot \frac{1}{5} \cdot 5 \cdot \frac{1}{7} \cdot 7, \frac{1}{5} \cdot 6 \cdot \frac{1}{2} \cdot 2 \cdot \frac{1}{7} \cdot 7, \frac{1}{7} \cdot \frac{1}{2} \cdot 2 \cdot \frac{1}{5} \cdot 5\right)$

$= \frac{1}{2} \cdot \frac{1}{5} \cdot \frac{1}{7} = \frac{1}{70}$ rad/s $\Rightarrow T_0 = 140\pi$ sek

h) Periodisk, $\omega_0 = SGD(4, 7, 3, 10) = 1$ rad/s $\Rightarrow T_0 = 2\pi$ sek

i) Periodisk, $\omega_0 = 2$ rad/s, $T_0 = \pi$ sek

Kommentar, deluppg. (g) & (f): Vid användande av SGD så ska det som återstår då varje ω_i delas med ω_0 vara ett heltal, dvs. $\omega_i = \text{heltal} \cdot \omega_0$. Därför behöver man i (g) och (f) hantera heltal som bråktal (ex. $3 = \frac{1}{4} \cdot 4 \cdot 3$) enligt metoden i lösningsförslagen. Se även Exempel 3 i det första videoklippet inför föreläsning 1.

6.3-4 a) $x(t) = 3 + \underbrace{\sqrt{3}\cos 2t + \sin 2t}_{\bigstar} + \underbrace{\sin 3t}_{=\cos(3t-\frac{\pi}{2})} \underbrace{- \frac{1}{2}\cos(5t + \frac{\pi}{3})}_{=\frac{1}{2}\cos(5t+\frac{\pi}{3}-\pi)}$

$\sin 2t = \cos(2t - \frac{\pi}{2})$

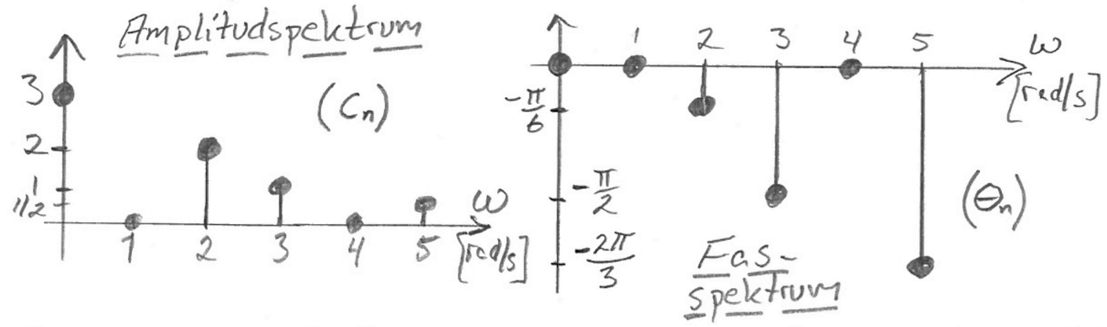
$$\bigstar = \sqrt{3}\,Re\{e^{j2t}\} + Re\{e^{j(2t-\frac{\pi}{2})}\} = Re\{(\sqrt{3} + e^{-j\frac{\pi}{2}})e^{j2t}\}$$

$$= /\ \sqrt{3} + e^{-j\frac{\pi}{2}} = \sqrt{3} - j = \sqrt{3+1}\, e^{j\arctan\frac{-1}{\sqrt{3}}} = 2e^{-j\frac{\pi}{6}}\ /$$

$$= Re\{2e^{j(2t-\frac{\pi}{6})}\} = 2\cos(2t - \frac{\pi}{6})$$

Dvs. $x(t) = 3 + 2\cos(2t - \frac{\pi}{6}) + \cos(3t - \frac{\pi}{2}) + \frac{1}{2}\cos(5t - \frac{2\pi}{3})$

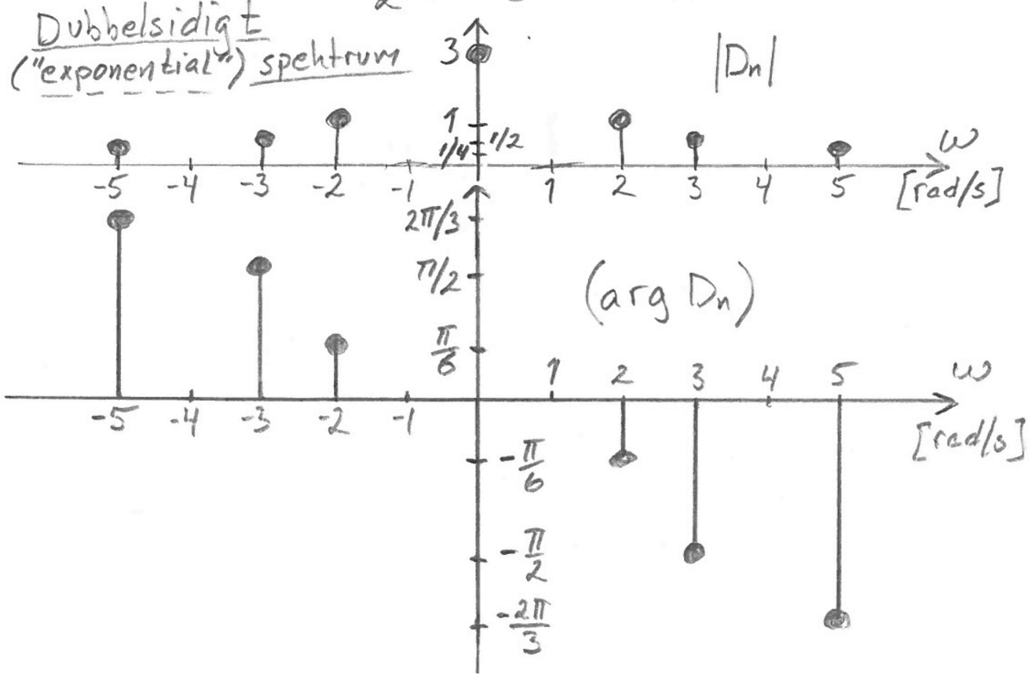
Enkelsidiga ("trigonometric") spektra:



(Anm: Ej nödvändigt att ange spektrumkomponenterna vid $\omega = 1$ & 4 rad/s eftersom de inte finns med i $x(t)$)

b) $A\cdot\cos(\omega_0 t + \theta) = A\cdot\dfrac{e^{j(\omega_0 t+\theta)} + e^{-j(\omega_0 t+\theta)}}{2} =$

$$= \frac{A}{2}e^{j\theta}\cdot e^{j\omega_0 t} + \frac{A}{2}e^{-j\theta}\cdot e^{-j\omega_0 t} \Longrightarrow$$



c) Från spektrumen i b) $\Rightarrow$

$$x(t)=\frac{1}{4}e^{j\frac{2\pi}{3}}\cdot e^{-j5t}+\frac{1}{2}e^{j\frac{\pi}{2}}\cdot e^{-j3t}+e^{j\frac{\pi}{6}}\cdot e^{-j2t}+3+$$
$$+\frac{1}{4}e^{-j\frac{2\pi}{3}}\cdot e^{j5t}+\frac{1}{2}e^{-j\frac{\pi}{2}}\cdot e^{j3t}+e^{-j\frac{\pi}{6}}\cdot e^{j2t}$$

d) Omskrivning av $x(t)$ i c) $\Rightarrow$

$$x(t)=3+2\cdot\frac{e^{j(2t-\frac{\pi}{6})}+e^{-j(2t-\frac{\pi}{6})}}{2}+$$
$$+\frac{e^{j(3t-\frac{\pi}{2})}+e^{-j(3t-\frac{\pi}{2})}}{2}+\frac{1}{2}\cdot\frac{e^{j(5t-\frac{2\pi}{3})}+e^{-j(5t-\frac{2\pi}{3})}}{2}=$$
$$=3+2\cos\left(2t-\frac{\pi}{6}\right)+\cos\left(3t-\frac{\pi}{2}\right)+\frac{1}{2}\cos\left(5t-\frac{2\pi}{3}\right)$$
$$\left(=3+2\cos\left(2t-\frac{\pi}{6}\right)+\sin(3t)-\frac{1}{2}\cos\left(5t+\frac{\pi}{3}\right)\right)$$

6.3-1 a) Figuren $\Rightarrow$ periodtid $T_0=4$ sek $\Rightarrow$ Grundvinkelfr. $\omega_0=\frac{2\pi}{T_0}=\frac{\pi}{2}$ rad/s

$$\underline{\underline{D_n}}=\frac{1}{T_0}\int_{T_0}x_d(t)e^{-jn\omega_0 t}dt=\frac{1}{4}\int_{-1}^{1}1\cdot e^{-jn\frac{\pi}{2}t}+\frac{1}{4}\int_{1}^{3}(-1)e^{-jn\frac{\pi}{2}t}dt=$$

← Låt signalen $=$ "$x_d(t)$"

$$=\frac{1}{4}\left[\frac{e^{-jn\frac{\pi}{2}t}}{-jn\frac{\pi}{2}}\right]_{-1}^{1}-\frac{1}{4}\left[\frac{e^{-jn\frac{\pi}{2}t}}{-jn\frac{\pi}{2}}\right]_{1}^{3}=$$

$n\neq 0$ ty div. med n

$$=\frac{2}{-4jn\pi}\left(e^{-jn\frac{\pi}{2}}-e^{jn\frac{\pi}{2}}-\left(\underbrace{e^{-jn\frac{\pi}{2}\cdot 3}}_{=e^{jn\frac{\pi}{2}}}-e^{-jn\frac{\pi}{2}}\right)\right)=$$

$$=\frac{2}{n\pi}\cdot\frac{e^{jn\frac{\pi}{2}}-e^{-jn\frac{\pi}{2}}}{2j}=\underline{\underline{\frac{2}{n\pi}\sin\left(n\cdot\frac{\pi}{2}\right)}}\quad n\neq 0$$

$$\underline{\underline{D_0}}=\frac{1}{T_0}\int_{T_0}x_d(t)\cdot\underbrace{e^{j0\cdot\omega_0 t}}_{=1}dt=\frac{1}{4}\int_{-1}^{1}1\,dt+\frac{1}{4}\int_{1}^{3}(-1)dt=\underline{\underline{0}}$$

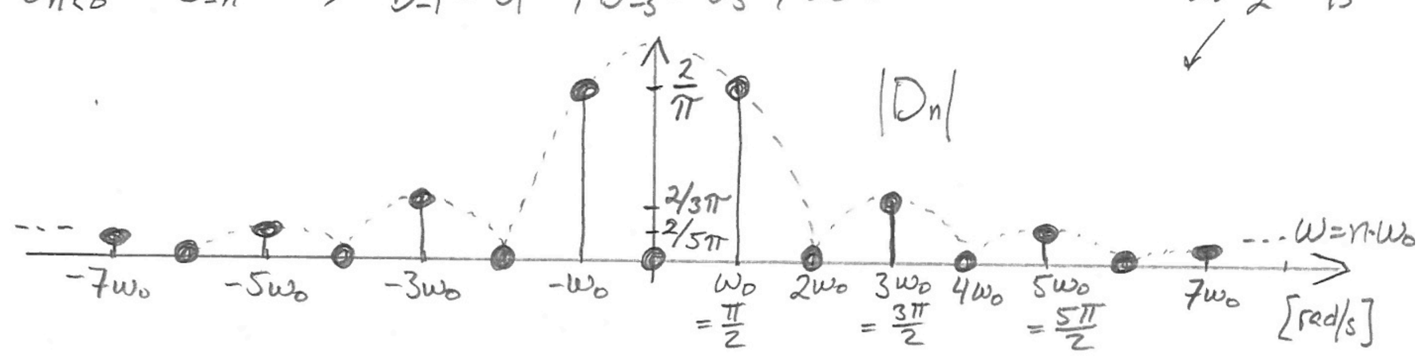
(vilket även ses direkt i figuren; $D_0=$ medelvärdet)

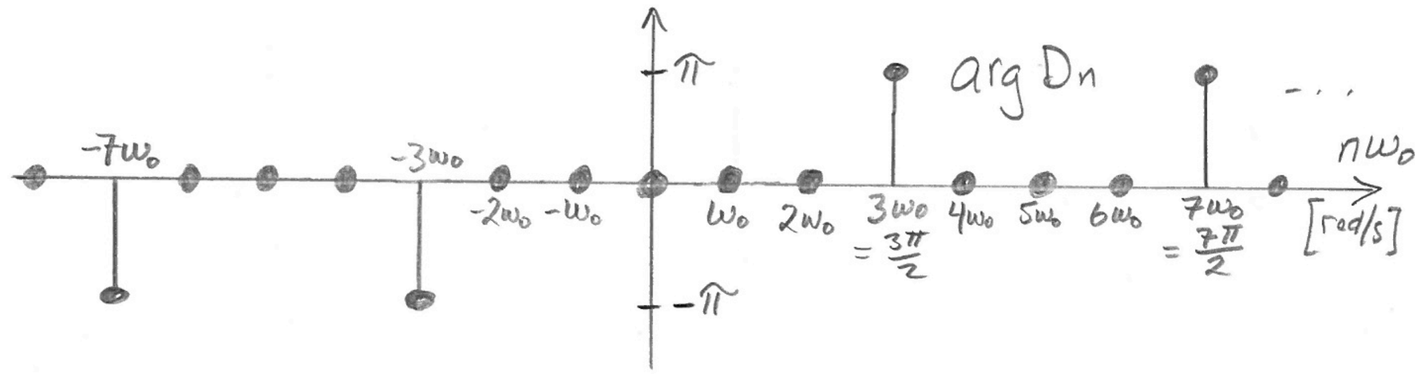
$$\underline{\underline{\text{Svar:}}}\quad D_n=\begin{cases}0; & n=0\\ \frac{2}{n\pi}\sin\left(n\frac{\pi}{2}\right); & n\neq 0\end{cases}$$

(← $D_n=0$ för jämna n)

$$D_n = \frac{2}{n\pi}\sin\left(n\cdot\frac{\pi}{2}\right) = \begin{cases} 0; & n=0 \quad (\text{även } n=0) \\ \frac{2}{n\pi} = \frac{2}{n\pi}e^{j0}; & n = 1, 5, 9, \ldots \\ \frac{-2}{n\pi} = \frac{2}{n\pi}\cdot e^{j\pi}; & n = 3, 7, 11, \ldots \end{cases}$$

$$D_{n<0} = D_{-n}^{*} \Rightarrow D_{-1} = D_1^{*}, \; D_{-3} = D_3^{*}, \text{ osv:}$$





b) Låt $x_b(t)$ = signalen, med periodtid $T_0 = 10\pi$ sek

$\Rightarrow$ Grundvinkelfrekvens $\omega_0 = \frac{2\pi}{T_0} = \frac{1}{5}$ rad/s

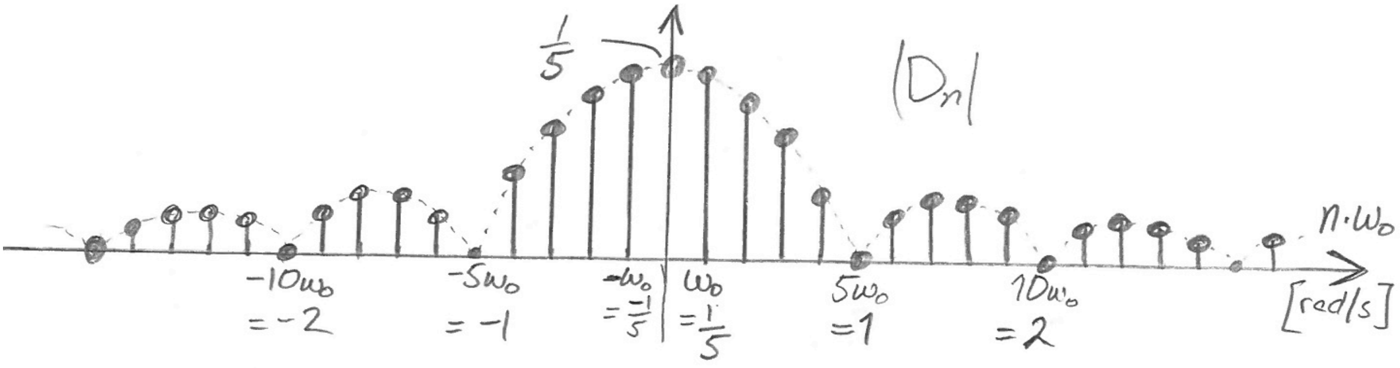
Dvs. $x_b(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t}$, där

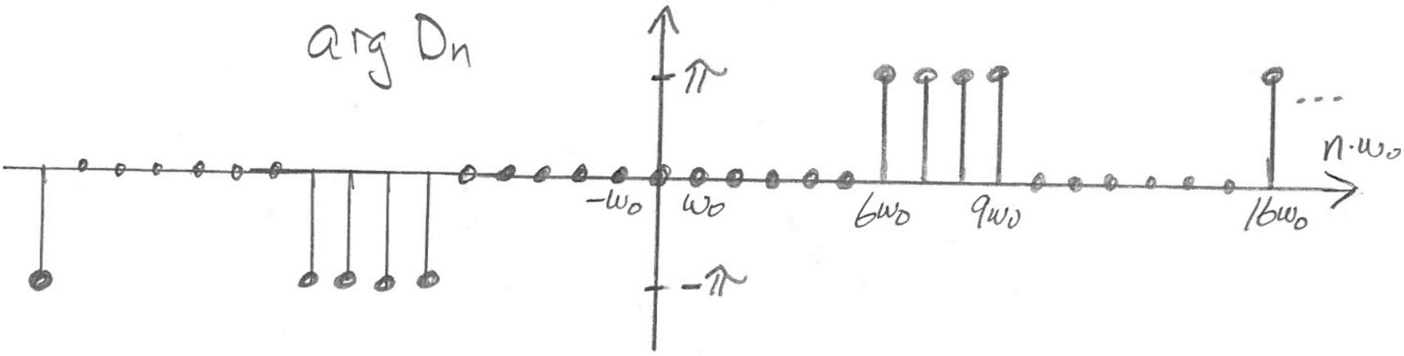
$$\underline{D_n} = \frac{1}{T_0}\int_{T_0} x_b(t)e^{-jn\omega_0 t}\,dt = \frac{1}{10\pi}\int_{-\pi}^{\pi} 1\cdot e^{-jn\frac{1}{5}t}\,dt = \left| \begin{matrix} n \neq 0 \\ \text{ty div.} \\ \text{med } n \end{matrix} \right|$$

$$= \frac{1}{10\pi}\left[\frac{e^{-jnt/5}}{-jn\frac{1}{5}}\right]_{-\pi}^{\pi} = \frac{1}{-jn2\pi}\left(e^{-jn\frac{\pi}{5}} - e^{jn\frac{\pi}{5}}\right)$$

$$= \frac{1}{n\pi}\,\frac{e^{jn\frac{\pi}{5}} - e^{-jn\frac{\pi}{5}}}{2j} = \underline{\frac{1}{n\pi}\cdot\sin\left(n\frac{\pi}{5}\right)}$$

$$\underline{D_0} = \frac{1}{T_0}\int_{T_0} x_b(t)e^{-j0\omega_0 t}\,dt = \frac{1}{10\pi}\int_{-\pi}^{\pi} 1\,dt = \underline{\frac{1}{5}}$$





c) $x_c(t)$ har period $T_0 = 2\pi$ sek. $\Rightarrow \omega_0 = \frac{2\pi}{T_0} = 1$ rad/s

$$x_c(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t}, \quad \text{där}$$

$$\underline{D_n} = \frac{1}{T_0}\int_{T_0} x_c(t)\, e^{-jn\omega_0 t}\,dt = \frac{1}{2\pi}\int_0^{2\pi} \frac{t}{2\pi}\cdot e^{-jnt}\,dt$$

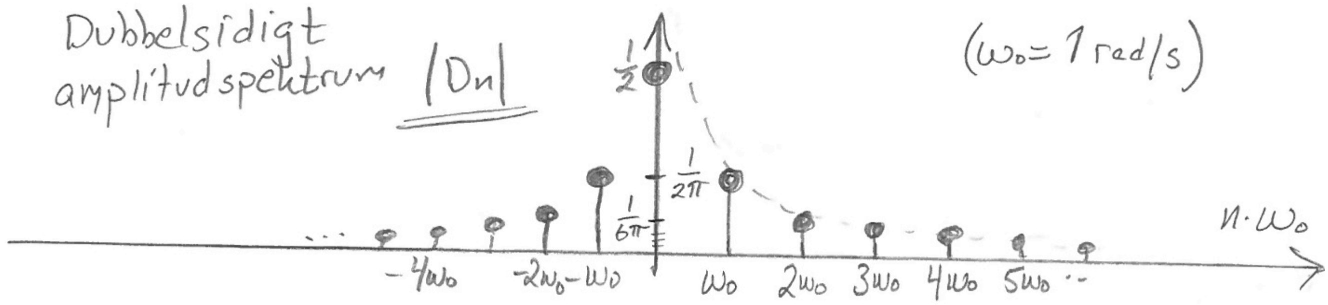
$$\underset{n\neq 0}{=} \frac{1}{4\pi^2}\left[\frac{t\cdot e^{-jnt}}{-jn}\right]_0^{2\pi} - \frac{1}{4\pi^2}\int_0^{2\pi}\frac{e^{-jnt}}{-jn}\,dt$$

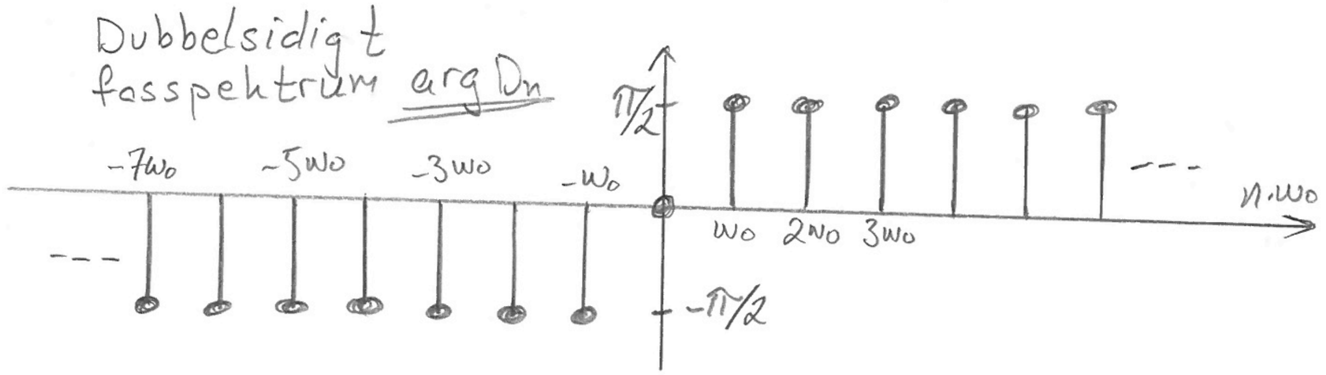
$$= \frac{j}{4\pi n}\left(2\pi\cdot e^{-jn2\pi} - 0\right) - \frac{1}{4\pi^2(-jn)}\underbrace{\left[e^{-jnt}\right]_0^{2\pi}}_{=e^{-jn2\pi} - e^0 = 1^n - 1 = 0}$$

$$= \underline{\underline{\frac{j}{2\pi n}}} \; ; \; n \neq 0$$

$$\underline{\underline{D_0}} = \frac{1}{2\pi}\int_0^{2\pi}\frac{t}{2\pi}\cdot e^0\,dt = \frac{1}{4\pi^2}\left[\frac{t^2}{2}\right]_0^{2\pi} = \underline{\underline{\frac{1}{2}}}$$ (vilket även ses direkt i figuren)

Svar: $D_n = \begin{cases} \frac{j}{2\pi n} \; ; & n \neq 0 \\ 1/2 \; ; & n = 0 \end{cases} \quad \left(\Rightarrow \arg D_n = \begin{cases} \frac{\pi}{2} \; ; & n > 0 \\ 0 \; ; & n = 0 \\ -\frac{\pi}{2} \; ; & n < 0 \end{cases} \right)$





6.1-3

a) Periodtid $T_0 = \frac{\pi}{2}$ sek $\Rightarrow$ Grundvinkelfrekv. $\omega_0 = \frac{2\pi}{T_0} = 4$ rad/s

$$y(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t}, \text{ där}$$

$$D_n = \frac{1}{T_0}\int_{T_0} y(t)e^{-jn\omega_0 t}\,dt = \frac{2}{\pi}\int_0^{\pi/2} e^{-t}\cdot e^{-jn4t}\,dt =$$

$$= \frac{2}{\pi}\int_0^{\pi/2} e^{-(1+j4n)t}\,dt = \frac{2}{\pi}\left[\frac{e^{-(1+j4n)t}}{-(1+j4n)}\right]_0^{\pi/2}$$

$$= \frac{2}{-\pi(1+j4n)}\Big(\underbrace{e^{-(1+j4n)\frac{\pi}{2}}}_{=e^{-\frac{\pi}{2}}\cdot e^{-jn2\pi} = e^{-\frac{\pi}{2}}} - e^0\Big) = \frac{2(1-e^{-\frac{\pi}{2}})}{\pi(1+j4n)}$$

$$C_0 = D_0 = \frac{2}{\pi}(1-e^{-\pi/2}) \approx 0{,}504$$

$$C_{n>0} = 2|D_n| = \frac{4(1-e^{-\pi/2})}{\pi\sqrt{1^2+(4n)^2}} \approx \frac{1{,}01}{\sqrt{1+16n^2}}$$

$$\theta_n = \arg D_n = -\arctan\frac{4n}{1}$$

Svar: $y(t) \approx 0{,}504 + \sum_{n=1}^{\infty} \frac{1{,}01}{\sqrt{1+16n^2}}\cos(4nt - \arctan(4n))$

b) Ekvation 6.15a, sid. 602:

$$x(t) = 0{,}504 + 0{,}504 \sum_{n=1}^{\infty} \frac{2}{\sqrt{1+16n^2}} \cos(2nt - \arctan 4n)$$

$\Rightarrow$ $x(2t) = y(t) \leftarrow$ se svaret på deluppgift a)

c) $x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot \cos(n\omega_0 t + \theta_n) \Rightarrow$

$$x(at) = C_0 + \sum_{n=1}^{\infty} C_n \cdot \cos(n(a\omega_0)t + \theta_n)$$

$\Rightarrow$ C_0, C_n & θ_n är oförändrade

- Om $\underline{a>1}$ $\Rightarrow$ grundvinkelfrekvensen hos signalen $x(at)$ är en faktor a <u>högre</u> än grundvinkelfrekv. hos $x(t)$ $\Rightarrow$ $x(at)$ är en <u>tidskomprimerad</u> version av $x(t)$; komprimerad en faktor a.
- Om $\underline{a<1}$ $\Rightarrow$ grundvinkelfrekvensen $a\omega_0$ hos $x(at)$ är <u>lägre</u> än grundvinkelfr. ω_0 hos $x(t)$ $\Rightarrow$ $x(at)$ är en <u>tidsexpanderad</u> version av $x(t)$ (expanderad en faktor $\frac{1}{a}$)

6.3-7

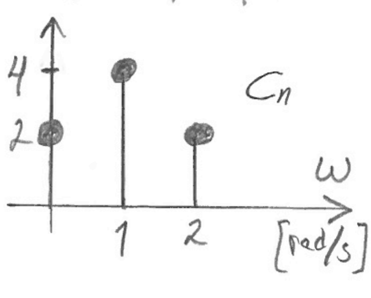
a) Fig. P6.3-7(a) & (b) $\Rightarrow$

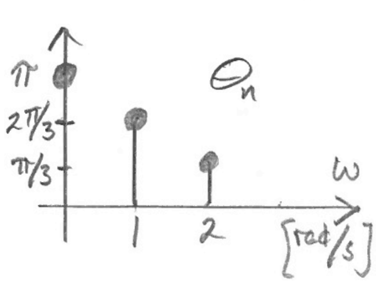
$$x(t) = 1\cdot e^{-j\frac{\pi}{3}}\cdot e^{-j2t} + 2\cdot e^{-j\frac{2\pi}{3}}\cdot e^{-jt} + 2\cdot e^{\pm j\pi} + 2\cdot e^{j\frac{2\pi}{3}}\cdot e^{jt} + 1\cdot e^{j\frac{\pi}{3}}\cdot e^{j2t}$$

$$= -2 + 4\cdot\frac{e^{j(t+\frac{2\pi}{3})} + e^{-j(t+\frac{2\pi}{3})}}{2} + 2\cdot\frac{e^{j(2t+\frac{\pi}{3})} + e^{-j(2t+\frac{\pi}{3})}}{2}$$

$$= -2 + 4\cos\left(t + \frac{2\pi}{3}\right) + 2\cos\left(2t + \frac{\pi}{3}\right)$$

b) $C_n = 2|D_n|$; $n>0$, $C_0 = D_0$, $\theta_n = \arg D_n$ $(\angle D_n)$ $\Rightarrow$





(Anm: $C_0 = -2 = 2\cdot e^{j\pi}$)

c) Lösningen till (grafen i) deluppgift b) $\Rightarrow$

$$x(t) = -2 + 4\cos\left(t + \frac{2\pi}{3}\right) + 2\cos\left(2t + \frac{\pi}{3}\right)$$

d) Trivialt – se $x(t)$ i deluppg. a) & c)

6.3-8

a) $x(t)$ är periodisk med periodtid $T_0 = 8$ sek $\Rightarrow$
grundvinkelfrekvensen är $\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{4}$ rad/s

$$x(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t}, \quad \text{där}$$

$$D_n = \frac{1}{T_0}\int_{T_0} x(t) e^{-jn\omega_0 t} dt = \frac{1}{8}\int_{-4}^{0}\left(\frac{t}{2}+1\right)e^{-jn\frac{\pi}{4}t} dt + \frac{1}{8}\int_{0}^{4}\left(\frac{-t}{2}+1\right)e^{-jn\frac{\pi}{4}t} dt$$

$$\underset{\substack{n\neq 0 \text{ ty} \\ \text{div. med } n}}{=} \left[\left(\frac{t}{2}+1\right)\frac{e^{-jn\frac{\pi}{4}t}}{-jn\frac{\pi}{4}}\right]_{-4}^{0} - \int_{-4}^{0}\frac{e^{-jn\frac{\pi}{4}t}}{-jn\frac{\pi}{4}\cdot 8\cdot 2} dt$$

$$+ \left[\left(\frac{-t}{2}+1\right)\frac{e^{-jn\frac{\pi}{4}t}}{-jn\frac{\pi}{4}}\right]_{0}^{4} - \int_{0}^{4}\frac{e^{-jn\frac{\pi}{4}t}}{-jn\frac{\pi}{4}\cdot 8\cdot(-2)} dt$$

$$= \underbrace{\frac{e^{0} + e^{jn\pi}}{-jn\cdot\frac{\pi}{4}} - \left[\frac{e^{-jn\frac{\pi}{4}t}}{(-jn\frac{\pi}{4})^2\cdot 16}\right]_{-4}^{0} + \frac{-e^{jn\pi} - e^{0}}{-jn\cdot\frac{\pi}{4}}}_{\Sigma = 0 \text{ (första och tredje termen)}} + \left[\frac{e^{jn\frac{\pi}{4}t}}{(-jn\frac{\pi}{4})^2\cdot 16}\right]_{0}^{4}$$

$$= \frac{e^{jn\pi} - e^{0}}{-n^2\cdot\pi^2} + \frac{e^{-jn\pi} - e^{0}}{-n^2\pi^2} = \frac{2(1-(-1)^n)}{n^2\pi^2} = \begin{cases} 0; & \text{jämna } n \neq 0 \\ \frac{4}{n^2\pi^2}; & \text{udda } n \end{cases}$$

$$D_0 = \frac{1}{T_0}\int_{T_0} x(t)\cdot e^{0} dt = /\text{framgår direkt av figuren}/ = 0$$

__Svar:__ $x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\frac{\pi}{4}t}$ där $D_n = \begin{cases} 0; & \text{jämna } n \\ \frac{4}{n^2\pi^2}; & \text{udda } n \end{cases}$

b) Vi ser att $\hat{x}(t) = x(t-2) = /\text{uppg. a)}/ = \sum_{n=-\infty}^{\infty} D_n e^{jn\frac{\pi}{4}(t-2)} =$

$$= \sum_{n=-\infty}^{\infty} D_n \cdot e^{-jn\frac{\pi}{2}} \cdot e^{jn\frac{\pi}{4}t} = \sum_{n=-\infty}^{\infty} \hat{D}_n \cdot e^{jn\frac{\pi}{4}t},$$

där $\hat{D}_n = D_n \cdot e^{-jn\frac{\pi}{2}} = \begin{cases} 0\ ; & \text{jämna } n \\ \dfrac{4 \cdot e^{-jn\frac{\pi}{2}}}{n^2\pi^2}\ ; & \text{udda } n \end{cases}$

c) Figur $\Rightarrow$ $\tilde{x}(t) = x(2t) = /a)/ = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\frac{\pi}{4}\cdot 2t} =$

$$= \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\frac{\pi}{2}t}$$

dvs. $\tilde{x}(t) = \sum_{n=-\infty}^{\infty} \tilde{D}_n \cdot e^{jn\frac{\pi}{2}t}$, där $\tilde{D}_n = D_n = \begin{cases} 0\ ; & \text{jämna } n \\ \dfrac{4}{n^2\pi^2}\ ; & \text{udda } n \end{cases}$

(Anm. grundvinkelfrekvensen för $\tilde{x}(t)$ är $\tilde{\omega}_0 = \frac{\pi}{2}$ rad/s, medan grundvinkelfrekvensen för $x(t)$ är $\omega_0 = \frac{\pi}{4}$ rad/s, dvs. $\tilde{x}(t) = x(2t) \Rightarrow \tilde{\omega}_0 = 2 \cdot \omega_0$)

6.1-2 a) $y(t) = 0{,}504 + \sum_{n=1}^{\infty} \frac{2 \cdot 0{,}504}{\sqrt{1+16n^2}} \cos(2nt + \arctan 4n)$

b) $y(t) = x(-t) \leftarrow$ se ekv. 6.15a

c) Om $x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot \cos(n\omega_0 t + \theta_n)$

$$\Rightarrow x(-t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot \cos(-n\omega_0 t + \theta_n)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \cdot \cos(n\omega_0 t - \theta_n)$$

6.1-4 a) $g(t) = \sum_{n=1}^{\infty} C_n \cdot \cos(n\pi t)$; $C_n = \begin{cases} 0; & \text{jämna } n \\ \frac{8A}{n^2\pi^2}; & \text{udda } n \end{cases}$

b) $g(t) = x(t+0{,}5) = \sum_{n=1}^{\infty} C_n \cdot \cos(n\pi(t+0{,}5))$

$$= \sum_{n=1}^{\infty} C_n \cdot \cos\left(n\pi t + n \cdot \frac{\pi}{2}\right)$$

Identifiera med uttrycket för $x(t)$ efter ekv. 6.16

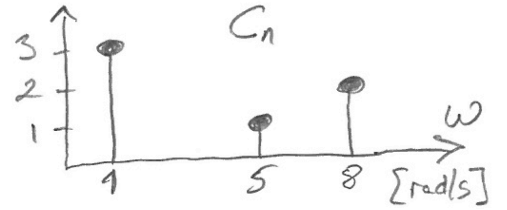
c) Om $x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot \cos(n\omega_0 t + \theta_n)$

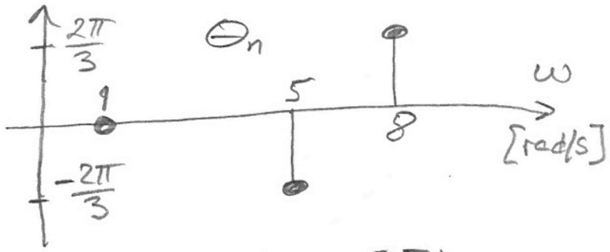
$$\Rightarrow x(t+T) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 (t+T) + \theta_n)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \cdot \cos(n\omega_0 t + (\theta_n + n\omega_0 T))$$

6.3-3

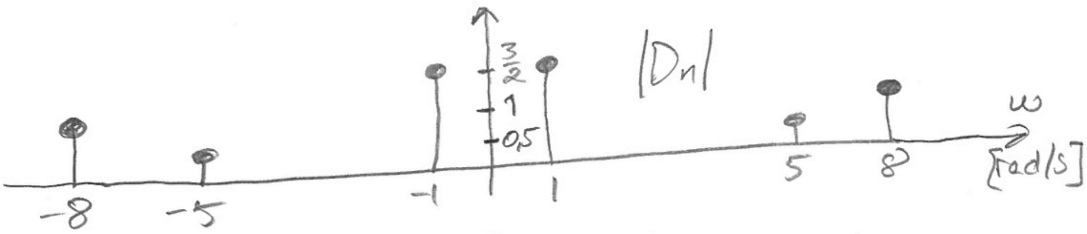
a)

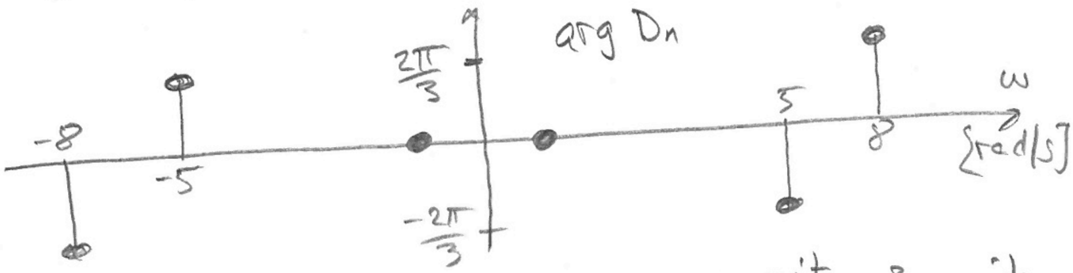




$$x(t) = 3\cos t + \cos\left(5t - \frac{2\pi}{3}\right) + 2\cos\left(8t + \frac{2\pi}{3}\right)$$

b)





c) $$x(t) = e^{-j\frac{2\pi}{3}} \cdot e^{-j8t} + \frac{1}{2} e^{j\frac{2\pi}{3}} \cdot e^{-j5t} + \frac{3}{2} e^{-jt} + \frac{3}{2} e^{jt} + \frac{1}{2} e^{-j\frac{2\pi}{3}} e^{j5t} + e^{j\frac{2\pi}{3}} \cdot e^{j8t}$$

d) $$x(t) = 3\,\frac{e^{jt} + e^{-jt}}{2} + \frac{e^{j(5t - \frac{2\pi}{3})} + e^{-j(5t - \frac{2\pi}{3})}}{2} + 2\,\frac{e^{j(8t + \frac{2\pi}{3})} + e^{-j(8t + \frac{2\pi}{3})}}{2}$$

$$= 3\cos t + \cos\left(5t - \frac{2\pi}{3}\right) + 2\cos\left(8t + \frac{2\pi}{3}\right)$$

6.3-9 $$x(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t}$$

a) $$\hat{x}(t) = x(t-T) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0(t-T)} = \sum_{n=-\infty}^{\infty} \underbrace{D_n \cdot e^{-jn\omega_0 T}}_{\hat{D}_n} \cdot e^{jn\omega_0 t}$$

$$\Rightarrow |\hat{D}_n| = |D_n|, \quad \arg \hat{D}_n = \arg D_n - n\omega_0 T$$

b) $$\tilde{x}(t) = x(at) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\underbrace{(a\cdot\omega_0)}_{=\tilde{\omega}_0} t}$$

[Anm: 6.3-8 b) & c) är specialfall av 6.3-9 a) resp. b)]