

(PASSIVA) FILTER

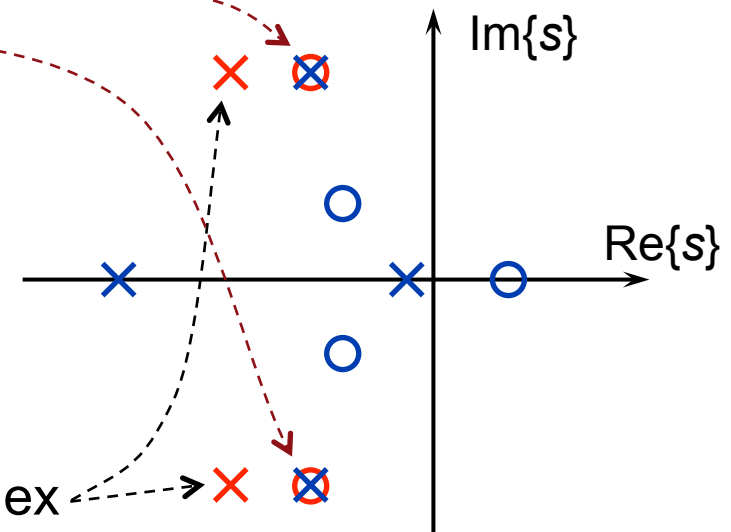
Bortfiltrering/eliminering av enskilda signalkomponenter:

$$Y(s) = X(s) \cdot H(s)$$

$$= \frac{T_X(s)}{(s-p_1)\cancel{(s-p_2)}\cancel{(s-p_3)}(s-p_4)} \cdot \frac{\cancel{(s-p_2)}\cancel{(s-p_3)}}{N_H(s)}$$

”Signalselektiv” filtrering:

De komplexkonjugerade polerna p_2 och p_3 hos $X(s)$ motsvarar en signalkomponent i $x(t)$ som skall helt filtreras bort:
Utförs m.h.a nollställen hos $H(s)$!



Krav för

stabilit system: #poler \geq #nollställen \Rightarrow t.ex

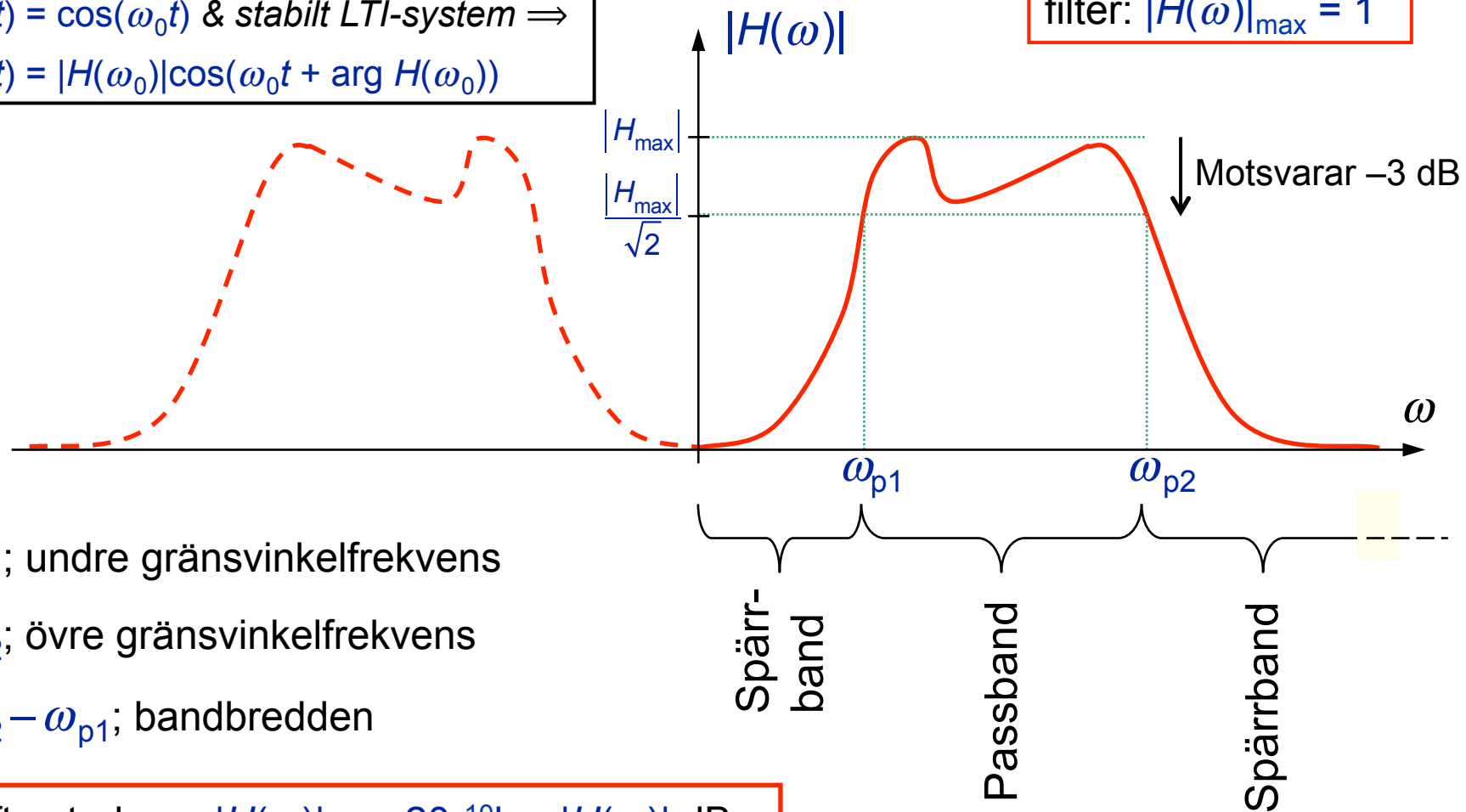
Frekvensselektiva filter

Frekvensselektiv filtrering:

$$x(t) = \cos(\omega_0 t) \text{ \& stabilt LTI-system } \Rightarrow$$

$$y(t) = |H(\omega_0)| \cos(\omega_0 t + \arg H(\omega_0))$$

Amplitudnormerat filter: $|H(\omega)|_{\max} = 1$



ω_{p1} ; undre gränsvinkelfrekvens

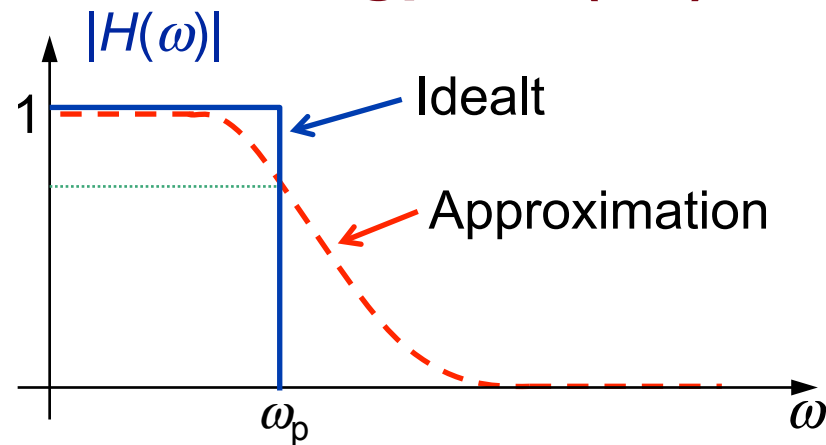
ω_{p2} ; övre gränsvinkelfrekvens

$\omega_{p2} - \omega_{p1}$; bandbredden

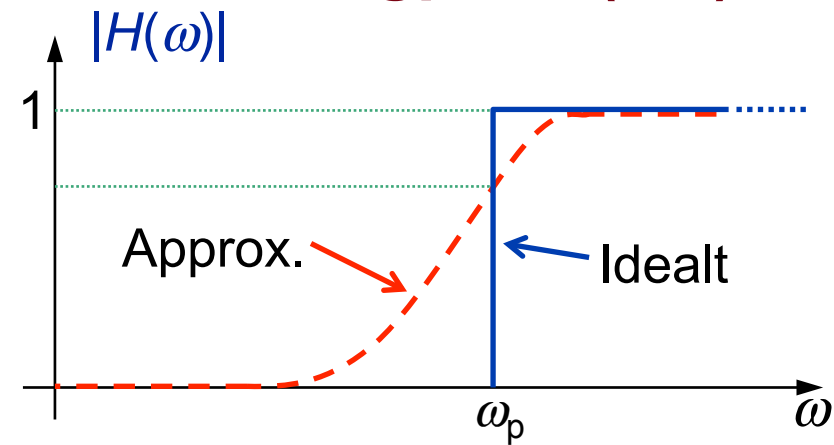
Ofta studeras $|H(\omega)|_{\text{dB}} = 20 \cdot 10 \log |H(\omega)|$ dB

Olika frekvensselektiva filtertyper

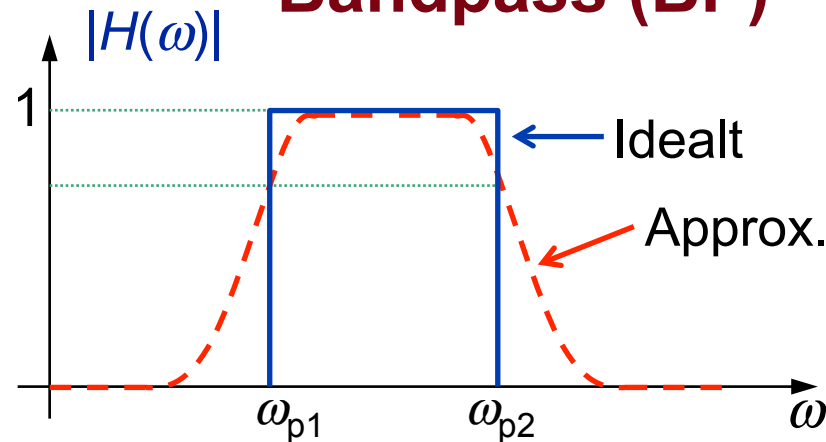
Lågpäss (LP)



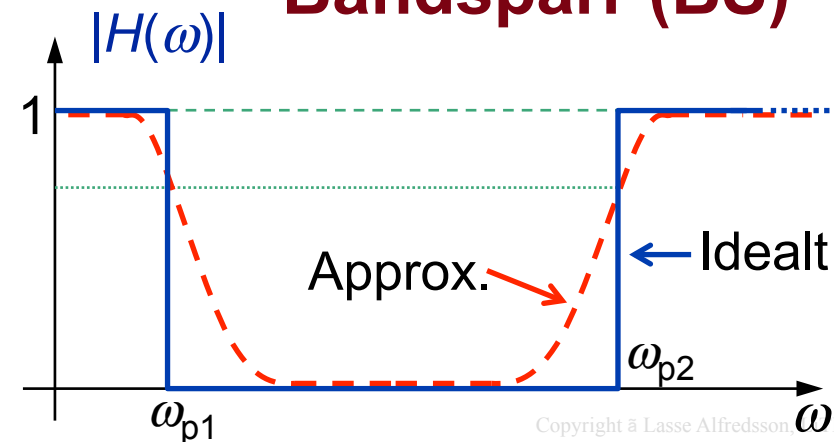
Högpäss (HP)



Bandpass (BP)

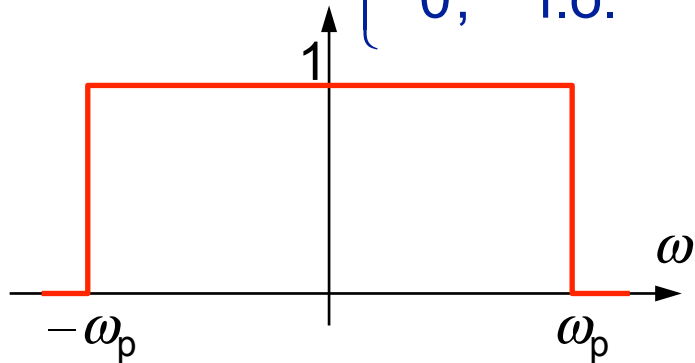


Bandspärr (BS)

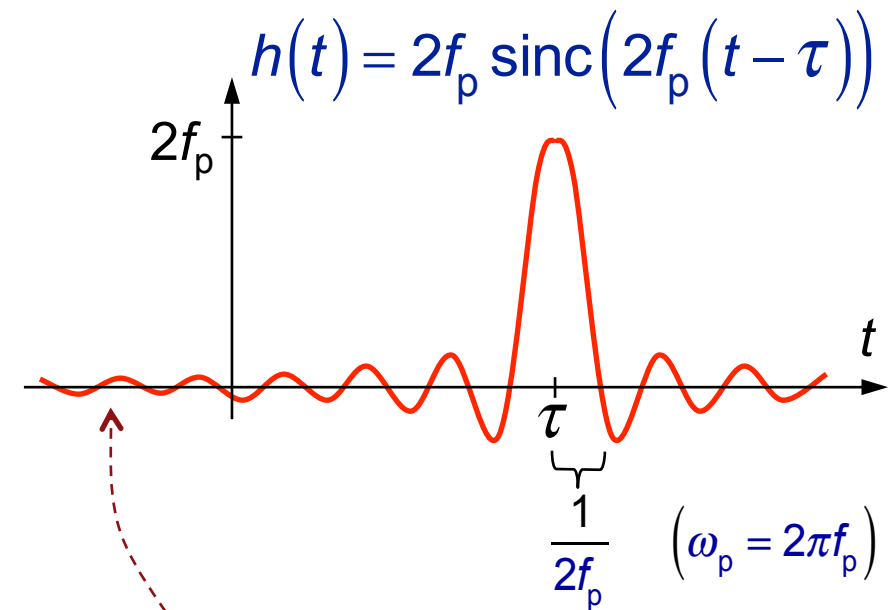
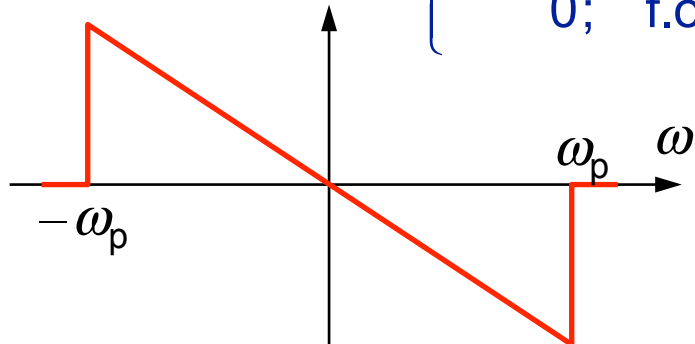


Exempel – Idealt LP-Filter:

$$|H(\omega)| = \begin{cases} 1; & |\omega| \leq \omega_p \\ 0; & \text{f.ö.} \end{cases}$$



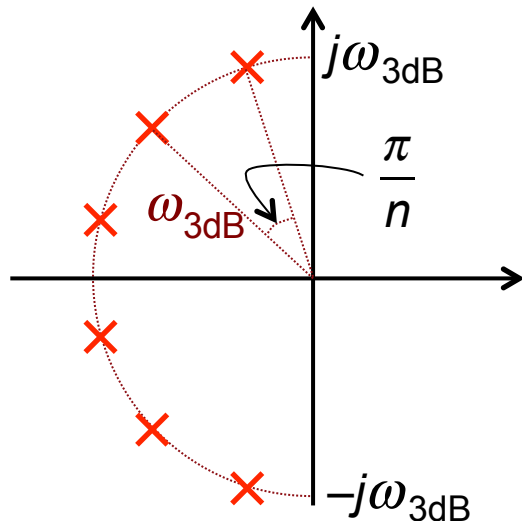
$$\arg H(\omega) = \begin{cases} -\omega\tau; & |\omega| \leq \omega_p \\ 0; & \text{f.ö.} \end{cases}$$



$h(t) \neq 0$ för $t < 0$,
dvs. icke-kausalt system
 \Rightarrow **ej realiserbart!**

Butterworthfilter – (LP-filter)

Poler hos $H(s)$ längs en halvcirkel:

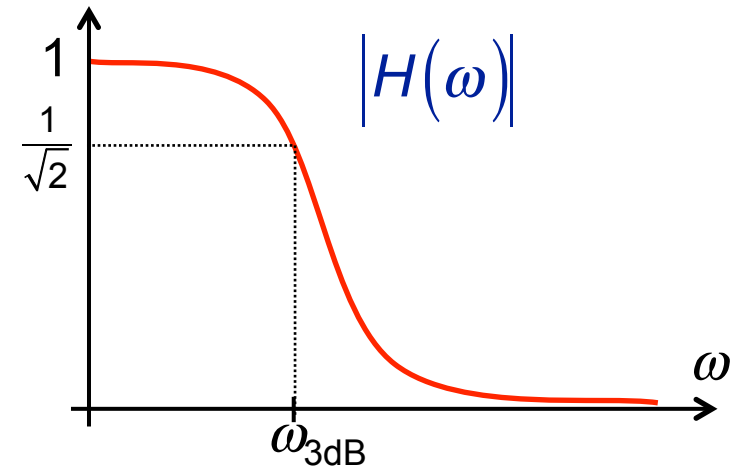


Poler:

$$p_k = \omega_{3\text{dB}} \cdot e^{j\left(\frac{\pi}{2} - \frac{1}{2n}\pi + k \cdot \frac{\pi}{n}\right)}$$

$$= \omega_{3\text{dB}} \cdot e^{j\left(\frac{2k+n-1}{2n}\right)\pi}$$

$$k = 1, 2, 3, \dots, n$$



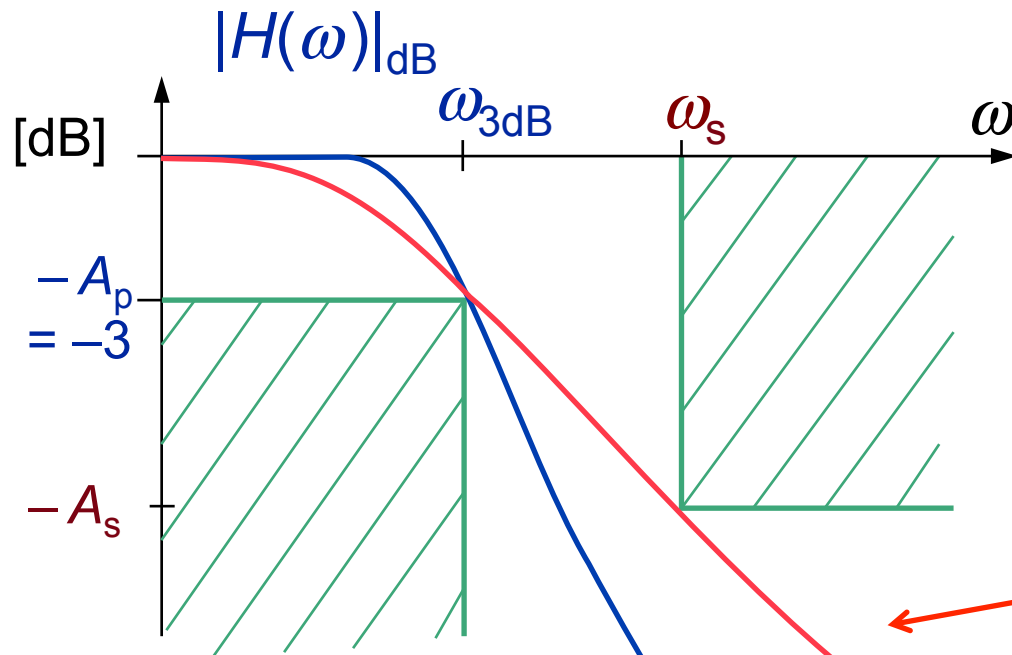
$\omega_{3\text{dB}}$ = 3 dB-gränsvinkelfrekvensen

$$H(s) = \frac{(\omega_{3\text{dB}})^n}{s^n + a_1 s^{n-1} + \dots + a_{n-1} s + (\omega_{3\text{dB}})^n}$$

a_i erhålls vanligen från tabell eller genom utveckling av $(s-p_1)(s-p_2)\dots(s-p_n)$

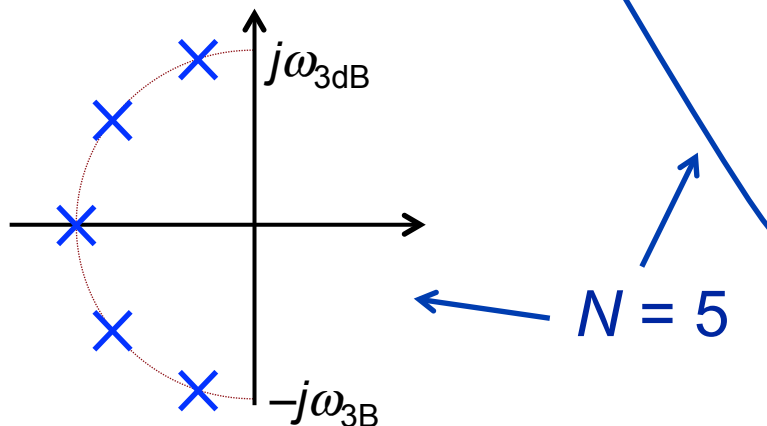
$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_{3\text{dB}}}\right)^{2n}}}$$

Butterworthfilter – (LP-filter)

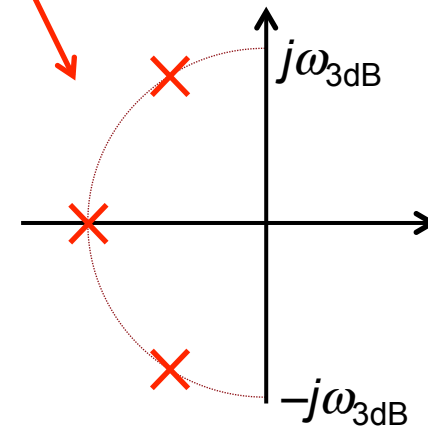


Butterworthfilter har maximalt flat amplitud-karaktäristik i passbandet!

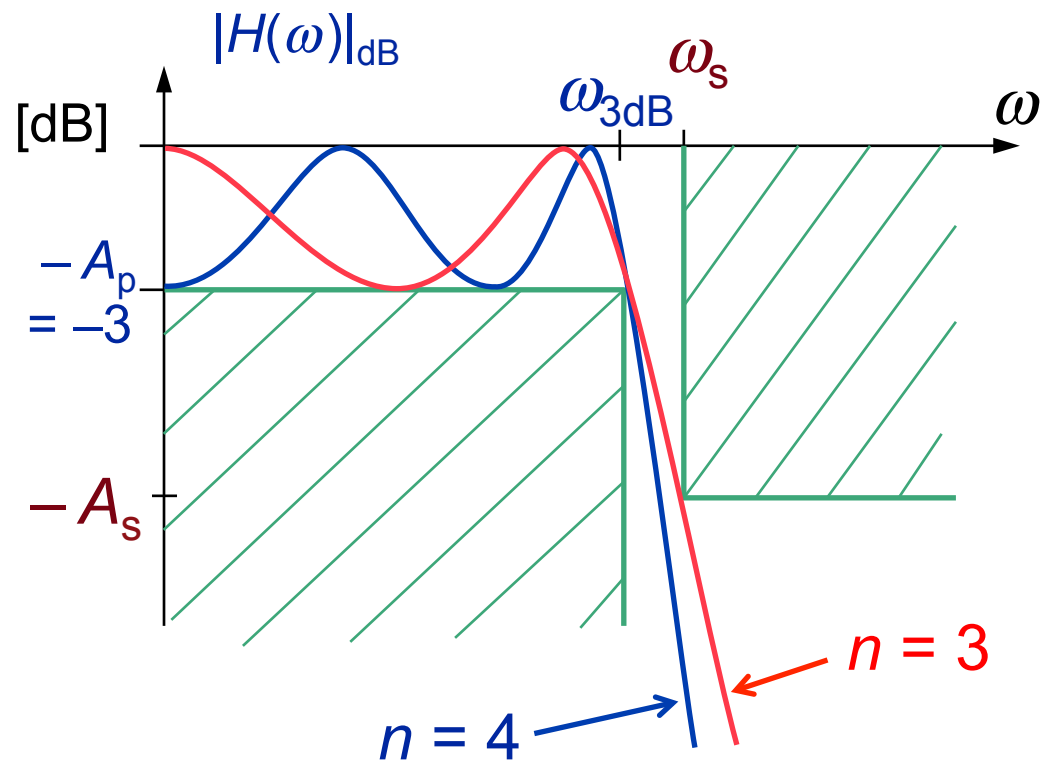
(Butt.filter ger *bästa möjliga passbands-approximationen*)



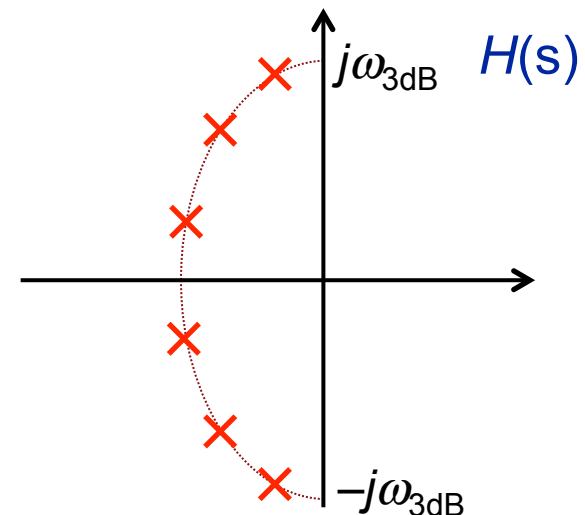
$n = 3$



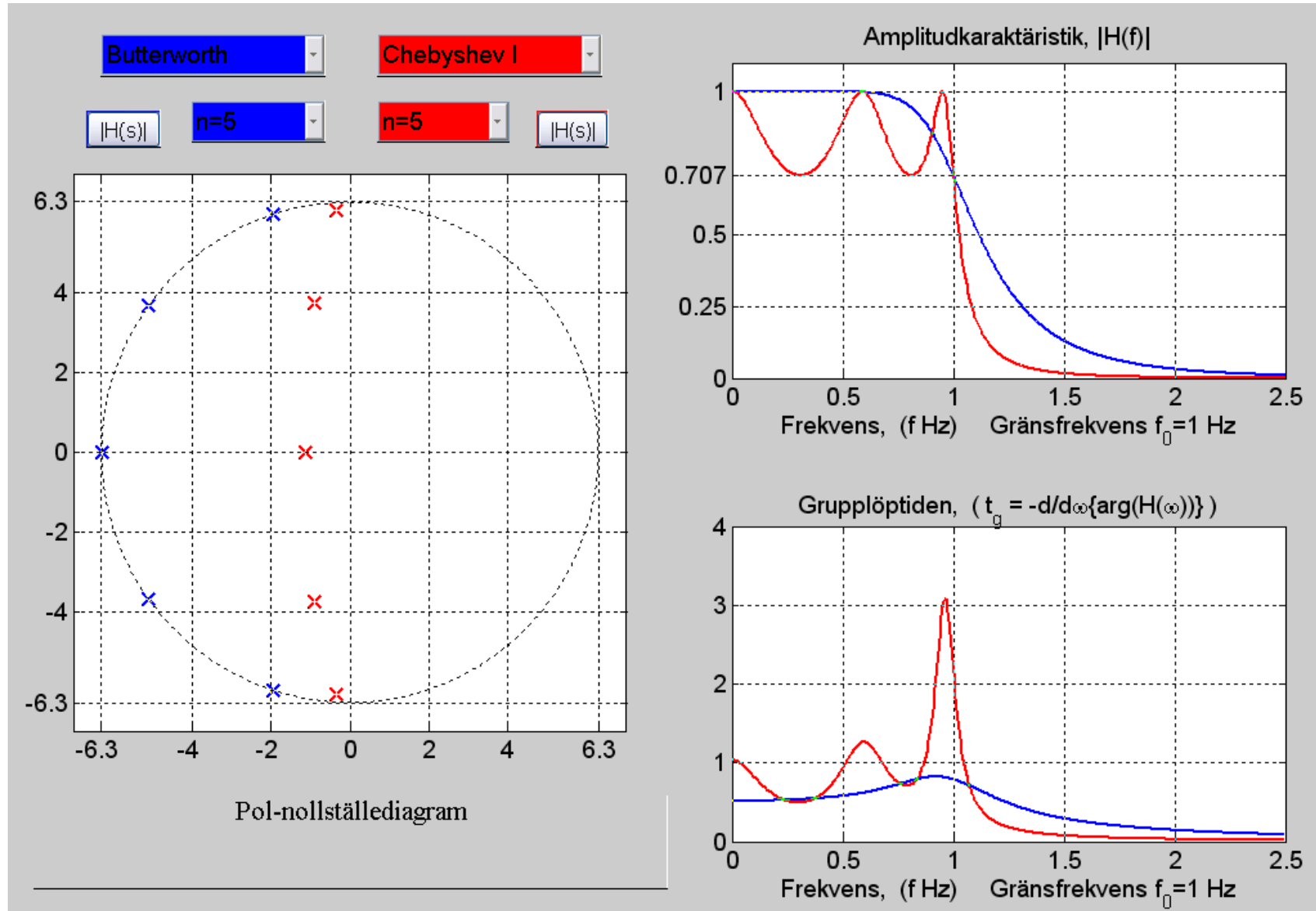
Chebyshev I-filter



- Rippel (A_p dB) i passbandet!
- Optimalt m.a.p. *brantheten* i övergångsbandet $\omega_{3dB} \rightarrow \omega_s$
- Polerna hos $H(s)$ ligger längs en halv-ellips i VHP

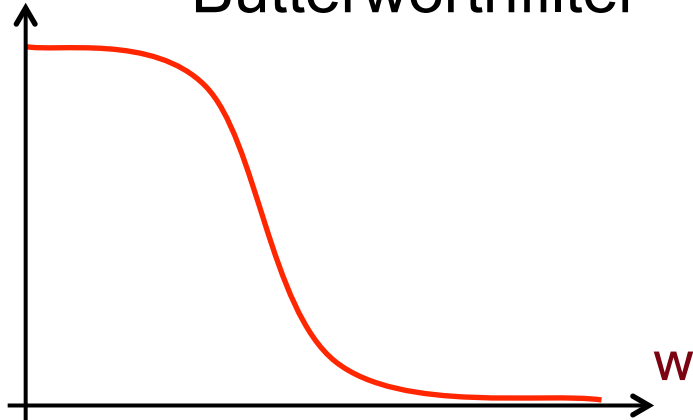


Matlabdemo, Butterworth- & Chebyshev I-filter:

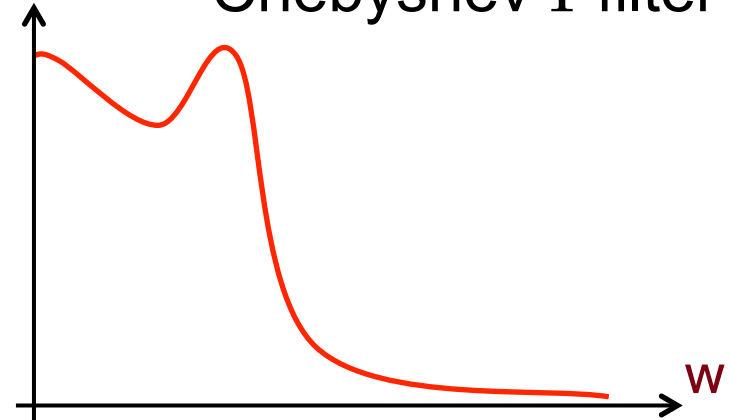


Klassiska ideala LP-approximationer

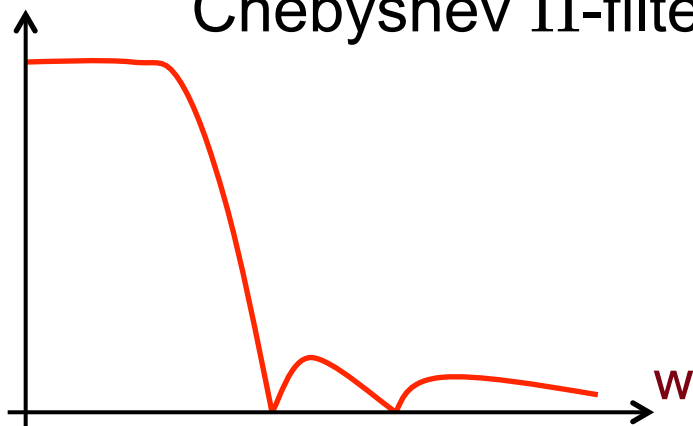
$|H(\omega)|$ Butterworthfilter



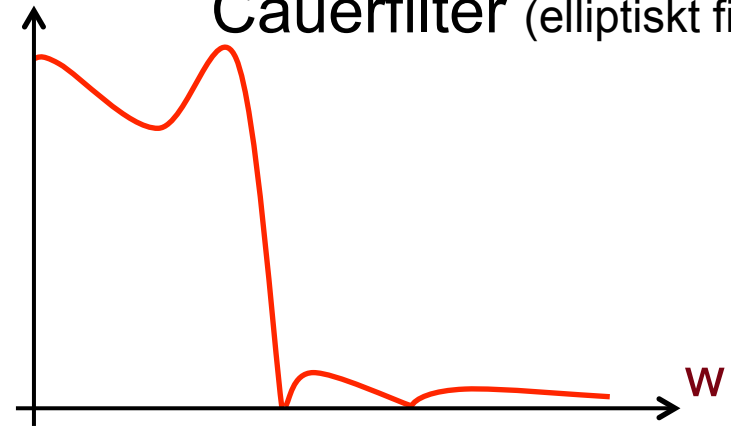
$|H(\omega)|$ Chebyshev I-filter



$|H(\omega)|$ Chebyshev II-filter

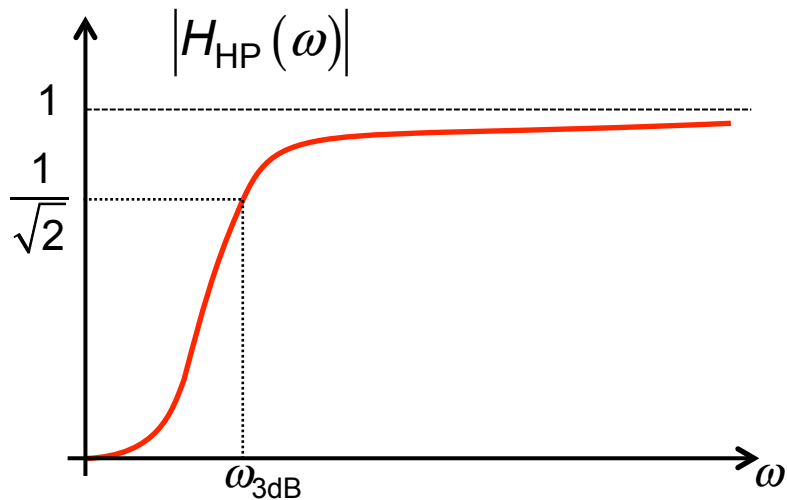
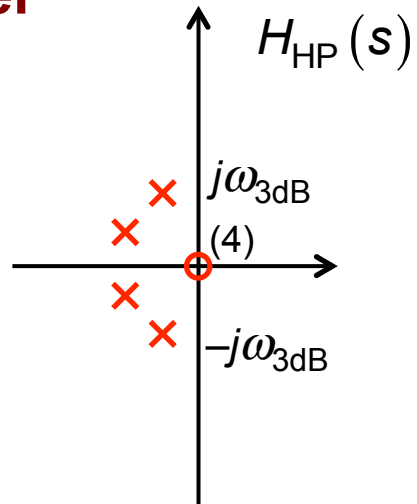


$|H(\omega)|$ Cauerfilter (elliptiskt filter)



Typiska HP- & BP-filter

Högpasfilter



Bandpassfilter

