Information page for written examinations at Linköping University

Examination date	2018-01-12
Room (1)	KÅRA(48)
Time	14-18
Course code	TSBB06
Exam code	TEN2
Course name **Exam name**	Multidimensional Signal Analysis (Multidimensionell signalanalys) Written Examination (Skriftlig tentamen)
Department	ISY
Number of questions in the examination	20
Teacher responsible/contact person during the exam time	Klas Nordberg
Contact number during the exam time	013-281634
Visit to the examination room approximately	4 pm
Name and contact details to the course administrator (name + phone nr + mail)	Carina Lindström 013-284423 carina.e.lindstrom@liu.se
Equipment permitted	Calculator
Other important information	Use cross-ruled paper
Number of exams in the bag	

Guide

The written examination consists of 4 parts, one part for each of the four course aims in the curriculum.

- Part I: Geometry
- Part II: Estimation
- Part III: Linear signal representation
- Part IV: Signal processing applications

Each part consists of 3 exercises where the student should demonstrate ability to explain concepts, phenomena, etc (type A exercises), and 2 additional exercises that test a deeper understanding of various topics in the course, for example, in terms of more detailed explanations or simpler calculations (type B exercises).

Type A exercises give at most 1 point each. Type B exercises give at most 2 points each.

To pass with grade 3: two parts must have at least 3p and two parts must have at least 4p, and there must be 2 B-type exercises passed with full 2p.

To pass with grade 4: two parts must have at least 4p and two parts must have at least 5p, and there must be 4 B-type exercises passed with full 2p.

To pass with grade 5: all parts must have at least 5p, and there must be 6 B-type exercises passed with full 2p.

The answers to the A-exercises should preferably be given in the blank spaces of this examination thesis, below the questions. Use additional sheets if necessary, with no more than one exercise per sheet

Write your anonymous examination ID (AID) at the top of the pages in this examination thesis and any sheet appended to the examination thesis.

Good luck!
Klas Nordberg

PART I: GEOMETRY

Exercise 1 (A, 1p) A 2D line has dual homogeneous coordinates $(1, 1, -2)$. Draw the line in a Euclidean coordinate system and specify its orientation and its distance to the origin.

Exercise 2 (A, 1p) A pinhole camera projects a 3D object onto a 2D image. The object is subject to a rigid 3D transformation. Does this mean that also the projected object points are rigidly transformed in the image? Motivate you answer.

Exercise 3 (A, 1p) A 3D rotation is represented by a rotation matrix $\mathbf{R}$. The rotation rotates the angle α about the axis $\hat{\mathbf{n}}$. What are the eigenvalues of $\mathbf{R}$?

Exercise 4 (B, 2p) Two 2D lines are parallel and two other lines are not. Explain why an affine 2D transformation preserves this property.

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 5 (B, 2p) The camera matrix is sometimes decomposed as $\mathbf{C} \sim \mathbf{K}[\mathbf{R}|\mathbf{t}]$, where $\mathbf{K}$ is the 3×3 internal calibration matrix (9 elements), $\mathbf{R}$ is a 3×3 matrix that represents the rotation of the camera coordinate system relative to the world coordinate system (9 elements), and $\mathbf{t}$ is a 3-dimensional vector that represents the translation of the camera relative the world coordinate system. Explain how it comes that we use $9 + 9 + 3 = 21$ elements to represent the $3 \times 4 = 12$ elements of the camera matrix.

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PART II: ESTIMATION

Exercise 6 (A, 1p) A 2D point has homogeneous coordinates $\mathbf{y} = (y_1, y_2, y_3)$ and a 2D line has dual homogeneous coordinates $\mathbf{l} = (l_1, l_2, l_3)$. This point and line should ideally intersect, but due to noise this may not be true. How can you measure the *geometric* error between the point and the line in terms of $\mathbf{y}$ and $\mathbf{l}$?

Exercise 7 (A, 1p) Many estimation problem can be stated: minimize $\|\mathbf{A}\,\mathbf{x}\|$ over $\mathbf{x}$, where $\mathbf{A}$ is a data dependent matrix and $\mathbf{x}$ is a projective representation of the geometric object that is estimated. This formulation requires a constraint to avoid the solution $\mathbf{x} = \mathbf{0}$. Describe two common constraints of this type.

Exercise 8 (A, 1p) Solving $\mathbf{A}\,\mathbf{x} = \mathbf{0}$ by means of SVD has some advantages compared to the inhomogeneous approach. Describe one such advantage.

Exercise 9 (B, 2p) OPP determines $\mathbf{R} \in SO(3)$ that minimizes $\|\mathbf{B} - \mathbf{R}\,\mathbf{A}\|$ as $\mathbf{R} = \mathbf{V}\,\mathbf{U}^\top$, where $\mathbf{U}\,\mathbf{S}\,\mathbf{V}^\top = \mathbf{A}\,\mathbf{B}^\top$ (SVD). This approach assumes that the data points are chosen such that $\det \mathbf{V}\,\mathbf{U}^\top = 1$. How would you interpret the case $\det \mathbf{V}\,\mathbf{U}^\top = -1$, and how can you modify OPP to take this case into account?

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Exercise 10 (B, 2p) A 3D plane is calculated from the positions of three 3D points. Explain what difference you get in the result if you make the estimation based on Hartley-normalized coordinates.

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PART III: LINEAR SIGNAL REPRESENTATIONS

Exercise 11 (A, 1p) Give an example of a scalar product that can be used for one-variable functions, e.g., in L^2.

Exercise 12 (A, 1p) Let $\mathbf{B}$ be a matrix hat contains a set of *frame vectors* in its columns, and let $\tilde{\mathbf{B}}$ be the corresponding matrix for the dual frame. How are the two matrices related, assuming that the scalar product is defined by $\mathbf{G}_0 = \mathbf{I}$?

Exercise 13 (A, 1p) Let $\mathbf{B}$ be matrix that, in its columns, holds a basis of subspace $U \subset V$ and let $\mathbf{G}_0$ be the scalar product matrix for V. How do you find the orthogonal projection of $\mathbf{v} \in V$ onto the subspace U?

Exercise 14 (B, 2p) A scalar product in $\mathbb{R}^2$ is specified by the metric

$$\mathbf{G}_0 = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}.$$

The space has a basis consisting of the two vector $\mathbf{b}_1 = (1, 1)$ and $\mathbf{b}_2 = (-1, 1)$. Determine the corresponding dual basis, $\tilde{\mathbf{b}}_1$ and $\tilde{\mathbf{b}}_2$, and verify that it is in a dual relationship with $\mathbf{b}_1$ and $\mathbf{b}_2$.

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Exercise 15 (B, 2p) The convolution operation can be interpreted either as computations of scalar products or as computing linear combinations. Explain these two distinct interpretations.

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PART IV: SIGNAL PROCESSING APPLICATIONS

Exercise 16 (A, 1p) In filter optimization the frequency domain is in practice sampled. Why?

Exercise 17 (A, 1p) Normalized convolution allows you to compute an expansion of the local signal, e.g., in terms of a polynomial basis. What interpretation can be made of the corresponding coordinates in this case?

Exercise 18 (A, 1p) Signal processing based on conversion from analog to digital representations, and vice versa, can use over-sampling to reduce the amount of noise that is introduced by this processing. Describe at least one such noise source that appears in a practical application.

Exercise 19 (B, 2p) A signal is represented by its projection onto a subspace spanned by the N largest principal components of the signal, i.e., the eigenvectors of the correlation matrix $\mathbf{C}$ that correspond to the N largest eigenvalues. If you increase the size of the subspace basis to use $N+1$ largest principal components, how much reduction in the representation error $\epsilon = \mathsf{E}[\mathbf{v} - \mathbf{B}\mathbf{B}^\top \mathbf{v}]^2$ do you get? Motivate your answer.

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Exercise 20 (B, 2p) In wavelet theory, two signal spaces $V_0 \subset V_1$ are defined from the scaling function. How are the signals in V_0 and V_1 related each other? Motivate your answer.

WRITE YOUR ANSWER ON A SEPARATE SHEET