1

a) SANT

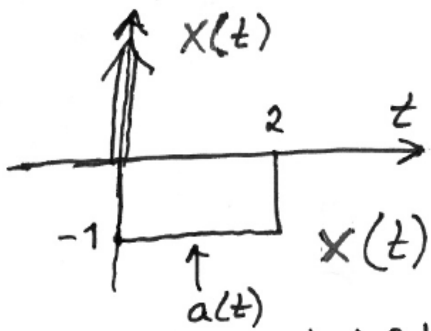
b) SANT

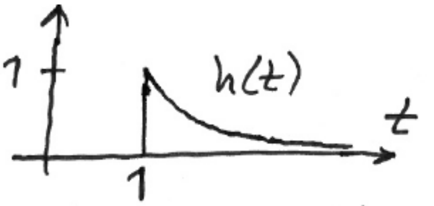
c) FALSKT – polernas placering omöjliggör detta

d) FALSKT – det måste vara absolutintegrerbart.

e) SANT

(2)





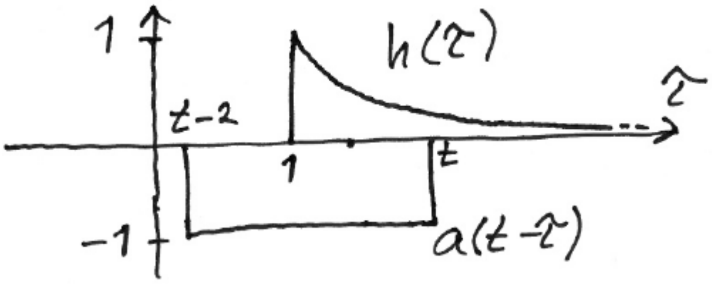
$x(t) = \delta(t) + a(t)$, där $a(t) = -(u(t) - u(t-2))$

LTI-system och både $x(t)$ & $h(t)$ absolutintegrerbara

$$\Rightarrow \text{Falta: } y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau =$$

$$= \underbrace{\int_{-\infty}^{\infty} \delta(t-\tau)h(\tau)\,d\tau}_{=h(t)} + \int_{-\infty}^{\infty} a(t-\tau)h(\tau)\,d\tau = h(t) + b(t)$$

$b(t) = (a * h)(t)$:



- $t < 0 \Rightarrow b(t) = 0$
- $t \geq 1$, $t-2 \leq 1$, dvs $1 \leq t < 3 \Rightarrow$

$$b(t) = \int_1^t (-1)\cdot e^{-(\tau-1)}\,d\tau = -\left[\frac{e^{-(\tau-1)}}{-1}\right]_1^t = e^{-(t-1)} - 1$$

- $t-2 \geq 1$, dvs. $t \geq 3 \Rightarrow b(t) = \int_{t-2}^{t} (-1)\cdot e^{-(\tau-1)}\,d\tau = e^{-(t-1)} - e^{-(t-3)}$

Alltså: $y(t) = h(t) + b(t) = \begin{cases} 0; & t < 1 \\ 2e^{-(t-1)} - 1; & 1 \leq t < 3 \\ 2e^{-(t-1)} - e^{-(t-3)}; & t \geq 3 \end{cases}$

LAPLACETRANSFORMLÖSNING:

$$y(t) = (x * h)(t) \Leftrightarrow Y(s) = X(s)\cdot H(s) = /\text{Transformtabell}/$$

$$= \left(1 - \frac{1}{s} + \frac{e^{-2s}}{s}\right)\cdot \frac{e^{-s}}{s+1} = /\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}/$$

$$= e^{-s}\cdot\frac{1}{s+1} - e^{-s}\cdot\frac{1}{s} + e^{-s}\frac{1}{s+1} + e^{-3s}\frac{1}{s} - e^{-3s}\frac{1}{s+1}$$

Konvergensområde för $\frac{1}{s}$: $\mathrm{Re}\{s\} > 0$, för $\frac{1}{s+1}$: $\mathrm{Re}\{s\} > -1$

$$\Rightarrow y(t) = 2e^{-(t-1)}\cdot u(t-1) - u(t-1) + u(t-3) - e^{-(t-3)}\cdot u(t-3)$$

(som är samma som $y(t)$ ovan!)

3

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_1 t}, \quad \text{där } \underline{\omega_1} = \frac{2\pi}{T_0} = \frac{\pi}{2} \text{ rad/s}$$

(Graf => $T_0 = 4$ sek)

$$\underline{\underline{C_k}} = \frac{1}{T_0}\int_{T_0} x(t) e^{-jk\omega_1 t} dt = \frac{1}{4}\int_0^3 4 \cdot e^{-jk\frac{\pi}{2}t} dt \left(+\frac{1}{4}\int_3^4 0 e^{-j\dots} dt\right)$$

$$= / k \neq 0 / = \left[\frac{e^{-jk\frac{\pi}{2}t}}{-jk\frac{\pi}{2}}\right]_0^3 = \frac{2\left(1 - e^{-jk\frac{3\pi}{2}}\right)}{jk\pi}$$

$$= / e^{-j\frac{3\pi}{2}} = e^{j\frac{\pi}{2}} = j / = \underline{\underline{\frac{2(1-j^k)}{jk\pi}}} \quad k \neq 0$$

$$\underline{\underline{C_0}} = \frac{1}{T_0}\int_{T_0} x(t) e^0 dt = \frac{1}{4}\int_0^3 4 dt = \underline{\underline{3}}$$

$$x(t) = X_0 + \sum_{k=1}^{\infty} \hat{X}_k \sin(k\omega_1 t + \varphi_1) \Longrightarrow y(t) = Y_0 + \sum_{k=1}^{\infty} \hat{Y}_k \sin(k\omega_1 t + \beta_k)$$

vid stabilt LTI-system: $x(t) \longrightarrow \boxed{H(\omega)} \longrightarrow y(t)$

där $\begin{cases} \underline{\hat{Y}_1 = \hat{X}_1 \cdot |H(\omega_1)|} \\ \underline{\beta_1 = \varphi_1 + \arg H(\omega_1)} \end{cases}$ där $\underline{\hat{X}_1 = 2|C_1|}$ och $\underline{\varphi_1 = \arg C_1 + \frac{\pi}{2}}$

$$C_1 = \frac{2(1-j)}{j\pi} = \frac{2\sqrt{1^2+1^2}\, e^{j\overbrace{\arctan\frac{-1}{1}}^{=-\pi/4}}}{\pi \cdot e^{j\pi/2}} = \frac{2\sqrt{2}}{\pi} \cdot e^{-j\frac{3\pi}{4}}$$

$$= |C_1| \cdot e^{j\arg C_1} \Rightarrow \underline{|C_1| = \frac{2\sqrt{2}}{\pi}}, \quad \underline{\arg C_1 = \frac{-3\pi}{4} \text{ rad.}}$$

$$h(t) = e^{-2t} u(t) \xRightarrow{\text{Tab. 17:2}} H(\omega) = \frac{1}{2 + j\omega}$$

$$\Rightarrow \underline{H(\omega_1)} = H\left(\frac{\pi}{2}\right) = \frac{1}{2 + j\frac{\pi}{2}} = \frac{2}{4 + j\pi} = \frac{2}{\sqrt{4^2+\pi^2} \cdot e^{j\arctan\frac{\pi}{4}}}$$

$$= \underline{\frac{2}{\sqrt{16+\pi^2}} \cdot e^{-j\arctan\frac{\pi}{4}}}$$

$$\underline{\hat{Y}_1} = \hat{X}_1 \cdot |H(\omega_1)| = 2|C_1| \cdot |H(\omega_1)| = 2 \cdot \frac{2\sqrt{2}}{\pi} \cdot \frac{2}{\sqrt{16+\pi^2}} \approx \underline{0{,}71}$$

$$\underline{\beta_1} = \varphi_1 + \arg H(\omega_1) = \arg C_1 + \frac{\pi}{2} + \arg H(\omega_1) = \frac{-3\pi}{4} + \frac{\pi}{2} - \arctan\frac{\pi}{4}$$

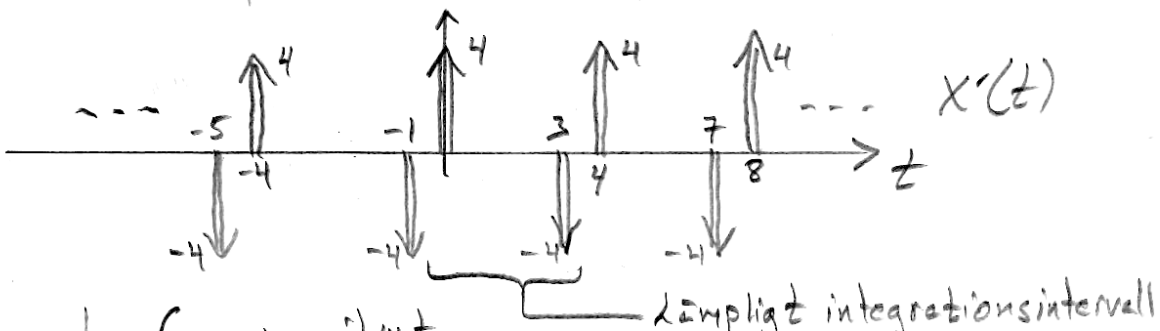
$$\underline{\approx -1{,}4 \text{ rad}}$$

$$\therefore \underline{\underline{y_1(t) \approx 0{,}71 \sin\left(\frac{\pi}{2}t - 1{,}4\right)}}$$

Alternativ beräkning av c_k:

c_k kan även beräknas som $c_k = \frac{c_{kx'}}{jk\omega_1}$,

där $c_{kx'}$ är de komplexa fourierseriekoefficienterna till $x'(t)$:



$$\underline{c_{kx'}} = \frac{1}{T_0}\int_{T_0} x'(t)\, e^{-jk\omega_1 t}\,dt$$

$$= \frac{1}{4}\int_{0^-}^{4^-}\left(4\delta(t) - 4\delta(t-3)\right) e^{-jk\frac{\pi}{2}t}\,dt$$

$$= \int_{0^-}^{0^+}\delta(t)\, e^{-jk\frac{\pi}{2}t}\,dt - \int_{3^-}^{3^+}\delta(t-3)\, e^{-jk\frac{\pi}{2}t}\,dt$$

$$= e^{-jk\frac{\pi}{2}0} - e^{-jk\frac{\pi}{2}\cdot 3} = 1 - e^{-jk\frac{3\pi}{2}} = \underline{1 - j^k}$$

Se föregående sida

$$\underline{\underline{c_k}} = \frac{c_{kx'}}{jk\omega_1} \underset{k \neq 0}{=} \frac{1-j^k}{jk\frac{\pi}{2}} = \underline{\underline{\frac{2(1-j^k)}{jk\pi}}}$$

(4) $\mathcal{L}\left\{\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t)\right\} = \mathcal{L}\left\{\frac{dx(t)}{dt} + x(t)\right\}$

($\mathcal{L}_{II}$ eller $\mathcal{L}_I$ med alla initialtillst. $=0$) Tab. 18:7/8 $\Rightarrow$

$$(s^2+2s+1)Y(s) = (s+1)X(s) \Rightarrow$$

$$\underline{\underline{H(s)}} = \left.\frac{Y(s)}{X(s)}\right|_{\text{Init.tillst.}=0} = \frac{s+1}{s^2+2s+1} = \frac{s+1}{(s+1)^2} = \underline{\underline{\frac{1}{s+1}}}$$

Kausalt system enl. uppg. $\Rightarrow$ högersidigt konv.omr: $\underline{\underline{\text{Re}\{s\} > -1}}$

(Till höger om polen i $s=-1$)

<u>Beräkning av $y(t)$:</u>

$$\mathcal{L}_I\left\{\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t)\right\} = \mathcal{L}_I\left\{\frac{dx(t)}{dt} + x(t)\right\} \xRightarrow{\text{Tab. 18:7/8}}$$

$$\left(s^2Y(s) - s\underbrace{y(0-)}_{=-1} - \underbrace{y'(0-)}_{=0}\right) + 2\left(sY(s) - \underbrace{y(0-)}_{=-1}\right) + Y(s) = \left(sX(s) - \underbrace{x(0-)}_{=0,\ \text{ty } x(t)=3u(t)}\right) + X(s)$$

$$\Rightarrow (s^2+2s+1)Y(s) + s+2 = (s+1)X(s)$$

$$\Rightarrow \underline{\underline{Y(s)}} = \underbrace{-\frac{s+2}{s^2+2s+1}}_{=Y_{zi}(s)} + \underbrace{X(s)\cdot\frac{s+1}{s^2+2s+1}}_{=Y_{zs}(s)=X(s)H(s)} =$$

$$= \left/ \begin{array}{c}\text{Tab 19:3} \\ \Rightarrow \\ X(s) = 3\frac{1}{s};\ \text{Re}\{s\}>0\end{array} \right/ = -\frac{s+2}{(s+1)^2} + 3\frac{1}{s}\cdot\frac{1}{s+1}$$

$$= -\frac{(s+1)+1}{(s+1)^2} + \frac{3}{s(s+1)} =$$

$$= -\underbrace{\frac{1}{s+1}}_{\text{Re}\{s\}>-1} - \frac{1}{(s+1)^2} + 3\left(\underbrace{\frac{1}{s}}_{\text{Re}\{s\}>0} - \underbrace{\frac{1}{s+1}}_{\text{Re}\{s\}>-1}\right)$$

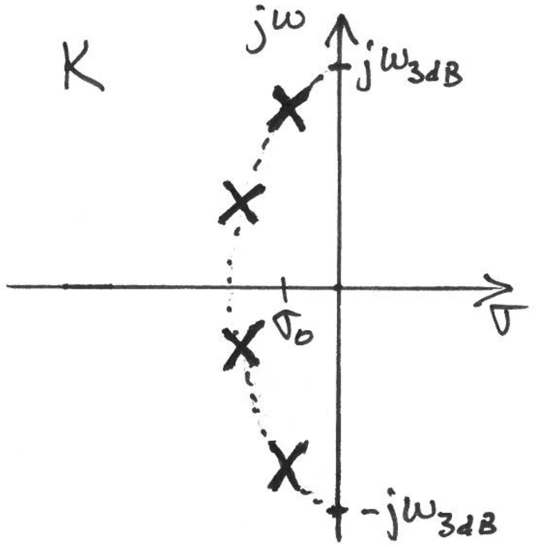
Formels. Tab 19:12, 19:15 & 19:3 $\Rightarrow$

$$\underline{\underline{y(t)}} = \underbrace{-e^{-t}u(t) - te^{-t}u(t)}_{=y_{zi}(t)} + \underbrace{3\left(u(t) - e^{-t}u(t)\right)}_{=y_{zs}(t)}$$

$$= \underline{\underline{\left(3 - (4+t)e^{-t}\right)u(t)}}$$

5 a) Chebyshev I, $n=4$, $\omega_{3dB} = 80$ rad/s $\Rightarrow$

Pol-nollställediagram:



$\Rightarrow$ 4 poler i s-planets VHP, längs en halvellips som skär $j\omega$-axeln vid ungefär $s = j\omega_{3dB}$

Konv.omr: $\underline{\text{Re}\{s\} > \sigma_0}$

där σ_0 = realdelen av polerna närmast $j\omega$-axeln.

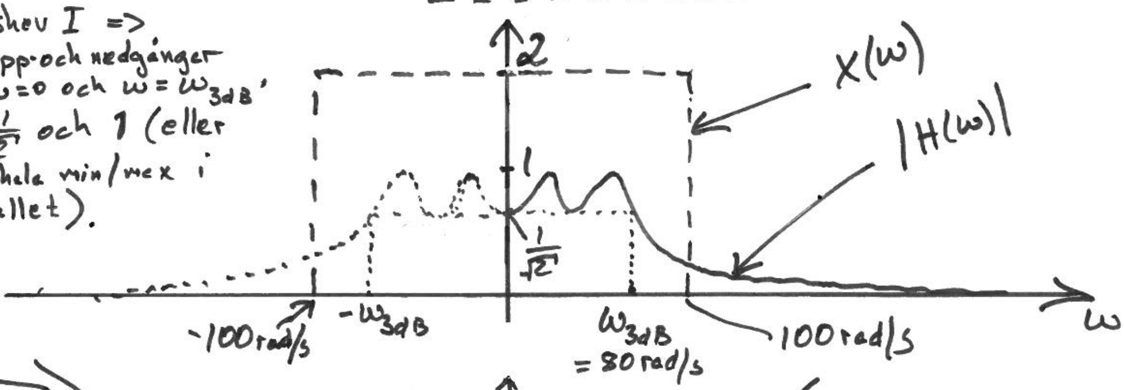
b) $x(t) = \underbrace{\frac{200}{\pi}}_{= 2\cdot\frac{100}{\pi}} \text{sinc}\left(\frac{100t}{\pi}\right)$

Formels. Tab 17:8 med $a = \frac{100}{\pi}$ $\Rightarrow$

$$X(\omega) = \begin{cases} 2\,; & |\omega| \le 100 \text{ rad/s} \\ 0\,; & |\omega| > 100 \text{ rad/s} \end{cases}$$

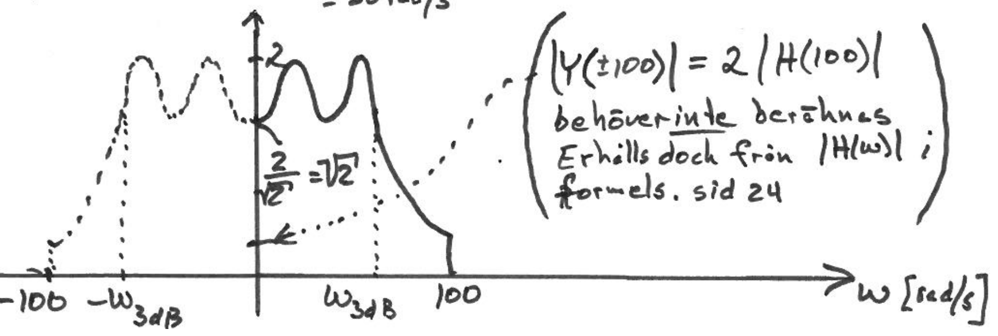
LTI-filter $\Rightarrow$ $\underline{Y(\omega) = X(\omega)\cdot H(\omega)}$

Chebyshev I $\Rightarrow$ $n=4$ "upp- och nedgånger mellan $\omega = 0$ och $\omega = \omega_{3dB}$" mellan $\frac{1}{\sqrt{2}}$ och 1 (eller $n=4$ lokala min/max i intervallet).



$\Rightarrow$ $|Y(\omega)|$

(Det är ok att rita enhelsidigt spektrum, dvs. bara för $\omega \ge 0$)



($|Y(\pm 100)| = 2\,|H(100)|$ behöver inte beräknas. Erhålls dock från $|H(\omega)|$ i formels. sid 24)