TSBB06 KTR1 Instructions

Welcome to KTR1, the voluntary midterm exam for TSBB06!

The written examination in TSBB06 has 2 parts: One part for the course content in HT1 and one for the content from HT2. This document is KTR1, the voluntary midterm exam, which corresponds to part 1 of the main exam.

There are 10 problems in KTR1 (and in part 1 of the final exam), and these are grouped under the two headings Geometry, and Estimation.

The exam problems are of two types: **Type A** are 1 point problems, and **Type B** are 2 point problems. Half points can be given.

The maximal score on KTR1 is 14p, and if you obtain 8p or more on KTR1 you are allowed to use that result on the final exam. (This is done by not handing in answers to **any** of the problems in part 1 of the exam. Your score from KTR1 will then automatically be added to the final exam score.)

The maximal score on the final exam is 28p, and grading of the exam is done based on your score as follows:

If you obtain **15p** or more you get **grade 3**.
If you obtain **18p** or more you get **grade 4**.
If you obtain **22p** or more you get **grade 5**.

All tasks should be answered on **separate sheets**. It is fine to answer multiple questions on one sheet.

Write your AID-number and the date on all paper sheets that you attach to the examination. In addition, these sheets should be numbered in consecutive order.

Good luck!
Per-Erik Forssén and Lasse Alfredsson

PART I, Geometry

Problem 1 (A, 1p) A 3D plane has dual homogeneous coordinates $\mathbf{p} = (-3, 1, 0, 0)$.

What is the distance from the plane to the origin?

Problem 2 (A, 1p) Two lines in the image plane have dual homogeneous coordinates given by $\mathbf{l}_1 = (-2, 4, 1)$ and $\mathbf{l}_2 = (1, -2, 1)$.

What is the point of intersection for the two lines?

Problem 3 (A, 1p) A 2D homography $\mathbf{H}$ is applied to a line $\mathbf{l}_1$ to obtain a new line $\mathbf{l}_2 = \mathbf{H}\mathbf{l}_1$.

Define a transformation that maps an arbitrary point $\mathbf{x}$ that lies on $\mathbf{l}_1$ to a point $\mathbf{y}$ that lies on $\mathbf{l}_2$?

Problem 4 (B, 2p) The homogeneous coordinates of three 3D points are given as the vectors $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \in \mathbb{R}^4$. In general, there is a unique plane that intersects all three points.

Describe a method that, from $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$, determines $\mathbf{p}$, the dual homogeneous coordinates of the plane.

Problem 5 (B, 2p) A 2D point has homogeneous coordinates $\mathbf{y} = (u, v, 0)$; it lies at infinity. This point is subject to a rigid transformation, represented by a 3×3 matrix that is applied to $\mathbf{y}$.

a) Derive and describe the result of this transformation: what point is it, and does it still lie at infinity? (1p)

b) How does the translation part of the transformation affect the point $\mathbf{y}$? Why does this result make sense? (1p)

PART I, Estimation

Problem 6 (A, 1p) Given a set of 2D points, how is Hartley normalization done on this data set?

Problem 7 (A, 1p) In general, algebraic errors lack intuitive geometric interpretations. Despite this, algebraic errors are often used for model estimation. Why?

Problem 8 (A, 1p) A model is estimated by minimizing an algebraic error, using the homogeneous method. This provides additional information in the form of the singular values of the data matrix.

Describe how you can use the singular values to characterize the data from which the model is estimated.

Problem 9 (B, 2p) The point $\mathbf{y}_k$ lies in one image, and the corresponding point $\mathbf{y}'_k$ lies in another image, $k = 1, \ldots, m$. The two images are related by a homography: $\mathbf{y}'_k \sim \mathbf{H}\,\mathbf{y}_k$.

Formulate an error function ϵ, based on *geometric errors*, for the estimation of $\mathbf{H}$ from these points. The function ϵ should be *symmetric*, in the sense that it measures the geometric errors in both images.

Problem 10 (B, 2p) The same data as in the previous exercise can be used to estimate $\mathbf{H}$ by minimizing an algebraic error (not necessarily a symmetric one).

Give the principal steps that lead to the formulation of the algebraic error, and suggest a method for minimization of the error.