TSBB15 Computer Vision

A comment on the 7-point algorithm

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Parametrisation of $\mathbf{F}$

The nullspace of the 7×9 data matrix is spanned by two vectors $\mathbf{f}_1, \mathbf{f}_2 \in \mathbb{R}^9$, which can be reshaped into matrices $\mathbf{F}_1, \mathbf{F}_2 \in \mathbb{R}^{3\times 3}$. The fundamental matrix $\mathbf{F}$ should be a linear combination of these.

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As an alternative to the form given in the lecture, we can use

$$\mathbf{F} = \mathbf{F}_1 + \alpha \mathbf{F}_2.$$

This has the advantage of being related to *generalised eigenvalue problems*, which makes it easier to solve the polynomial equation $\det(\mathbf{F}_1 + \alpha \mathbf{F}_2) = 0$.

Generalised eigenvalue problems

For $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{n \times n}$ (or $\mathbb{C}^{n \times n}$), the *linear matrix pencil* $\mathbf{A} - \lambda\mathbf{B}$ arises in relation to the *generalised eigenvalue problem*

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{B}\mathbf{v}, \quad \lambda \in \mathbb{C}, \quad \mathbf{v} \neq \mathbf{0}.$$

(Ordinary eigenvalue problems are special cases where $\mathbf{B} = \mathbf{I}$.)

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The *characteristic equation* for this problem is $\det(\mathbf{A} - \lambda\mathbf{B}) = 0$, which is a polynomial equation of order n in λ.

Generalised eigenvalue problems (cont.)

Generalised eigenvalue problems are well studied, and thus implemented in many software packages for linear algebra:

SciPy:	`scipy.linalg.eig` NOTE: *not* `numpy.linalg.eig`!
Eigen3:	`Eigen::GeneralizedEigenSolver`
Armadillo:	`eig_pair`
Matlab:	`eig`
LAPACK:	`xGGEV`
...	