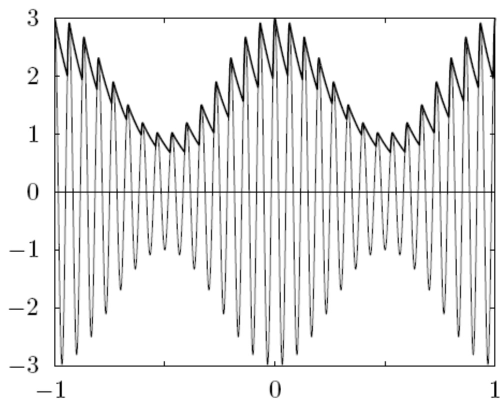
Lektionsuppgifter – ANALOG MODULATION

Uppgifterna relateras till kurskompendiet
Analog och Digital Kommunikation av Mikael Olofsson

1. Describe AM-SC and account for its spectral properties. Also, describe a way to demodulate AM-SC.
2. Describe AM-SSB. Why would you prefer to use AM-SSB instead of ordinary AM?
3. Define the term *deviation* for angle modulation.
4. Describe SSB modulation. How is it generated? What are its spectral properties?
5. Describe AM-SC. How is it generated? What are its spectral properties?
6. Describe FM. How is it generated? What are its spectral properties?
7. Describe PM. How is it generated? What are its spectral properties?
8. What property of the Fourier transform explains the spectral behaviour of amplitude modulation?
9. The graph below shows a modulated signal (the thin line) and the corresponding demodulated signal (the thick line).



What modulation method is used to get the modulated signal? Describe that modulation method. What demodulation method is used?

10.
Consider the AM sender that, given an input $m(t)$, produces the output

$$y(t) = (m * h_1)(t) \cdot \cos(2\pi f_c t).$$

The filter with impulse response $h_1(t)$ is an ideal low-pass filter with cut-off frequency f_0. A possible receiver starts by producing

$$w(t) = y(t) \cdot \cos(2\pi f_c t + \phi).$$

Its output, $z(t)$, is the given by filtering $w(t)$ using an ideal low-pass filter with impulse response $h_2(t)$. The cut-off frequency of that filter is f_c. Assume that we have $f_0 \ll f_c$. The input is $m(t) = e^{-at}u(t)$ with $a > 0$.

a. Scetch the amplitude spectrum of $y(t)$.

b. Ideally, the reference signal of the receiver should be the same as of the sender. Therefore, scetch the amplitude spectrum of $z(t)$ for $\phi = 0$.

c. We are now going to check what happens if the reference signal of the receiver is out of phase. Scetch the amplitude spectrum of $z(t)$ for $\phi = \pi/2$.

Hint:
$\mathcal{F}\{a(t)b(t)\} = (A * B)(f)$
$\mathcal{F}\{(a * b)(t)\} = A(f)B(f)$
$\cos(2\pi f_c t)\sin(2\pi f_c t) = \frac{1}{2}\sin(4\pi f_c t)$

Svar till lektionsuppgifter – ANALOG MODULATION

Uppgifterna & svaren relateras till kurskompendiet
Analog och Digital Kommunikation av Mikael Olofsson
Nedanstående är författarens uppgifter och lösningsförslag.

1.

AM-SC is short for Amplitude Modulation with Suppressed Carrier. In the time domain:

$$x(t) = Am(t)\cos(2\pi f_c t)\,,$$

where $x(t)$ is the AM-SC signal corresponding to the message $m(t)$, f_c is the carrier frequency, A is some non-zero constant. The corresponding spectrum is

$$X(f) = \frac{A}{2}\left(M(f - f_c) + M(f + f_c)\right).$$

Demodulation of AM-SC can be done by modulating once more, which gives us the signal

$$y(t) = x(t)\cos(2\pi f_c t) = \frac{A}{2}m(t)\left(1 + \cos(4\pi f_c t)\right),$$

and the corresponding spectrum is

$$Y(f) = \frac{A}{2}M(f) + \frac{A}{4}\left(M(f - 2f_c) + M(f + 2f_c)\right).$$

Under the reasonable assumption that the bandwidth of of the message is smaller than the carrier frequency, we have regained $M(f)$, but we also have copies of $M(f)$ centered around $\pm 2f_c$. Those copies can finally be filtered out using an LP filter.

2. One way of generating AM-SSB is to first generate ordinary AM, and then filter out one of the sidebands. The reason we would prefer AM-SSB over ordinary AM is that bandwidth of AM-SSB is half of that of ordinary AM. Still, AM-SSB carries the same information as ordinary AM.

3.

Let $f_{\mathrm{mom}}(t)$ be the momentary frequency of the angle modulation, and define the maximum frequency

$$f_{\max} = \max f_{\mathrm{mom}}(t)$$

and the minimum frequency

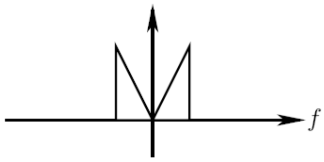
$$f_{\min} = \min f_{\mathrm{mom}}(t).$$

The deviation f_d is then defined as

$$f_d = \frac{f_{\max} - f_{\min}}{2}.$$

4\.

SSB (single side-band) modulation is one form of AM (amplitude modulation). One way to generate SSB is to start by generating DSB (double side-band) AM. Let $m(t)$ be the message signal that we want to communicate, with the spectrum $X(f)$ below.



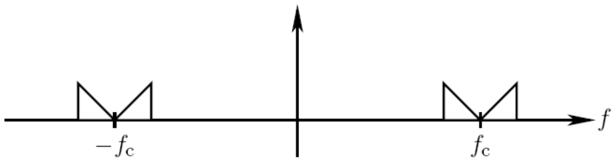
The DSB signal $x_{\mathrm{DSB}}(t)$ is generated as

$$x_{\mathrm{DSB}}(t) = m(t)\cos(2\pi f_{\mathrm{c}} t)$$

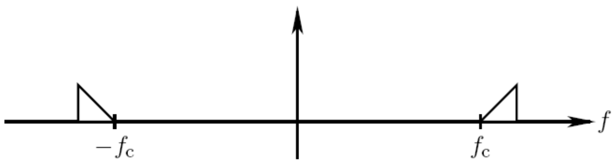
and the corresponding spectrum $X_{\mathrm{DSB}}(f)$ is given by

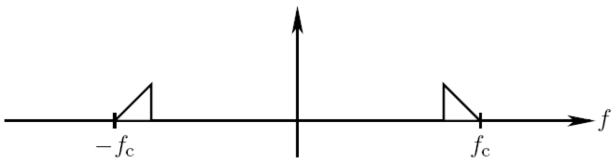
$$F_{\mathrm{DSB}}(t) = \frac{1}{2}\left(M(f - f_{\mathrm{c}}) + M(f + f_{\mathrm{c}})\right),$$

which is displayed below.



To get an SSB signal, the DSB signal is filtered with a suitable pass-band filter that filters out the wanted side-band. Below is the spectrum of the two possible variants of SSB signals, the first one using the upper side-band and the second one using the lower side-band.





5.

AM-SC is *Amplitude Modulation with Suppressed Carrier.* Let $m(t)$ be the message, whith Fourier transform $M(f)$. Then the AM-SC signal $x(t)$ is generated as

$$x(t) = m(t)\cos(2\pi f_c t),$$

where f_c is the carrier frequency. The corresponding spectrum is

$$X(f) = \frac{1}{2}\left(M(f - f_c) + M(f + f_c)\right).$$

In other words, the spectrum of AM-SC is the (double-sided) spectrum of the message, centered around the carrier frequency, and scaled by the factor $1/2$. There is no carrier present in $X(f)$, which is the reason for the name of this modulation method.

6.

FM is frequency modulation, an analog modulation technique where the analog message $m(t)$ modulates the frequency of the carrier as

$$x(t) = \cos\left(2\pi f_c t + a\int m(t)dt\right).$$

The momentary frequency $f_{\mathrm{mom}}(t)$ is given by

$$f_{\mathrm{mom}}(t) = \frac{1}{2\pi}\cdot\frac{d}{dt}\left(2\pi f_c t + a\int m(t)dt\right) = f_c + \frac{a}{2\pi}\cdot m(t).$$

The deviation is given by

$$f_{\mathrm{d}}(t) \triangleq f_{\mathrm{mom}}(t) - f_c,$$

and the peak deviation

$$f_{\mathrm{d,max}} \triangleq \max|f_{\mathrm{d}}(t)| = \frac{a}{2\pi}\cdot\max|m(t)|,$$

is therefore proportional to

$$\max|m(t)|.$$

The bandwidth of $x(t)$ is approximately $2f_{\mathrm{d,max}}$. Thus, the bandwidth of $x(t)$ is proportional to the amplitude of $m(t)$, and we can achieve different bandwidths by amplifying $m(t)$.

7.

PM is short for Phase Modulation. Let $m(t)$ be the message to be sent. Then the corresponding PM signal $x(t)$ is generated as

$$x(t) = \cos(2\pi f_c t + m(t)).$$

The momentary frequency f_{mom} is defined as

$$f_{\text{mom}} = \frac{1}{2\pi}\frac{d}{dt}\left(2\pi f_c t + m(t)\right),$$

from which we get

$$f_{\text{mom}} = f_c + \frac{1}{2\pi}\frac{d}{dt}m(t).$$

Let $f_{\min}$ and $f_{\max}$ denote the minimum and maximum values of f_{mom}. The spectrum of $x(t)$ occupies approximately the frequency band between $f_{\min}$ and $f_{\max}$ and thus, the bandwidth B of $x(t)$ is approximately given by

$$B = f_{\max} - f_{\min} = \frac{1}{2\pi}\left(\max\frac{d}{dt}m(t) - \min\frac{d}{dt}m(t)\right).$$

Thus, scaling $m(t)$ by a factor a results in a scaling of B by the same factor. In all reasonable situations, the spectrum of the signal is spread symmetrically around the carrier frequency f_c.

8.

The Fourier transform of a multiplication:

Let $a(t)$ and $b(t)$ be signals with Fourier transforms $A(f)$ and $B(f)$. Then we have

$$\mathcal{F}\{a(t)b(t)\} = (A * B)(f).$$

9.

The modulation method is standard AM - Amplitude Modulation, where the modulated signal $x(t)$ is given from the message $m(t)$ as

$$x(t) = A(C + m(t))\cos(2\pi f_c t),$$

where we should have $C > |m(t)|$ for all t. The demodulation method used is an envelope detector.

10.

a. Define $X(f) = \begin{cases} \frac{1}{a+j2\pi f}, & |f| \le f_0 \\ 0, & \text{elsewhere.} \end{cases}$
Then $Y(f) = \frac{1}{2}[X(f+f_c) + X(f-f_c)]$.

b. $Z(f) = \frac{1}{2}X(f)$.

c. $Z(f) = 0$.