Information page for written examinations at Linköping University

Examination date	2016-01-12
Room (3)	R34 R35 R36
Time	14-18
Course code	TSBB06
Exam code	TEN2
Course name **Exam name**	Multidimensional Signal Analysis (Multidimensionell signalanalys) Written Examination (Skriftlig tentamen)
Department	ISY
Number of questions in the examination	20
Teacher responsible/contact person during the exam time	Klas Nordberg
Contact number during the exam time	013-281634
Visit to the examination room approximately	16 pm
Name and contact details to the course administrator (name + phone nr + mail)	Carina Lindström 013-284423 carina.e.lindstrom@liu.se
Equipment permitted	Calculator
Other important information	Use cross-ruled paper
Number of exams in the bag	

Guide

The written examination consists of 4 parts, one part for each of the four course aims in the curriculum.

- Part I: Geometry
- Part II: Estimation
- Part III: Linear signal representation
- Part IV: Signal processing applications

Each part consists of 3 exercises where the student should demonstrate ability to explain concepts, phenomena, etc (type A exercises), and 2 additional exercises that test a deeper understanding of various topics in the course, for example, in terms of more detailed explanations or simpler calculations (type B exercises).

Type A exercises give at most 1 point each. Type B exercises give at most 2 points each.

To pass with grade 3: two parts must have at least 3p and two parts must have at least 4p, and there must be 2 B-type exercises passed with full 2p.

To pass with grade 4: two parts must have at least 4p and two parts must have at least 5p, and there must be 4 B-type exercises passed with full 2p.

To pass with grade 5: all parts must have at least 5p, and there must be 6 B-type exercises passed with full 2p.

The answers to the A-exercises should preferably be given in the blank spaces of this examination thesis, below the questions. Use additional sheets if necessary, with no more than one exercise per sheet

Write your anonymous examination ID (AID) at the top of the pages in this examination thesis and any sheet appended to the examination thesis.

Good luck!
Klas Nordberg

AID:

PART I: GEOMETRY

Exercise 1 (A, 1p) A 2D point has homogeneous coordinates $(1, 2, 3)$. Specify the dual homogeneous coordinates of a line that intersects this point.

Exercise 2 (A, 1p) Give the elements of a transformation matrix that transforms the homogeneous coordinates of a 2D point, *first* by a translation with $(1, 1)$, and *then* by a anti-clockwise rotation of 90°.

Exercise 3 (A, 1p) Draw a figure that illustrates a homography transformation between the points of two 3D planes.

Exercise 4 (B, 2p) Given a rotation axis $\hat{\mathbf{n}}$ and angle α, the corresponding rotation matrix is computed by Rodrigues' formula:

$$\mathbf{R} = \mathbf{I} + \big(1 - \cos\alpha\big)(\mathbf{I} - \hat{\mathbf{n}}\hat{\mathbf{n}}^\top) + \sin\alpha\,[\,\hat{\mathbf{n}}\,]_\times .$$

How can you determine the axis $\hat{\mathbf{n}}$ and the angle α from the matrix $\mathbf{R}$?.

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 5 (B, 2p) An ellipse in the 2D plane is defined by all points (u, v) that satisfy the equation

$$4\,(u - 4)^2 + 9\,(v - 2)^2 = 1.$$

Based on homogeneous coordinates $\mathbf{y}$, the ellipse can alternatively be define by the equation $\mathbf{y}^\top\mathbf{Q}\,\mathbf{y} = 0$ for some symmetric 3×3 matrix $\mathbf{Q}$. What is $\mathbf{Q}$ in this case?

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PART II: ESTIMATION

Exercise 6 (A, 1p) Why do we apply normalization, e.g., the Hartley normalization, to data before a model that fits the data is estimated?

Exercise 7 (A, 1p) If we apply DLT to a relation like $\mathbf{a} \sim \mathbf{T}\mathbf{b}$, we get $[\mathbf{a}]_\times \mathbf{T}\mathbf{b} = \mathbf{0}$. The left hand side of this last expression contains 3 linear equations in $\mathbf{T}$, but it only generates 2 constraints in $\mathbf{T}$? Why is this so?

Exercise 8 (A, 1p) An $n \times 3$ data matrix $\mathbf{A}$, consisting of the homogeneous coordinates of 2D points, is used for estimating a line that intersects the points. $\mathbf{A}$ has singular values $\sigma_1 = 1.7, \sigma_2 = 0.1, \sigma_3 = 0.08$. How do you interpret this situation?

Exercise 9 (B, 2p) You are estimating a homography from image points that contain some amount of noise. You can determine the homography either by minimizing an algebraic error or a geometric error. Explain these two errors, and describe advantages and disadvantages of both approaches.

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Exercise 10 (B, 2p) Two sets of m image points $\bar{\mathbf{y}}_k$ and $\bar{\mathbf{y}}'_k$ are approximately related by a rigid transformation $(\mathbf{R}, \bar{\mathbf{t}})$. Formulate a geometric error ϵ such that $(\mathbf{R}, \bar{\mathbf{t}})$ can be determined by minimizing ϵ. Also, show that this error is symmetric: it gives the same value independent of if the two points sets are exchanged.

WRITE YOUR ANSWER ON A SEPARATE SHEET

PART III: LINEAR SIGNAL REPRESENTATIONS

Exercise 11 (A, 1p) $\mathbf{B}$ and $\tilde{\mathbf{B}}$ hold a basis and the corresponding dual basis in their columns. $\mathbf{G}_0$ defines the scalar product. How are the three matrices related?

Exercise 12 (A, 1p) The scalar product between vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ can be implemented as $\langle \mathbf{u} \,|\, \mathbf{v} \rangle = \mathbf{v}^\top \mathbf{G}_0 \, \mathbf{u}$ for some $n \times n$ matrix $\mathbf{G}_0$. Here, we require that $\mathbf{G}_0$ is symmetric: $\mathbf{G}_0^\top = \mathbf{G}_0$. Why is the necessary? What property of the scalar product relates to the symmetry of $\mathbf{G}_0$?

Exercise 13 (A, 1p) $\mathbf{B}$ holds a basis for a subspace $U \subset V$ in its columns. $\mathbf{G}_0$ defines the scalar product in V. How can you compute the orthogonal distance from $\mathbf{v}$, a vector in V, to the subspace U?

Exercise 14 (B, 2p) Let $\mathbf{b}_k \in V$ be a set of N vectors that forms a frame for V, with $\mathbf{F}$ as the corresponding frame operator. $\mathbf{F}$ is a *self-adjoint operator*. Describe how $\tilde{\mathbf{b}}_k$, the corresponding dual frame vectors, are computed. Show that it is possible to write any $\mathbf{v} \in V$ as

$$\mathbf{v} = \sum_k \langle \mathbf{v} \mid \mathbf{b}_k \rangle \, \tilde{\mathbf{b}}_k .$$

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Exercise 15 (B, 2p) The discrete Fourier transform maps a set of N samples $f[k]$ in the time domain to N samples $F[l]$ in the frequency domain, and it is defined according to the equation below. This expression suggests that the set of all $F[l]$ as the coordinates of the sequence $f[k]$ relative to some set of basis functions. What are the basis functions in this case, and which scalar product is used?

$$F[l] = \sum_{k=0}^{N-1} f[k] \, e^{-i2\pi kl/N}$$

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AID:

PART IV: SIGNAL PROCESSING APPLICATIONS

Exercise 16 (A, 1p) Explain why it is often not reasonable to use a weighting function $W = 1$ in filter optimization.

Exercise 17 (A, 1p) What is the primary application of Principal Component Analysis (PCA)?

Exercise 18 (A, 1p) Let V_0 and V_1 be two vector spaces such that the scaling functions $\phi(t-k), k \in \mathbb{Z}$, is an ON-basis of the space V_0 and $2^{1/2}\phi(2t-k), k \in \mathbb{Z}$ is an ON-basis of the space $V_1 \supset V_0$. Then the wavelet functions $\psi(t-k), k \in \mathbb{Z}$ form a basis for the space W_0. Characterize W_0 in terms of V_0 and V_1.

Exercise 19 (B, 2p) Filter optimization can be described as a linear optimization problem. In what space is the problem formulated and how? What is the scalar product in this space?

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Exercise 20 (B, 2p) A special case of normalized convolution is normalized averaging, where only a single, constant, basis function is used. Let $s[k]$ denote the signal, let $a[k]$ denote the corresponding symmetric applicability function, and let $c[k]$ denote the signal certainty. What is the filter $f[k]$ that is applied to the signal in this case? What is the metric $\mathbf{G}[k]$, at point k, in this case? Motivate!

WRITE YOUR ANSWER ON A SEPARATE SHEET